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THE DISTRIBUTED CONTROL SYSTEM SYNTHESIS OF ELASTIC STRUCTURE

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Abstract. *For an elastic distributed plant with parameters depending on the spatial variable, based on the spectral method of distributed systems analysis and synthesis and considering the internal resistance according to Voigt the transition is made from PDE to an infinite system of ODE in state space form. The boundary conditions are additively included to the obtained spectral representation, which enables to control from the boundaries. The control law for suppression of oscillations is synthesized and the closed system is analyzed. The obtained results can be used in the control systems synthesis for aircraft with active dynamic compensation of elastic vibrations.*

Keywords: *elastic beam, oscillation equation, Fourier series, spectral method, operational matrix, dynamic regulator, synthesis, analysis.*

Introduction

Elastic structures are widely used in the fields of aviation and rocket engineering to increase flight speed, reduce weight, and increase aircraft length. Under the appropriate flight conditions and particularly in terms of fuel consumption, elastic oscillations of the carrying structure occur; these are commensurate with the angular oscillations of an aircraft in terms of frequency and amplitude.

Elastic oscillations affect control system sensors and consequently, the control elements. These disturbances can result in the loss of accuracy and stability of the flight control [1, 2]. This creates an issue in the development of the aircraft control law to counteract external disturbances and elastic oscillations of the aircraft's body.

Modern spacecraft are equipped with both rigid members and elastic structures, such as antennae, solar batteries, and outboard rods with metering instruments. These devices require passive or active stabilization for normal operation of the spacecraft.

The major control operations of elastic structures and their stabilization are based on the modern theory of analytical design of the best regulators for the systems with distributed parameters [3, 4].

It should be noted that for the objects described with the help of the system of differential or integro-differential equations with partial derivatives, the optimization system of the control-determining equations comprises a non-linear system of differential and integro-differential equations with partial derivatives [3].

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The resolution of this system is a rather complicated task in both the implementation of computation procedures and the search for algorithms that provide a good convergence of the obtained solutions.

This study's objectives are first, to perform a transition from differential equations with partial derivatives to an infinite system of standard differential equations in the form of the state space. This uses a spectral method of the theory of control [5, 6] for an elastic distributed object (an aircraft fuselage or a rocket body) with parameters dependent on the spatial variable and with regard to the inner resistance according to Voigt [2]. The second objective is to synthesize the law of control for the suppression of oscillations and perform an analysis of the closed system.

1. Mathematical model of the controlled object

It is assumed in this study that the elastic oscillations of both the aircraft fuselage and the rocket body are sufficiently accurately described by the equation of the flexible beam of the variable cross section with regard to the inner resistance as per Voigt, which, according to [2], appears as follows:

$$\bar{\mu}(\bar{x})\frac{\partial^2 \bar{y}(\bar{x},\bar{t})}{\partial \bar{t}^2}+\frac{\partial^2}{\partial \bar{x}^2}\left(\overline{EJ}(\bar{x})\frac{\partial^2 \bar{y}(\bar{x},\bar{t})}{\partial \bar{x}^2}\right)+$$
$$+\bar{h}(\bar{x})\frac{\partial}{\partial \bar{t}}\left(\frac{\partial^2}{\partial \bar{x}^2}\left(\overline{EJ}(\bar{x})\frac{\partial^2 \bar{y}(\bar{x},\bar{t})}{\partial \bar{x}^2}\right)\right)=\bar{f}(\bar{x},\bar{t}),$$
$$\bar{\mu}\in(0,\mu_0),\ \bar{x}\in(0,l),\ \bar{y}\in(0,l),\ \overline{EJ}\in(0,E_0J_0),$$
$$\bar{F}\in(0,F_0),\ \bar{h}\in(0,h_0),\ \bar{t}\in(0,t_0), \tag{1}$$

where $\bar{x}$ is the spatial variable,
$\bar{t}\geq 0$ is time,
$\bar{y}(\bar{x},\bar{t})$ is the beam-axis bending measured in a perpendicular direction to the non-deformed beam axis,
$\bar{\mu}(\bar{x})$ is the length unit weight,
$\overline{EJ}(\bar{x})$ is the flexural stiffness,
$\bar{E}$ is the elastic modulus,
$\bar{J}$ is the moment of inertia of the beam cross section relative to the section's neutral axis perpendicular to the oscillation plane,
$\bar{f}(\bar{x},\bar{t})$ is the external distributed transversal load attributed to the beam-length unit, and
$\bar{h}(\bar{x})$ is the coefficient of the inner resistance, as per Voigt.

