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Original Study Article

The study of forced oscillations in the non-linear system of an individual traction drive

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ABSTRACT

BACKGROUND: The processes taking place in the 'traction electric drive – wheel – road' system during acceleration and braking cause increased dynamic loads on drive components, which may lead to a breakdown. Therefore, it is important to control the drive in a way to minimize and to suppress the given processes. To make it possible, the control system is to be equipped with a resistance torque observer at the electric motor shaft. In addition, in any system, there is a heighten interest in study of arise of resonances, which come with abrupt increase of oscillations amplitudes. Therefore, the features of oscillation phenomena in the given non-linear system are to be studied.

AIM: Identification of the peculiarities of oscillatory processes, resonance phenomena in the systems of electromechanical drive of vehicles, which are nonlinear technical systems.

METHODS: The study of features of the oscillating processes and the study of capabilities of arise of resonant phenomena were conducted using analysis of the differential equations system describing the operation of the non-linear system.

RESULTS: The features of the oscillating phenomena in non-linear systems of interaction between an elastic wheel and road as well as capabilities of arise of resonant phenomena were considered. It is defined that arise of the resonant phenomena in the considered systems is not possible due to breakdown of them. The behavior of modes of interaction between an elastic wheel and road during intensive acceleration and braking was analyzed. The abrupt shock behavior of change rate of wheel torque and current consumed by a drive as well as features of lowering of them when using the self-oscillating phenomena suppression were found.

CONCLUSION: The practical value of the study lies in ability of using the proposed conclusions at development of units of a traction electric drive and at synthesis of vehicle motion control systems.

Keywords: self-oscillations; resonance; near-resonance mode; resonance breakdown; traction electric drive.

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Оригинальное исследование

Исследование вынужденных колебаний в нелинейной системе индивидуального тягового электропривода

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АННОТАЦИЯ

Обоснование. Процессы, проходящие в системе тяговый электромеханический привод-колесо-дорога при разгоне/торможении, вызывают повышенные динамические нагрузки на элементы привода, что может привести к выходу их из строя. Колебания возникают из-за изменения скоростного режима движения транспортного средства, сцепных свойств опорного основания, его неровностей, характеристик эластичных шин и сопровождаются изменениями угловых скоростей, крутящих моментов, продольных, поперечных, нормальных сил. Повышенный интерес в любых системах, работа которых является сложным, случайным, колебательным процессом, вызывает изучение возникновения резонансных явлений, сопровождаемые резким увеличением амплитуд колебаний. Поэтому необходимо исследовать особенности колебательных явлений, условий возникновения резонансов в данных нелинейных системах для последующего определения методов борьбы с ними.

Цель работы — выявление особенностей протекания колебательных процессов, возникновения резонансных явлений в системах электромеханического привода транспортных средств, представляющих собой нелинейные технические системы.

Материалы и методы. Исследование динамики движения ведущих колёс машины на предмет особенностей колебательных процессов и возможности возникновения резонанса проведено методами экспериментальных исследований процессов разгона и торможения транспортного средства. Исследование особенностей колебательных процессов. Изучение возможностей зарождения резонансных явлений при работе систем машины в любых условиях (отличных от тех, которые приведены в предыдущем разделе) проведено с помощью математического анализа систем дифференциальных уравнений, описывающих функционирование нелинейной систем.

Результаты. В системе электромеханического привода ведущих колёс транспортного средства при движении имеют место быть колебательные процессы по крутящим моментам. Поскольку данная система является нелинейной, то ярко выраженных резонансных явлений с неконтролируемым ростом амплитуд вплоть до бесконечности в ней наблюдаться не может. Наблюдается срыв резонанса, который может сопровождаться как уменьшением амплитуд колебаний при росте их частот в случае разгона машины, так и увеличении при уменьшении частот в случае замедления машины. Выявлен резкий ударный характер скорости изменения крутящего момента на колесе, тока, потребляемого приводом, а также особенности их снижения при применении подавления автоколебательных явлений. В работе рассмотрены особенности колебательных явлений в нелинейных системах взаимодействия

Заключение. Практическая ценность исследования заключается в возможности использования предложенных выводов при разработке агрегатов тягового электромеханического привода и при синтезе систем управления движением транспортных средств.

Ключевые слова: колебания; резонанс; околорезонансный режим; срыв резонанса; тяговый электромеханический привод.

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INTRODUCTION. PURPOSE. MATERIALS AND METHODS

The global adoption of traction electric vehicles on the road is gradually increasing in various indus-tries. These vehicles utilize traction electric motors, which vary in design, as the power source for wheel drives [1]. Modern electric motors operate across a wide speed range, with rapid frequency changes occurring during vehicle acceleration [2]. During electrodynamic regenerative braking, the torque can reverse, which is a crucial aspect of their operation. The dynamics of the traction electric drive and its transmission control are highly complex and exhibit oscillatory behavior. Understand-ing these phenomena is essential, as they can lead to resonant and self-oscillatory conditions [3-6], increasing the dynamic load on system elements owing to rising oscillation amplitudes and potential-ly causing failures. Therefore, examining the conditions and causes of these oscillations is very im-portant for developing methods to manage them effectively.