Equation (1) is the following:

$$\bar{\mu}(\bar{x})\frac{\partial^2\bar{y}(\bar{x},\bar{t})}{\partial\bar{t}^2}+\frac{\partial^2\overline{EJ}(\bar{x})}{\partial\bar{x}^2}\frac{\partial^2\bar{y}(\bar{x},\bar{t})}{\partial\bar{x}^2}+2\frac{\partial\overline{EJ}(\bar{x})}{\partial\bar{x}}\frac{\partial^3\bar{y}(\bar{x},\bar{t})}{\partial\bar{x}^3}+\overline{EJ}(\bar{x})\frac{\partial^4\bar{y}(\bar{x},\bar{t})}{\partial\bar{x}^4}+$$
$$\bar{h}(\bar{x})\frac{\partial}{\partial\bar{t}}\left(\frac{\partial^2\overline{EJ}(\bar{x})}{\partial\bar{x}^2}\frac{\partial^2\bar{y}(\bar{x},\bar{t})}{\partial\bar{x}^2}+2\frac{\partial\overline{EJ}(\bar{x})}{\partial\bar{x}}\frac{\partial^3\bar{y}(\bar{x},\bar{t})}{\partial\bar{x}^3}+\overline{EJ}(\bar{x})\frac{\partial^4\bar{y}(\bar{x},\bar{t})}{\partial\bar{x}^4}\right)=\bar{f}(\bar{x},\bar{t}).$$

(2)

Let (2) be considered as a mathematical model of the controlled object with initial conditions

$$\bar{y}(\bar{x},0)=\bar{f}_1(\bar{x}),\ \partial\bar{y}(\bar{x},0)/\partial\bar{t}=\bar{f}_2(\bar{x}),\quad \bar{x}\in[0,l] \tag{3}$$

and boundary conditions

$$\partial^2\bar{y}(l,\bar{t})/\partial\bar{x}^2=\bar{U}(\bar{t}),\ \partial^2\bar{y}(0,\bar{t})/\partial\bar{x}^2=\partial^3\bar{y}(0,\bar{t})/\partial\bar{x}^3=\partial^3\bar{y}(l,\bar{t})/\partial\bar{x}^3=0,\quad t\geq 0. \tag{4}$$

Let differential equation (2), initial conditions (3), and boundary conditions (4) be reduced to a dimensionless form. The dimensionless variables are introduced as

$$\mu=\bar{\mu}/\mu_0,\ x=\bar{x}/l,\ y=\bar{y}/l,\ EJ=\overline{EJ}/(E_0J_0),\ f=\bar{f}/f_0,$$
$$h=\bar{h}/h_0,\ t=\bar{t}/t_0,\ f_1=\bar{f}_1/l,\ f_2=\bar{f}_2t_0/l,\ U=\bar{U}/l, \tag{5}$$

where $\mu_0,\ E_0J_0,\ f_0,\ h_0,\ t_0$ are some nominal values of appropriate variables.

In the new variables in (5), the differential equation of the controlled object will be the following:

$$\mu\frac{\partial^2 y}{\partial t^2}+a_1\frac{\partial^2 EJ}{\partial x^2}\frac{\partial^2 y}{\partial x^2}+a_2\frac{\partial EJ}{\partial x}\frac{\partial^3 y}{\partial x^3}+a_3EJ\frac{\partial^4 y}{\partial x^4}+$$
$$+h\frac{\partial}{\partial\bar{t}}\left(b_1\frac{\partial^2 EJ}{\partial x^2}\frac{\partial^2 y}{\partial x^2}+b_2\frac{\partial EJ}{\partial x}\frac{\partial^3 y}{\partial x^3}+b_3EJ\frac{\partial^4 y}{\partial x^4}\right)=cf,$$
$$\mu\in(0,1),\ x\in(0,1),\ y\in(0,1),\ EJ\in(0,1),\ h\in(0,1). \tag{6}$$

The coefficients of equation (6) are determined with expressions as

$$a_1=a_3=E_0J_0t_0^2/(l^4\mu_0),\ a_2=2E_0J_0t_0^2/(l^4\mu_0),$$
$$b_1=b_3=h_0E_0J_0t_0^2/(l^4\mu_0),\ b_2=2h_0E_0J_0t_0^2/(l^4\mu_0),\ c=f_0t_0^2/(\mu_0 l).$$

(7)

The initial conditions are

$$y(x,0)=f_1(x),\ \partial y(x,0)/\partial t=f_2(x),\quad x\in[0,1]. \tag{8}$$

The boundary conditions are

$$\partial^2 y(1,t)/\partial x^2=U(t),\ \partial^2 y(0,t)/\partial x^2=\partial^3 y(0,t)/\partial x^3=\partial^3 y(l,t)/\partial x^3=0,\ t\geq 0. \tag{9}$$

Then, based on the properties of the spectral features in [5], the expressions for the matrix of spectral representation of the controlled object is obtained.

2. Spectral representation of the task

Let it be assumed that the function describing the controlled object's condition $y(x,t)$ is a substantial one-valued limited function with square integrability in the area of the spatial variable $x \in [a,b]$, and boundary conditions are applied at the points $\bar{a} = a + 0$, $\bar{b} = b - 0$, and $t \in [0, t_0],\ t_0 \to \infty$.

The function $y(x,t)$ with regard to the boundary conditions can be presented as follows:

$$y(x,t) = y_0(x,t) + \varphi_a^0(t) \cdot 1(\bar{a} - x) + \varphi_b^0(t) \cdot 1(x - \bar{b}), \quad (10)$$

where $y_0(x,t)$ is the function coinciding with the function $y(x,t)$ at the interval $x \in [a+0, b-0]$,

$\varphi_a^0(t)$ is the value of the single jump function at the boundary $x = a + 0$,

$\varphi_b^0(t)$ is the value of the single jump function at the boundary $x = b - 0$, and

$$1(a+0-x) = \begin{cases} 1, & x \le a+0, \\ 0, & x > a+0, \end{cases} \quad 1(x-b+0) = \begin{cases} 1, & x \ge b-0, \\ 0, & x < b-0. \end{cases}$$

The generalized variable [7] of function (10) on *x* will be as follows:

$$\frac{\partial y(x,t)}{\partial x} = \frac{\partial y_0(x,t)}{\partial x} - \varphi_a^0(t) \cdot \delta(\bar{a} - x) + \varphi_b^0(t) \cdot \delta(x - \bar{b}).$$

For the *m*- variable, the following expression can be written as

$$\frac{\partial^m y(x,t)}{\partial x^m} = \frac{\partial^m y_0(x,t)}{\partial x^m} - \sum_{j=1}^{m-2} \left(\varphi_a^j(t) \frac{\partial^{m-j} \delta(\bar{a} - x)}{\partial x^{m-j}} - \varphi_b^j(t) \frac{\partial^{m-j} \delta(x - \bar{b})}{\partial x^{m-j}} \right).$$

Function $y_0(x,t)$ shall be decomposed in the Fourier series according to the system of orthonormal functions $\{P(\bar{h}, x)\},\ \bar{h} = \overline{1, \infty}$ on the variation interval $x \in [a,\ b]$

$$y_0(x,t) = \sum_{\bar{h}=1}^{\infty} \Phi_0(\bar{h}, t) P(\bar{h}, x), \qquad \Phi_0(\bar{h}, t) = \int_a^b y_0(x,t) P(\bar{h}, x) dx. \quad (11)$$

Using spectral-feature properties and with regard to $a = 0,\ b = 1$, the transition from the differential equation with partial variables (2) at the initial conditions (3) and the boundary conditions (4) is performed to the system of standard differential equations as follows:

$$P_\mu \ddot{\Phi}_0 + a_1 \left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi} \right) P_2 \Phi_0 + a_2 \left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi} \right) \left(P_3 \Phi_0 + \Gamma_3^{21} \right) +$$

$$+ a_3 P_{EJ} \left(P_4 \Phi_0 + \Gamma_4^{21} \right) + b_1 P_h \left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi} \right) P_2 \dot{\Phi}_0 +$$