In the traction electromechanical drive–wheel–road system, acceleration and braking cause in-creased dynamic loads that can jeopardize drive elements. Oscillations stem from changes in vehicle speed, traction properties of the supporting base, its unevenness, and the elastic characteristics of tires, affecting angular velocities, torques, and longitudinal, transverse, and normal forces. Investi-gating resonance phenomena, which can significantly amplify oscillations, is crucial in systems where operations are inherently complex, random, and oscillatory.

Therefore, this study aims to investigate the features of oscillatory phenomena and the conditions under which resonances occur in these nonlinear systems to devise effective methods to combat them.

Experimental investigation of oscillatory processes in the electromechanical wheel drive system during the frictional interaction of an elastic tire with a solid support base

When the oscillation frequency matches the system's natural frequency, resonance occurs, leading to increased oscillation amplitudes and heightened dynamic loads on system elements. The latter leads to their failure. The dynamics of a vehicle's driving wheels during acceleration and braking have been experimentally studied to understand oscillatory processes and potential resonances.

During intense acceleration and braking, vehicles may experience self-oscillations that do not damp out [7-9]. Self-oscillations can continue without external influence and can increase oscillation am-plitude. A torque graph for a vehicle with a specific traction electric drive during intensive accelera-tion [10] on a low-friction surface (like wet basalt) captured using Kistler–Rim RoaDyn strain gauge wheels (Fig. 1), is shown in Fig. 2.

The torque graph demonstrates the origin of oscillations owing to rapid torque changes and indicates a torque sign change when the traction control system is activated, causing gear teeth to shift at high speeds.

During intensive braking of the vehicle [10], oscillatory phenomena may also occur [11]. A torque graph, depicted in Fig. 3, illustrates the relationship between torques on the driving wheel of a vehi-cle equipped with a specific traction electric drive during intensive braking on a low-friction surface (wet basalt).

Fig. 1. Kistler-Rim RoaDyn strain gauge wheels.
Рис. 1. Тензометрические колёса Kistler-Rim RoaDyn.

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The torque graph reveals the origin of oscillatory processes characterized by rapid torque changes. Similarly, to acceleration, the operation of the anti-lock braking system results in a torque sign reversal, leading to the same phenomena as those observed during acceleration.

The processes depicted in Figs. 3 and 4 increase dynamic loads on drive elements, potentially leading to their failure, though no resonant phenomena are observed. Therefore, it is important to control the drive to minimize these effects [12]. This requires a control system equipped with a resistive torque observer on the electric motor shaft [13].

The origin of self-oscillating processes in the drive can elevate dynamic loads on components to the point of failure. Table 1 displays the rate of torque changes on the wheel and the direct current consumed by the wheel drive during intensive acceleration (Fig. 2) and braking (Fig. 3) tests [10] on a low-friction surface (Figs. 2, 3). These current values were recorded using the Vector VN1630A adapter and a computer (Fig. 4) from the vehicle's CAN bus.

From data in Table 1, we observe a high rate of torque change, indicating shock effects in the drive's mechanical parts and high dynamic loads. Current fluctuations increase the load on the current source, the traction battery, affecting its lifespan. Therefore, understanding the origin of self-oscillatory phenomena is vital to reducing increased dynamic effects. For these purposes, it is possible to apply the approach proposed in [12].

Notably, the rapid changes in consumed current and the oscillatory nature of the process highlight the influence of variable current components on the vehicle's energy storage system. Figure 5 shows the torque and current consumption for intensive twofold acceleration of a similar vehicle on a low-friction road.

The feature of the electric drive's direct torque control aids in suppressing oscillatory processes. Reducing self-oscillations in the contact zone of the wheel [12] with the road positively affects current consumption and torque fluctuations on the wheel.

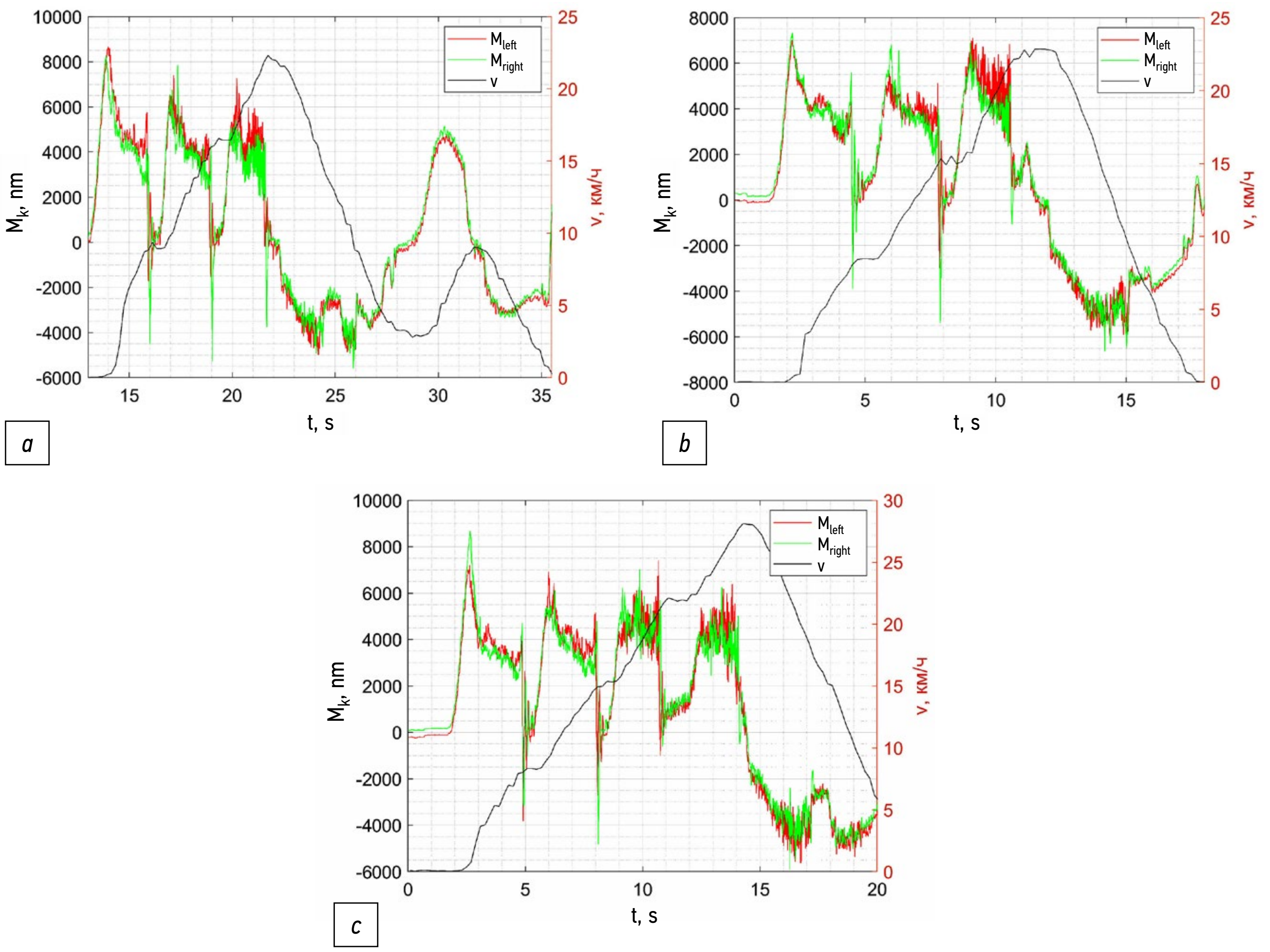

Fig. 2. Torque at traction wheels at abrupt intensive acceleration when both left and right side are on wet basalt road: *a*) ride 1; *b*) ride 2; *c*) ride 3.

Рис. 2. Крутящие моменты на ведущих колёсах при резком интенсивном разгоне и нахождении правого и левого борта на мокром базальте: *a*) 1 заезд; *b*) 2 заезд; *c*) 3 заезд.

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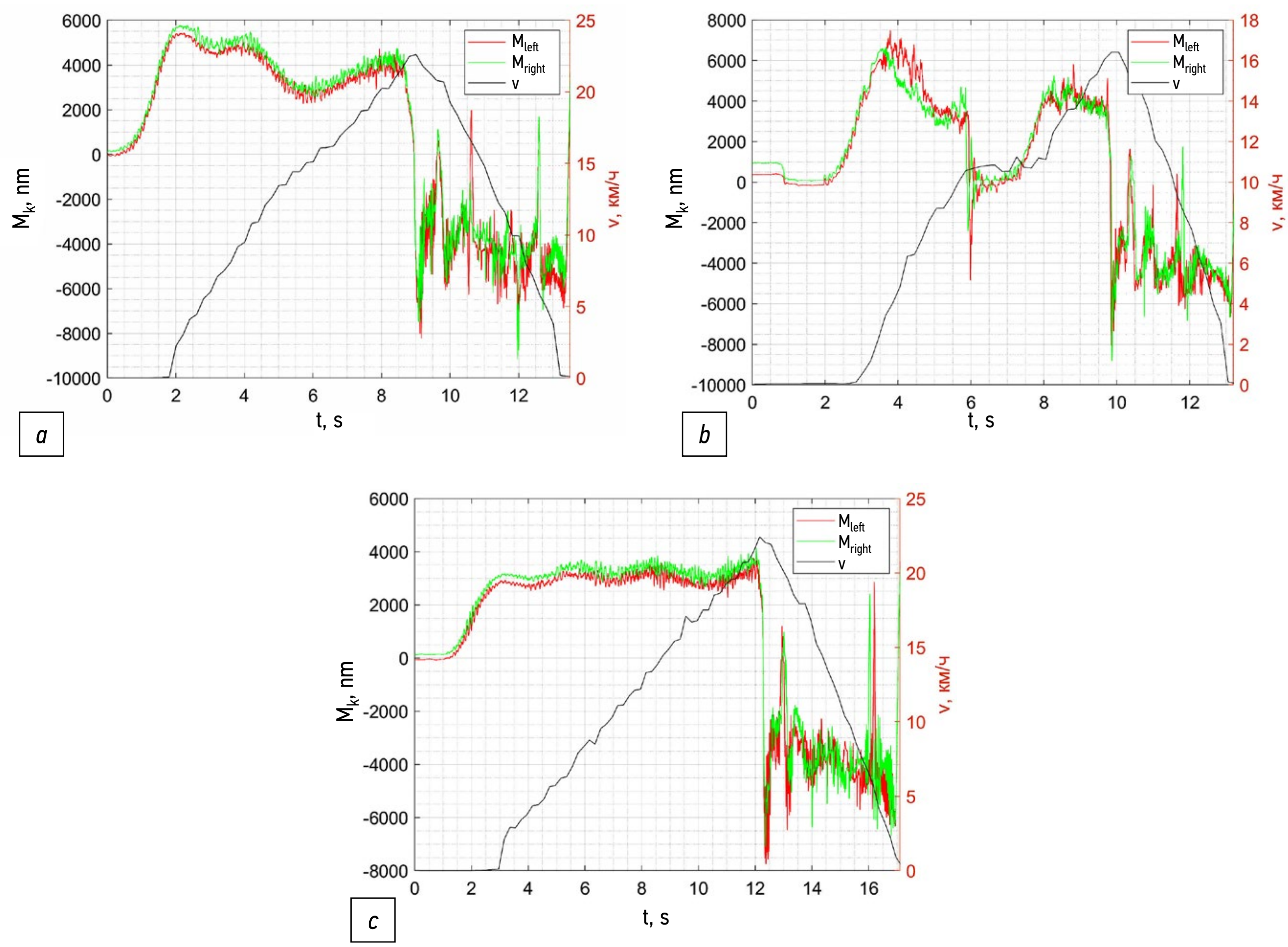