$$+ b_2 P_h \left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi} \right) \left(P_3 \dot{\Phi}_0 + \dot{\Gamma}_3^{21} \right) + b_3 P_h P_{EJ} \left(P_4 \dot{\Phi}_0 + \dot{\Gamma}_4^{21} \right) = c \Phi_f \, , \tag{12}$$

where $\Phi_0 \in \mathrm{R}^{\bar{h}}$, $\bar{h} = \overline{1, \infty}$ is the vector of the spectral feature of function $y_0(x,t)$ with components

$$\Phi_0(\bar{h}, t) = \int_0^1 y_0(x,t) P(\bar{h}, x) dx, \ \bar{h} = \overline{1, \infty};$$

P_μ, P_{EJ}, P_h are the infinite-dimensional square operational matrices of the first multiplier of the spectral properties of functions $\mu(x)$, $EJ(x)$, $h(x)$ accordingly, whose elements are calculated with the expressions

$$P_\mu(\bar{h}, h) = \int_0^1 P(\bar{h}, x) P(h, x) \mu(x) dx, \quad P_{EJ}(\bar{h}, h) = \int_0^1 P(\bar{h}, x) P(h, x) EJ(x) dx,$$

$$P_h(\bar{h}, h) = \int_0^1 P(\bar{h}, x) P(h, x) h(x) dx, \qquad \bar{h}, h = \overline{1, \infty};$$

P_{EJ}^1, P_{EJ}^2 are the infinite-dimensional square operational matrices of the multipliers $\partial EJ(x) / \partial x$, $\partial^2 EJ(x) / \partial x^2$, whose elements are determined with the expressions

$$P_{EJ}^k(\bar{h}, h) = \int_0^1 \frac{\partial^k}{\partial \tau^k} \left[P(\bar{h}, \tau) P(h, \tau) \right] EJ(\tau) \, d\tau, \ k = 1, 2, \ \bar{h}, h = \overline{1, \infty};$$

$\Phi_f \in \mathrm{R}^{\bar{h}}$, $\bar{h} = \overline{1, \infty}$ is the vector of the spectral property of function $f(x,t)$ with the components

$$\Phi_f(\bar{h}, t) = \int_0^1 f(x,t) P(\bar{h}, x) dx, \ \bar{h} = \overline{1, \infty};$$

P_m is the infinite-dimensional square operational matrix of differentiation with the elements calculated according to the expression

$$P_m(\bar{h}, h) = \int_0^1 P(\bar{h}, x) \frac{\partial^m P(h, x)}{\partial x^m} dx, \qquad \bar{h}, h = \overline{1, \infty}, \quad m = \overline{1, 4},$$

$\Gamma_m^{21} \in \mathrm{R}^n,\ n = \overline{1, \infty},\ m = 3, 4$ are vectors of the spectral properties of the boundary conditions with the elements

$$\Gamma_m^{21}(\overline{h}) = \int_0^1 \varphi_1^2 \frac{\partial^{m-1}\delta(x-1)}{\partial x^{m-1}} P(\overline{h}, x)\, dx, \quad \overline{h} = \overline{1, \infty},\ m = 3, 4.$$

$P_{\nu EJ\, i},\ i = \overline{1, k},\ \nu = 1, 2$ are operational matrices describing the jumps of function $EJ(x)$ at interval $x \in (0, 1)$ calculated with the expression

$$P_{\nu EJ\, i}(\overline{h}, h) = \int_0^1 P(\overline{h}, x) P(h, x)\, \varphi_i \frac{\partial^{\nu-1}\delta(x - x_i)}{\partial x^{\nu-1}} dx, \quad \overline{h} = \overline{1, \infty},\ i = \overline{1, k},\ \nu = 1, 2.$$

Expression (12) is reduced to the following:

$$\begin{aligned}\ddot{\Phi}_0 = &- P_\mu^{-1}\left[a_1\left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi}\right)P_2 + a_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)P_3 + a_3 P_{EJ} P_4\right]\Phi_0 + \\ &+ a_2 P_\mu^{-1}\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)\Gamma_3^{21} + a_3 P_{EJ}\,\Gamma_4^{21} + \\ &+ P_\mu^{-1} P_h\left[b_1\left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi}\right)P_2 + b_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)P_3 + b_3 P_{EJ} P_4\right]\dot{\Phi}_0 + \\ &+ P_\mu^{-1} P_h\left[b_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)\dot{\Gamma}_3^{21} + b_3 P_{EJ}\dot{\Gamma}_4^{31}\right] + c P_\mu^{-1}\Phi_f.\end{aligned}$$