Fig. 3. Torque at traction wheels at abrupt intensive braking when both left and right side are on wet basalt road: *a*) ride 1; *b*) ride 2; *c*) ride 3.
Рис. 3. Крутящие моменты на ведущих колёсах при экстренном торможении при нахождении правого и левого борта на мокром базальте: *a*) 1 заезд; *b*) 2 заезд; *c*) 3 заезд.

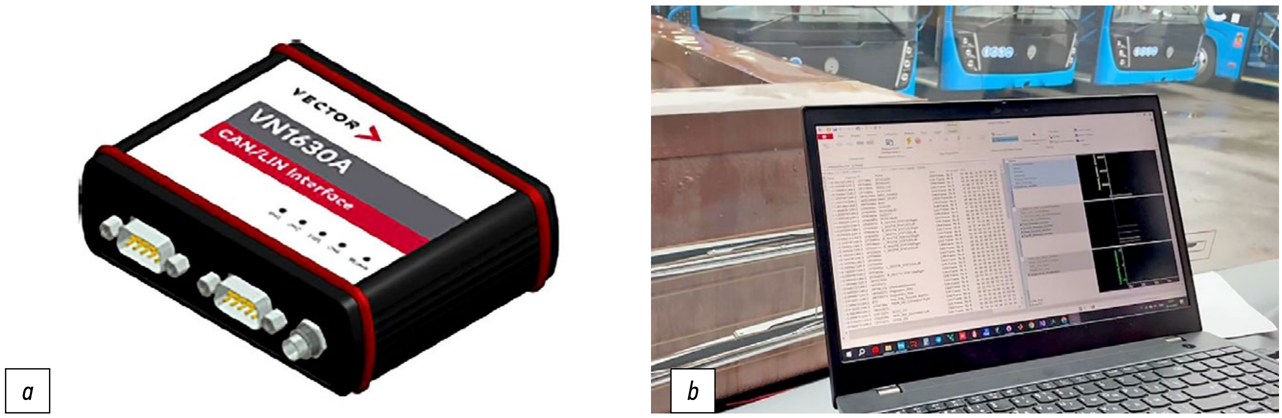

Fig. 4. Equipment for data recording: *a*) Vector VN1630A; *b*) Computer.
Рис. 4. Оборудование для фиксации данных: *a*) Vector VN1630A; *b*) ЭВМ.

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Таблица 1. Величина крутящего момента на колесе и потребляемого тока приводом
Table 1. Values of wheel torque and current consumed by the drive

Parameters		ride 1	ride 2	ride 3
Intensive acceleration (Fig. 1)				
Rate of change of the torque, Nm/s	Left side drive	$3{,}85 \cdot 10^6$	$3{,}85 \cdot 10^6$	$2{,}75 \cdot 10^6$
	Right side drive	$5{,}92 \cdot 10^6$	$3{,}85 \cdot 10^6$	$3{,}75 \cdot 10^6$
Rate of change of current, A/s	Left side drive	$3{,}20 \cdot 10^5$	$4{,}85 \cdot 10^6$	$2{,}06 \cdot 10^5$
	Right side drive	$3{,}69 \cdot 10^5$	$6{,}31 \cdot 10^6$	$2{,}24 \cdot 10^5$
Intensive braking (Fig. 2)				
Rate of change of the torque, Nm/s	Left side drive	$1{,}20 \cdot 10^7$	$1{,}17 \cdot 10^7$	$6{,}01 \cdot 10^6$
	Right side drive	$8{,}35 \cdot 10^6$	$9{,}31 \cdot 10^6$	$3{,}01 \cdot 10^6$
Rate of change of current, A/s	Left side drive	$2{,}05 \cdot 10^5$	$3{,}16 \cdot 10^6$	$2{,}45 \cdot 10^5$
	Right side drive	$2{,}50 \cdot 10^5$	$3{,}72 \cdot 10^6$	$2{,}96 \cdot 10^5$

Analytical study of oscillatory processes in the electromechanical wheel drive system during the frictional interaction of an elastic tire with a solid support base

The study of resonance and near-resonance phenomena in complex systems is crucial because these phenomena can cause a sharp increase in oscillatory amplitudes, leading to elevated dynamic loads on drive units, reduced durability, and potential destruction.

Investigating the features of oscillatory processes and the potential for resonant phenomena during vehicle system operations, beyond previously discussed scenarios, involves using a mathematical analysis of nonlinear systems.

The frictional interaction between an elastic tire and a support base in the general formulation is typically described by a system of nonlinear differential equations with six degrees of freedom, making analytical solutions challenging. To better understand the nature of the interaction and determine the possibility for resonant or self-oscillating phenomena, a simplified model with three degrees of freedom can be used.

Studying the dynamics of friction systems through the lens of nonlinear differential equations reveals that analyzing frictional self-oscillations in complex systems, especially those with several degrees of freedom, is often an analytically intractable problem. In this regard, it is reasonable to start by studying frictional self-oscillations in a system with three degrees of freedom [3–10]. Figure 6 shows the calculation scheme for the frictional interaction between the elastic tire and the road, as well as the effects on the vehicle's sprung part and the wheel drive's electric motor [3-10].

Using theorems on the conservation of the momentum and angular momentum for the system under consideration (Fig. 6), describing tire–road interactions, allows forming systems of differential equations applicable to both traction and driven rolling modes of the wheel:

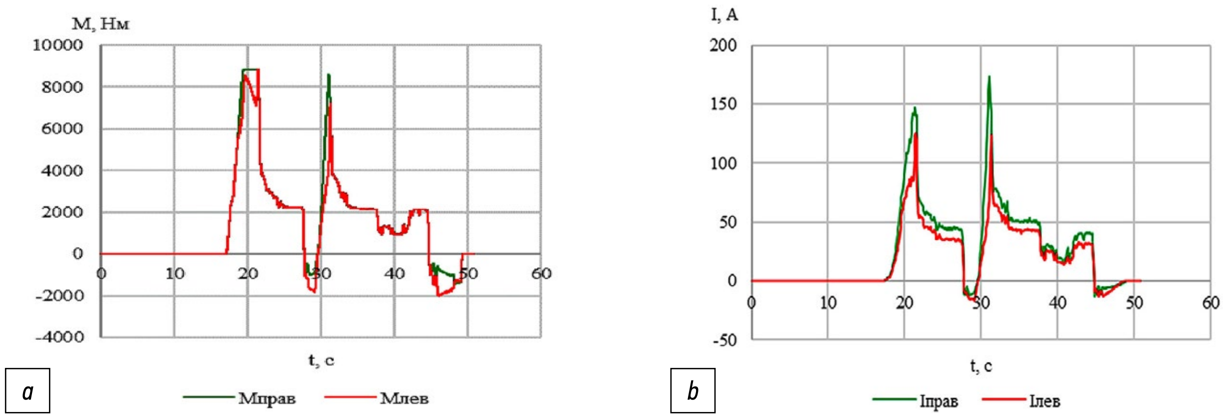


Fig. 5. Curves of wheel torque (*a*) and current consumed by the drive (*b*).
Рис. 5. Значение крутящего момента на колесе (*a*) и тока, потребляемого приводом (*b*).

$$\begin{aligned}
&\dot{x}_1 = v_1;\\
&\dot{v}_1 = \frac{c}{M}\left(x_1 - x_2\right);\\
&\dot{x}_2 = v_2;\\
&\dot{v}_2 = \frac{1}{m}\left(F - cx_1 + cx_2\right);\\
&\dot{\varphi}_\text{к} = \omega_\text{к};\\
&\dot{\omega}_\text{к} = \frac{1}{J_\text{к}}\left[c_m\left(\varphi_m - \varphi_\text{к}\right) - Fr_\text{к}\right];\\
&\dot{\varphi}_m = \omega_m;\\
&\dot{\omega}_m = \frac{1}{J_m}\left[-c_m\left(\varphi_m - \varphi_\text{к}\right) + M_t\right].
\end{aligned} \tag{1}$$

where $J_\text{к}$ and J_m denote the axial moments of inertia. The first term represents the moment of inertial of the wheel, while the second one the moment of inertial of the transmission, as referenced to the rotor of the traction motor. Both are measured relative to their rotational axes.

Using the conservation theorems for momentum and angular momentum, we can derive the following differential equations for the braking mode:

$$\begin{aligned}
&\dot{x}_1 = v_1;\\
&\dot{v}_1 = \frac{c}{M}\left(-x_1 + x_2\right);\\
&\dot{x}_2 = v_2;\\
&\dot{v}_2 = \frac{1}{m}\left(-F + cx_1 - cx_2\right);\\
&\dot{\varphi}_\text{к} = \omega_\text{к};\\
&\dot{\omega}_\text{к} = \frac{1}{J_\text{к}}\left[-c_m\left(\varphi_m - \varphi_\text{к}\right) + Fr_\text{к} - M_\text{к}\right];\\
&\dot{\varphi}_m = \omega_m;\\
&\dot{\omega}_m = \frac{1}{J_m}\left[c_m\left(\varphi_m - \varphi_\text{к}\right) - M_t\right].
\end{aligned} \tag{2}$$

where $M_\text{к}$ is the braking torque developed by the wheel brake mechanism.