(13)

The new variable $\Phi_1 = \dot{\Phi}_0$ is introduced and represented with equation (13) in the form of the system of vector–matrix equations in Cauchy's integral formula as

$$\begin{aligned}\dot{\Phi}_0 = &\ \Phi_1, \\ \dot{\Phi}_1 = &- P_\mu^{-1}\left[a_1\left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi}\right)P_2 + a_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)P_3 + a_3 P_{EJ} P_4\right]\Phi_0 + \\ &+ a_2 P_\mu^{-1}\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)\Gamma_3^{21} + a_3 P_{EJ}\,\Gamma_4^{21} + \\ &+ P_\mu^{-1} P_h\left[b_1\left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi}\right)P_2 + b_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)P_3 + b_3 P_{EJ} P_4\right]\Phi_1 + \\ &+ P_\mu^{-1} P_h\left[b_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)\dot{\Gamma}_3^{21} + b_3 P_{EJ}\dot{\Gamma}_4^{21}\right] + c P_\mu^{-1}\Phi_f.\end{aligned}$$

(14)

The value of moment $u_1(t)$ and its derivative in time $u_2(t) = \partial u_1(t)/\partial t$ are considered as control activities at the object's right boundary. The following designations are introduced:

$$\overline{A} = a_1\left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi}\right)P_2 + a_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)P_3 + a_3 P_{EJ} P_4,$$

$$\tilde{A} = b_1\left(P_{EJ}^2 + \sum_{i=1}^{k} P_{2EJi}\right)P_2 + b_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right)P_3 + b_3 P_{EJ} P_4,$$

$$\overline{B} = a_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right) P\big|_{x=1} + a_3 P_{EJ}\ \partial P/\partial x\big|_{x=1},$$

$$\tilde{B} = b_2\left(P_{EJ}^1 + \sum_{i=1}^{k} P_{1EJi}\right) P\big|_{x=1} + b_3 P_{EJ}\ \partial P/\partial x\big|_{x=1}. \qquad (15)$$

Expressions (15) use the designations of vectors $P\big|_{x=1} = \text{colon}\{P(1, x),\ P(2, x), \ldots\}\big|_{x=1}$, $\partial P/\partial x\big|_{x=1} = \text{colon}\left\{\frac{\partial P(1, x)}{\partial x}, \frac{\partial P(2, x)}{\partial x}, \ldots\right\}\Big|_{x=1}$.

With regard to designations (15), the system (14) can be written in the vector–matrix form as

$$\dot{\Phi} = A\Phi + Bu + Mv, \qquad (16)$$

where $\Phi = \text{colon}\{\Phi_0, \Phi_1\} \in R^n,\ n = 2\overline{h},\ \overline{h} = \overline{1, \infty}$ is the vector of the condition,
$u = \text{colon}\{u_1,\ u_2\}$ is the vector of control, and
$v = \text{colon}\{0, \Phi_f\}$ is the vector of disturbance.

Matrices *A*, *B*, and *M* are the following:

$$A = \begin{bmatrix} 0 & I \\ -P_\mu^{-1}\overline{A} & -P_\mu^{-1}P_h\tilde{A} \end{bmatrix},\ B = \begin{bmatrix} 0 & I \\ -P_\mu^{-1}\overline{B} & -P_\mu^{-1}P_h\tilde{B} \end{bmatrix},\ M = \begin{bmatrix} 0 & 0 \\ 0 & cP_\mu^{-1} \end{bmatrix}. \qquad (17)$$

Thus, the transition from the description of the controlled object of equation with partial derivatives (2) with the set initial and boundary conditions (3) and (4) is performed to the system of standard differential equations (16) in the form of the state space with permanent coefficients.

Expression (16) is added with the following expression:

$$\theta = D\Phi,\ D = [\overline{D}\quad 0], \qquad (18)$$

where $\theta \in \mathrm{R}^r$ is the vector of measured variables – values $y(x, t)$ at the points where the sensors are installed, and *D* is the matrix, whose lines are composed of the orthonormal functions $P(\overline{h}, x),\ \overline{h} = \overline{1, \infty}$ used for splitting into the Fourier series (11) and calculated at the points of measurement.