The motion of the nonlinear system shown in Fig. 6 and described by differential equations (1) or (2) in general form can be represented as follows:

$$\ddot{\varphi} + h(\varphi, \dot{\varphi}) + f(\varphi) = Q(t), \tag{3}$$

where φ is the generalized coordinate (referring to the rotation angle of the shaft); $f(\varphi)$, $h(\varphi, \dot{\varphi})$, and $Q(t)$ represent conservative forces (which are restoring or potential forces whose operation does not depend on the motion trajectory), dissipative forces (which are damping forces that reduce the total energy

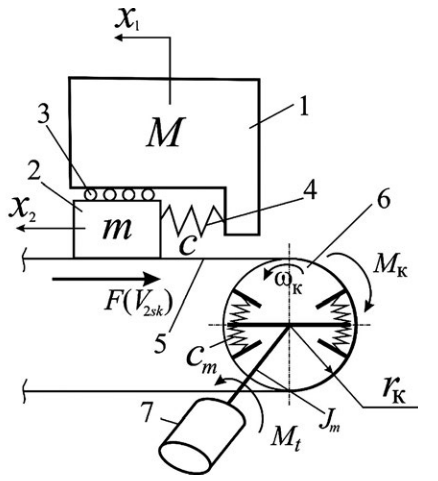


Fig. 6. The analytical model of interaction of an elastic wheel with solid ground surface: *1* — the vehicle sprung mass M given to a wheel; *2* — the wheel mass m; *3* — rollers; *4* — a spring showing longitudinal compliance of a tire; *5* — a ground surface; *6* — a rolling wheel; *7* — a traction electric motor; c — spring stiffness; x_1, x_2 — longitudinal displacements of masses 1 and 2; $F\left(V_{2sk}\right)$ — friction force dependent on slip rate V_{2sk} of the wheel relatively to the ground surface; $\omega_\text{к}$ — wheel rotation velocity; $r_\text{к}$ — distance between the wheel center and the ground surface; M_t — traction or braking torque of the traction electric drive; c_m — angular 'electromagnetic' stiffness of the traction permanent magnet synchronous machine; J_m — inertia moment of rotating parts of the electric motor given to the rotor.

Рис. 6. Расчётная схема взаимодействия эластичного колеса с твёрдым опорным основанием: *1* — масса M подрессоренных частей автомобиля, приходящаяся на колесо; *2* — масса m колеса; *3* — ролики; *4* — упругий элемент, характеризующий податливость шины в продольном направлении; *5* — опорное основание; *6* — вращающееся колесо; *7* — тяговый электродвигатель; c — жёсткость пружины, эквивалентная продольной жёсткости шины; x_1, x_2 — продольные перемещения масс 1 и 2 соответственно; $F\left(V_{2sk}\right)$ — сила трения, зависящая от скорости V_{2sk} скольжения колеса относительно опорного основания; $\omega_\text{к}$ — угловая скорость вращения колеса; $r_\text{к}$ — радиус колеса; M_t — крутящий или тормозной момент, развиваемый тяговым электродвигателем; c_m — угловая жёсткость тягового электродвигателя, деталей трансмиссии и колеса; J_m — момент инерции вращающихся частей электродвигателя, приведённый к ротору.

of the system), and external driving forces attributed to the moment of inertia of the system, respectively.

We assume that $f(0) = 0$; $f'(0) > 0$. At the same time, $\varphi = 0$ is a stable equilibrium position. Let the dissipative force satisfy the following condition [13]:

$$h(\varphi, \dot{\varphi}) = -h(\varphi, -\dot{\varphi}),$$

and is small in magnitude compared to the restoring force (a system with weak dissipation, damping).

In nonlinear systems, large-amplitude oscillations are commonly referred to as resonant. During these oscillations, the maximum values of $\ddot{\varphi}$ and $f(\varphi)$ significantly exceed the values of $h(\varphi,\dot{\varphi})$ and $Q(t)$, respectively [13]. This indicates that in system (3), the sum $\ddot{\varphi}+f(\varphi)$ becomes relatively small compared to their individual maximum values. Consequently, resonant oscillations in a nonlinear system resemble the free oscillations of a corresponding conservative system, in which $\ddot{\varphi}+f(\varphi)\equiv 0$ [14].

In the system under consideration, possible resonant oscillations can be viewed as free oscillations, with frequency ω remaining constant irrespective of amplitude, but supported by external forces, like those arising in the wheel–road contact point.

For gear pairs, splined joints, and other mechanical components within the transmission system of the traction wheel drive operating in an oil bath, the characteristics of the restoring force and the angular frequency ω of free oscillations as a function of the half-range of oscillations A are shown in Fig. 7 [14].