An analysis of the spectral representation of the controlled object (14) demonstrates that the boundary conditions are included in the object's equation, which allows control to be performed from the object's boundaries. Both bending moment and lateral force can be applied on the beam ends as the control activities; therefore, expressions (14) consider the members proportional to the second and third derivatives of function $y(x,t)$ per the spatial variable x at the object's right boundary.

3. Calculation of the matrix of the controlled object's spectral representation and regulator synthesis

Let the distributed controlled object (2)–(4) with coefficients that are the functions of spatial coordinates with the following baseline data be considered as:

$$\bar{l}=1,\ \bar{h}(\bar{x})=0.001(1+\bar{x}),\ \bar{f}(\bar{x},\bar{t})=e^{-0.5\bar{t}}\delta(\bar{x}-\bar{x}^*),\ \bar{x}^*=0.8,$$
$$\bar{y}(\bar{x},0)=-0.84+\sin\pi\bar{x},\ \partial\bar{y}/\partial\bar{t}\big|_{\bar{t}=0}=0. \qquad (19)$$

The distribution of weight and rigidity is

$$\overline{\mu}(\bar{x})=\begin{cases}105, & 0<\bar{x}\le 0.45,\\ 705, & 0.45<\bar{x}\le 0.65,\\ 141, & 0.65<\bar{x}<1.\end{cases}\qquad \overline{EJ}(\bar{x})=\begin{cases}88, & 0<\bar{x}\le 0.45,\\ 146, & 0.45<\bar{x}\le 0.65,\\ 117, & 0.65<\bar{x}<1,\end{cases} \qquad (20)$$

The dimensionless coefficients are obtained from (7) after the selection of the following nominal values:

$$\mu_0=146,\ E_0J_0=705,\ f_0=1,\ h_0=1,\ t_0=1.$$

The relative distribution of weight and rigidity will be the following for the selected values (20):

$$\mu(x)=\begin{cases}0.60, & 0<x\le 0.45,\\ 1, & 0.45<x\le 0.65,\\ 0.80, & 0.65<x<1,\end{cases}\qquad EJ(x)=\begin{cases}0.15, & 0<x\le 0.45,\\ 1, & 0.45<x\le 0.65,\\ 0.2, & 0.65<x<1,\end{cases}$$

and the numerical values of the coefficients in (7) will be the following:

$$a_1=a_3=b_1=b_3=0.2071,\ a_2=b_2=0.4142,\ c=0.0014.$$

The following will be used as the system of orthonormal functions:

$$P=\sqrt{2}\{\cos(2i-1)\pi x/2\},\ i=\overline{1,\infty}. \qquad (21)$$

The values of the controlled-object matrix are provided, which consider (17). The calculations of matrix D also consider the fact that measurements are conducted at point $x=0.7$:

$$A = \left[\begin{array}{c:c} 0_{n\times n} & I_n \\ \hdashline A_{21} & A_{22} \end{array}\right],\ B = \left[\begin{array}{c:c} 0_{n\times 1} & 0_{n\times 1} \\ \hdashline B_{21} & B_{22} \end{array}\right],\ M = \left[\begin{array}{c:c} 0_{n\times 1} & 0_{n\times 1} \\ \hdashline 0_{n\times 1} & M_{22} \end{array}\right],\ D = \left[\begin{array}{c:c} D_{11} & 0 \end{array}\right].$$

(22)

$$A_{21} = \begin{bmatrix} 5.71 & -74.46 & 52.84 & \dots \\ -0.31 & -233.71 & 872.12 & \dots \\ 0.39 & 10.92 & 383.40 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix},\ A_{22} = \begin{bmatrix} 0.01 & -0.05 & -0.10 & \dots \\ 0 & -0.35 & 1.58 & \dots \\ 0 & 0.09 & -2.55 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} -2.84 \\ 8.40 \\ -10.95 \\ \dots \end{bmatrix},\ B_{22} = \begin{bmatrix} -0.01 \\ 0.02 \\ -0.02 \\ \dots \end{bmatrix},\ M_{22} = 10^{-2} \cdot \begin{bmatrix} 0.76 & 0.24 & 0.09 & \dots \\ 0.24 & 0.58 & 0 & \dots \\ 0.09 & 0 & 0.68 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0.64 & -1.40 & 1.00 & \dots \end{bmatrix}.$$

(23)

The synthesis of the continuous control unit is conducted for the controlled object (16) with matrices (22) and (23) based on LQ-optimization and the theory of supervising devices in accordance with the procedure described in [6]. The control unit equation is expressed as

$$\begin{aligned} \dot{\xi} &= A_r \xi + B_r \theta, \\ u &= C_r \xi + D_r \theta, \end{aligned} \tag{24}$$

where $\xi \in \mathrm{R}^{\alpha}$, $\alpha = n - r$ is the vector of the control unit conditions and A_r, B_r, C_r, D_r constitute the numerical matrix. The calculations consider the five amplitudes of the spatial modes.

The values of the control unit matrix are provided as

$$A_r = \left[\begin{array}{c:c} 0_{\alpha\times\alpha} & A_{r12} \\ \hdashline A_{r21} & A_{r22} \end{array}\right],$$

$$A_{r\,12} = \begin{bmatrix} -0.50 & 2.08 & -0.78 & -0.17 & 0.98 \\ 0.55 & -1.20 & 1.86 & 0.19 & -1.08 \\ -0.27 & 0.59 & -0.43 & 0.91 & 0.54 \\ -0.40 & 0.87 & -0.63 & -0.14 & 1.79 \end{bmatrix},$$

$$A_{r\,21} = 10^3 \cdot \begin{bmatrix} -0.16 & 0.02 & 3.58 & 11.96 \\ 0.07 & 0.96 & -0.83 & 14.05 \\ -0.36 & -1{,}56 & 1.80 & 3.41 \\ 0.37 & 0.94 & -7.27 & 7{,}70 \\ -0.34 & -0.39 & 0.61 & -14.54 \end{bmatrix},$$

$$A_{r\,22} = \begin{bmatrix} 50.32 & 98.08 & -72.84 & -14.11 & 94.65 \\ 2.07 & 28.75 & -11.88 & 3.36 & 62.72 \\ -51.07 & 67.38 & -60.32 & -19.58 & 38.05 \\ 2.51 & 33.25 & -13.76 & -4.70 & 63.60 \\ 11.69 & -75.50 & 42.49 & -0.56 & -118.97 \end{bmatrix}$$

$$B_r^{\mathrm{T}} = 10^3 \cdot \begin{bmatrix} -0.09 & 0.17 & -0.04 & -0.14 & -1.74 & 4.72 & -3.97 & -1.95 & -1.26 \end{bmatrix},$$

$$C_r = \begin{bmatrix} 36.04 & 10.05 & 2.83 & 67.15 & 1.66 & 0.40 & 0.59 & 1.17 & 2.62 \\ 0.06 & 0.01 & 0 & 0.11 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; D_r = \begin{bmatrix} 119.74 \\ 0.21 \end{bmatrix}.$$

Figures 1 and 2 present the results of the closed-system analysis.