Figure 4 depicts parameters $\omega_0^2=\operatorname{tg}\alpha_0$ and $\omega_1^2=\operatorname{tg}\alpha_1$. The restoring force $f(\varphi)$ (Fig. 6, a) is represented by a polyline with three rectilinear sections.

$$f(\varphi)=\begin{cases} c_1\varphi, \quad -a\le\varphi\le a \\ c_1\varphi+c_2(\varphi-a), \quad \varphi>a \\ -c_1\varphi+c_2(\varphi+a), \quad \varphi<-a \end{cases} \tag{4}$$

When an external harmonic driving force acts on the system, it induces oscillatory processes whose frequency matches that of the driving force. The amplitude of these oscillations is determined by equations (4) [12]:

$$\omega_{\text{в}}=\begin{cases} \sqrt{\dfrac{c_1}{J}}, \quad 0<h<h_0=\dfrac{c_1a^2}{2} \\ \dfrac{\pi}{2}\sqrt{\dfrac{c_1}{J}}\left\{\arcsin\left(\sqrt{\dfrac{h_0}{h}}\right)+\sqrt{\dfrac{c_1}{c_2}}\operatorname{arctg}\left(\sqrt{\dfrac{c_1}{c_2}\left[\dfrac{h_0}{h}-1\right]}\right)\right\}, \quad h>h_0 \end{cases}$$

$$A=\begin{cases} \dfrac{2h}{c_1}, \quad 0<h\le h_0 \\ a\cdot\left(1-\dfrac{c_1}{c_2}\right)+\sqrt{\dfrac{2}{c_2}\left[h-h_0\left(1-\dfrac{c_1}{c_2}\right)\right]}, \quad h>h_0 \end{cases}$$

where h is the total energy (potential and kinetic) of the conservative system.

Figure 6 shows the resonance and backbone curves for a system (3) that includes a nonlinear restoring force (4). From Fig. 8, it is evident that the system can exhibit two stable modes of forced oscillations, each with different amplitudes, such as A'' and A' at frequency ω_1.

In the system under consideration, when the external driving force increases, the oscillation amplitude rises until it matches the frequency of its natural oscillations. However, in this case, the resonance is disrupted, leading to a decrease in amplitude even as the frequency continues to increase. Conversely, when the frequency of the external force decreases, an amplitude jump occurs.

Analytical study of the possibility of resonant modes in the electromechanical wheel drive sistem during the frictional interaction of an elastic tire with a solid support base

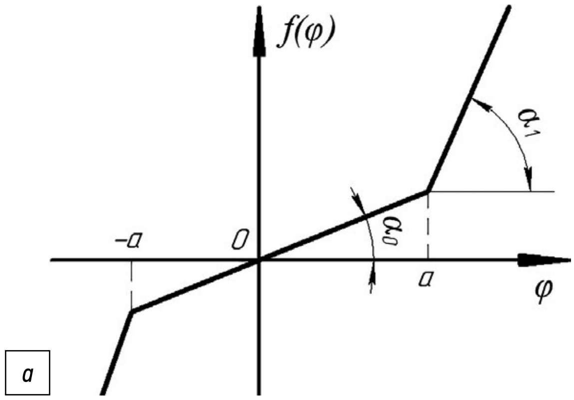
Let us examine the resonant modes in a nonlinear system with one degree of freedom, specifically the “traction electric motor-transmission-drive wheel” set-up (Fig. 5). As a friction model, we use a simplified analog of a polynomial of the third degree $F\left(v_{2sk}\right)=kv_{2sk}\left(1-g_1v_{2sk}^2+g_2v_{2sk}^4\right)$. The system’s oscillatory processes are then described by the Duffing equation [19]:

$$J\ddot{\varphi}+b\dot{\varphi}+c_1\varphi+c_2\varphi^3=P\cdot\cos vt, \tag{5}$$

where J is the moment of inertia, b denotes the damping coefficient, P and v represent the amplitude and frequency of the external disturbing force, respectively, and t indicates the time.

Solving this system generally poses significant challenges. Nevertheless, during the resonant regime, it is possible to derive a sufficiently accurate analytical solution for the system of equations (6), despite the inherent complexity [13]. At resonance, the system of equations (5) can be written as follows [13]:

$$\begin{cases} J\ddot{\varphi}+c_1\varphi+c_2\varphi^3=0 \\ b\dot{\varphi}-P\cdot\cos vt=0 \end{cases} \tag{6}$$



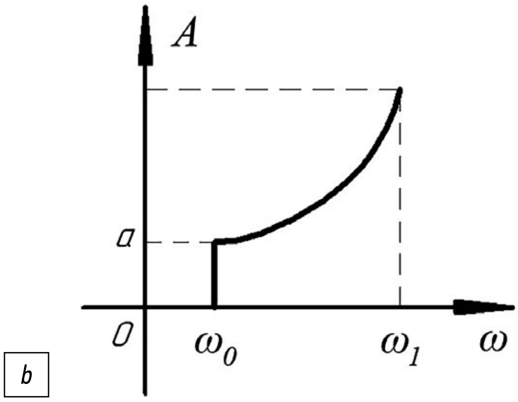


Fig. 7. Curve of restoring force (*a*) and dependence of angular frequency ω of free oscillations on half-range A (*b*).
Рис. 7. Характеристика восстанавливающей силы (*a*) и зависимость угловой частоты ω свободных колебаний от полуразмаха A (*b*).

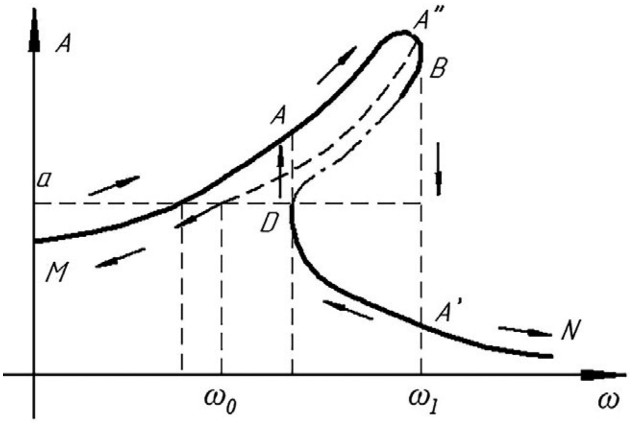


Fig. 8. Resonant and skeleton curves for the non-linear system.
Рис. 8. Резонансная и скелетная кривые для нелинейной системы.

The first equation, after a series of simple transformations, can be represented as follows:

$$\ddot{\varphi} + \omega^2\varphi + \beta\varphi^3 = 0\,, \tag{7}$$

where $\omega^2 = \dfrac{c_1}{J}$ is the natural frequency of the generating system, when $\beta = 0$; $\beta = \dfrac{c_1}{J}$ denotes the coefficient determining the degree of nonlinearity in the elastic characteristic. In the resonant regime, the solutions of the equations decompose into trigonometric functions.

In a linear system without damping, the amplitude of oscillations A is determined as follows:

$$|A| = \frac{P}{\left|v^2 - \omega^2\right|}$$

In the resonant mode, when the frequencies of the external influence v and the natural frequency ω of the generating system $A \to \infty$ coincide. For a nonlinear system [16]

$$|A| \approx \frac{P}{\left|v^2 - \omega^2 - 3\beta A^2\omega^2/4\right|}$$

that is, the amplitude no longer tends to infinity.

In system (5), the presence of dissipative losses leads to what is known as resonance disruption, as illustrated in Fig. 9.

Research [17, 18] has shown that in nonlinear systems, whose dynamic behavior is described by equations (3), (4), and (5), modes close to resonant, known as near-resonant modes, can arise. In such systems, resonance is a mode that the system may approach but never fully reach owing to resonance disruption. No other resonant subharmonic, superharmonic and combined modes can arise in a nonlinear system [19-21].

However, studies [3-10] indicate that in nonlinear systems (Fig. 5) describing the interaction of the wheel and the road, self-oscillating modes can occur in both traction and braking modes of wheel rolling. These processes can be either “soft,” occurring at any system parameter values, or "hard" emerging only at certain parameter settings. Moreover, the self-oscillating mode can be characteristic of various parts: the wheel (both rotational and translational motions), the suspension system, or the rotor shaft of the traction motor. These processes are often triggered by increased sliding of elastic tires on the relative support base, which reduces friction as sliding speed increases. At the same time, the design of the electric motors used in drives does not facilitate damping the oscillations of the rotor shaft arising from tire oscillations. Additionally, the design features and control system can lead to oscillations in the torque and angular velocity of the shaft. Consequently, electromechanical drives are particularly susceptible to self-oscillating phenomena, even in scenarios where mechanical drives would not experience them.

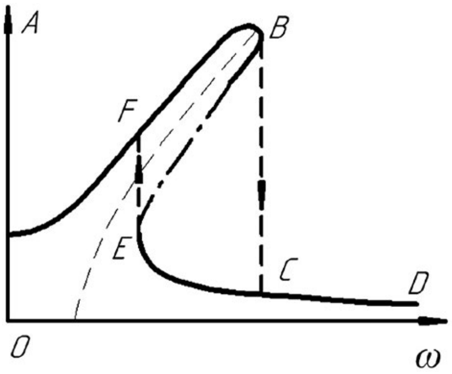


Fig. 9. Resonance 'breakdown' in the systems with non-linearly increasing stiffness.
Рис. 9. «Срыв» резонанса в системах с нелинейно увеличивающейся жёсткостью.

RESULTS. CONCLUSIONS

In the electromechanical drive system for the traction wheels of the vehicle, t oscillatory processes related to toque occur during movement (Figs. 2, 3). Since this system is nonlinear, pronounced resonant phenomena with uncontrolled amplitude increases to infinity cannot are not observed. Instead, resonance disruption occurs. This can manifest as a decrease in oscillation amplitudes with increased frequencies during vehicle acceleration, and an increase in amplitudes with decreased frequencies during deceleration (Fig. 8, 9). The sharp, abrupt changes in wheel torque and drive current, as well as their reduction when suppressing self-oscillatory phenomena, are revealed.

However, in nonlinear drive systems in electromechanical wheel drives, self-oscillating phenomena can be triggered by the frictional interaction of an elastic tire with a solid support base. These phenomena are characterized by frequent changes in torque and current, resulting in high dynamic loads and even reversal of torque direction. Using self-oscillation suppression algorithms reduces the amplitudes and the rates of change, thereby reducing dynamic loads.

The practical value of the study lies in in applying these findings to design traction electromechanical drive units and synthesize vehicle traffic control systems.

ADDITIONAL INFORMATION

Author's contribution. The author confirms the compliance of their authorship with the international ICMJE criteria (the author made a significant contribution to the development of the concept, research and preparation of the article, read and approved the final version before publication).

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Вклад автора. Автор подтверждает соответствие своего авторства международным критериям *ICMJE* (автор внёс существенный вклад в разработку концепции, проведение исследования и подготовку статьи, прочёл и одобрил финальную версию перед публикацией).

Конфликт интересов. Автор декларирует отсутствие явных и потенциальных конфликтов интересов, связанных с публикацией настоящей статьи.

Источник финансирования. Автор заявляет об отсутствии внешнего финансирования при проведении исследования.

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