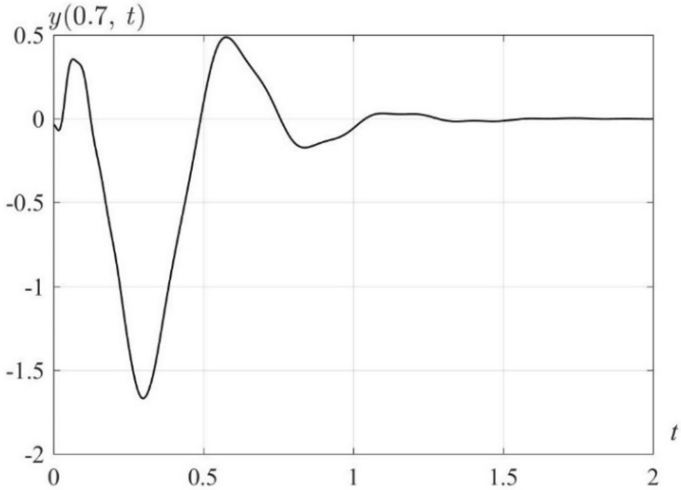
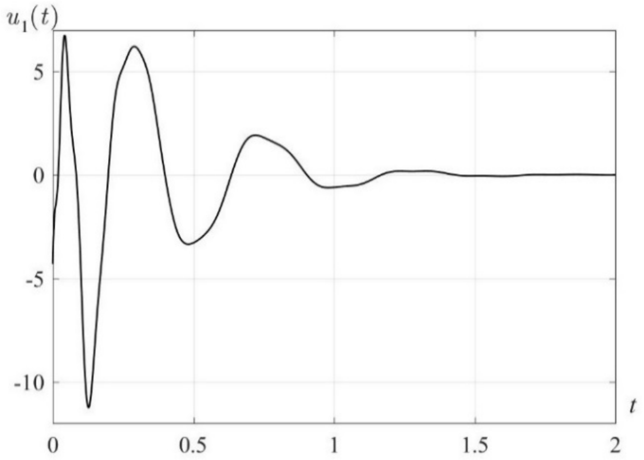
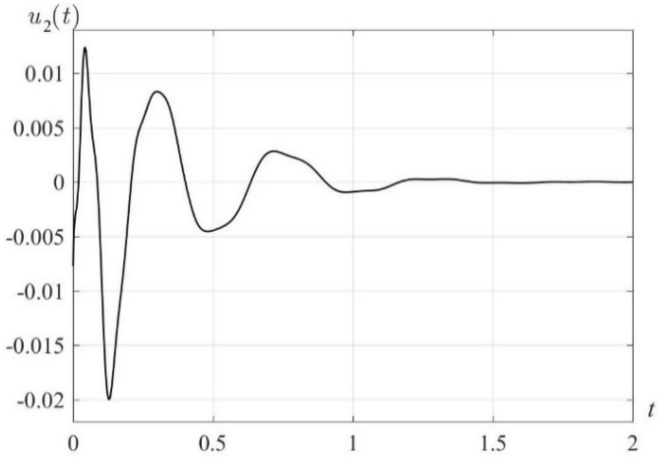


Fig. 1. Value of regulated variable at point $x = 0.7$

The diagram of the transition process presented in Fig. 1 demonstrates that the disturbance effect is compensated with error that does not exceed 3%. The control activities applied at the elastic object’s right boundary are active for 1.5 sec and do not exceed the module’s allowable values.



а



б

Fig. 2. The control actions. *а*: moment $u_1(t)$ at the object's right boundary and *б*: time derivative $u_2(t) = \partial u_1(t)/\partial t$ at the object's right boundary

Conclusion

Based on the spectral method of the theory of control, this study performed a transition from an equation with partial derivatives to an infinite system of standard differential equations in Cauchy's form. The partial derivatives describe an aircraft's elastic oscillations with regard to the inner resistance as per Voigt with irregular distribution of weight and rigidity in the structure.

Using LQ-optimization and the theory of supervising devices, the control unit was synthesized, and the watch unit was constructed with correction of the recovery error.

The obtained results can be used for the construction of aircraft control systems with active dynamic compensation of elastic oscillations. The application of these findings can facilitate an improvement in aircraft dynamics, a decrease in navigation errors, and a reduction in structural loads and stresses.

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СИНТЕЗ РАСПРЕДЕЛЕННОЙ СИСТЕМЫ УПРАВЛЕНИЯ УПРУГОЙ КОНСТРУКЦИЕЙ

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Аннотация. *Для упругого распределенного объекта управления с параметрами, зависящими от пространственной переменной, и с учетом внутреннего сопротивления по Фойгхту на основе спектрального метода анализа и синтеза распределенных систем выполнен переход от дифференциальных уравнений с частными производными к бесконечной системе обыкновенных дифференциальных уравнений в форме пространства состояний. В векторно-матричные уравнения полученного спектрального представления аддитивно входят граничные условия задачи, что позволяет осуществить управление с границ. Синтезирован закон управления для подавления колебаний и выполнен анализ замкнутой системы. Полученные результаты могут быть использованы при построении систем управления летательными аппаратами с активной динамической компенсацией упругих колебаний.*

Ключевые слова: *упругая балка, уравнение колебаний, ряд Фурье, спектральный метод, операционная матрица, динамический регулятор, синтез, анализ.*

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