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OPTIMIZATION OF MAINTENANCE PERIOD FOR THE ELEMENTS OF RESOURCE-SUPPLYING NETWORKS WITH BRANCHING STRUCTURE

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A restorable system with linear branching structure and finite reliability is considered. Preventive maintenance of its elements is carried out. Operation and restoration periods are assumed to be random values of general kind. Iteration processes of calculation of stationary reliability and economical characteristics of the network are constructed. Optimal intervals between elements' maintenance are obtained as the functions of their times-to-failure. The examples of electricity supply network and industrial circuit-switched net are given. It is shown that the optimal choice of elements' preventive maintenance results in economical network indexes improvement.

Keywords: *resource-supplying networks, branching structure, maintenance, stationary operating efficiency, average specific income, average specific expenses, optimization of preventive maintenance intervals.*

There has been considerable expansion in research concerning complex engineering system reliability theory, particularly resource-supplying networks, as a result of practical demands. Reliability factors include different aspects: technological, engineering, and organizational ones. It is necessary to develop the fundamental mathematical tools that are suitable for these issues. The reference guide [1] presents methods, algorithms, and mathematical models to solve the practical problems of ensuring reliability of electric power, gas, oil, and heat supply systems. However, predominantly, the formulae for calculation system reliability characteristics are obtained under the assumption of the exponential law of distribution of restoration time and time between failures of the system elements, as the apparatus of Markov random processes is suitable in this case.

Assumption of the general distribution law of the above-mentioned random values significantly complicates the determination of the reliability and economical characteristics of the system. And this is the problem setting of the present work. Recurrent processes of calculations of the stationary characteristics of resource-supplying networks with branching-structure are applied. Preventive maintenance of system elements is taken into account, its optimal frequency is obtained.

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Problem-setting

Herein, we consider the resource-supplying network with multilevel branching linear structure, in which every element of a certain level directly controls several elements of the lower level. And every element is connected with the ones of adjacent levels only [2–4]; such a network structure is shown in Fig. 1.

The head element a_0 is connected with a_1 elements of the first level 1, each of which, in turn, is connected with a_2 elements of second level, etc. Each element of the $(n-1)$-th level is connected with a_n elements of the last n-th level. Elements of the final level are called the outlet ones.

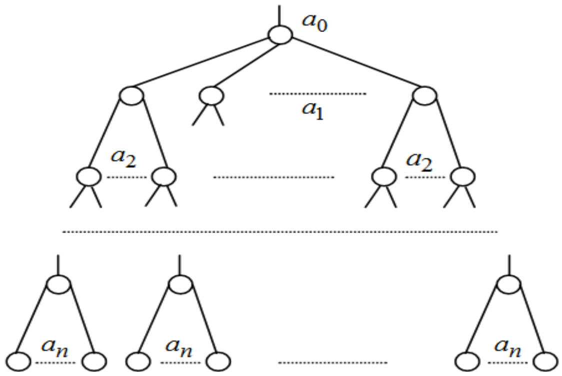

Fig. 1. Structural diagram of resource-supplying network

The network is described as follows. Failure of any element in the network is detected immediately, and its restoration begins at once. When there is an element emergency failure, operation and restoration of all the succeeding elements cease. All the links that succeed the faulty one, which therefore no longer form part of a serviceable path, are also disconnected. The serviceable path means the chain of functionally connected working elements from the head element to one of the outlet ones. When the element is restored, simultaneously, the serviceable links previously disconnected restart working; this constitutes the serviceable path together with the restored element. Their time-to-failure is the same as it was before disconnection. Additionally, restoration of disconnected elements functionally connected with the recovered one is continued.

Assuming the uniformity of network, i.e., the elements of the same level are similar: time-to-failure of elements the i-th level is the random value α_i with distribution function $F_i(t)=\mathrm{P}(\alpha_i \le t)$, i=$\overline{0,n}$; the restoration time is the random value β_i, with distribution function $G_i(t) = P(\beta_i \le t), i = \overline{0,n}$.

Network preventive maintenance is made according to the strategy known as "maintenance by age" [3]. If after the restoration completion, the element's time-to-failure is τ_i, then preventive maintenance is performed; this completely restores the element. Preventive maintenance duration is the random value β_i^p with distribution function $G_i^p(t)=\mathrm{P}(\beta_i^p \le t)$, i=$\overline{0,n}$. As at the moment of emergency failure, disconnection and connection of the functionally connected elements occur when preventive maintenance of the element begins and ends.

A characteristic of network failure is the absence of at least one serviceable path from the head element to the outlet one. When system failure occurs, all serviceable elements are disconnected.

The following economic parameters of the elements of the *i*-th level are considered known in ($i=\overline{0,n}$) network: where c_i is the income per the fault-free operation time unit, c_i^0 is the expense per time unit of restoration, and c_i^p is the expense per time unit of maintenance.

The goal of this research is to construct iteration processes to calculate the stationary factor of the technical use of $K(\tau_0, \tau_1, \dots \tau_n)$ network, the average specific income $S(\tau_0, \tau_1, \dots, \tau_n)$ per the unit of calendar time, and the average specific expense $C(\tau_0, \tau_1, \dots, \tau_n)$ per unit of the network operation, as well as the optimum frequency of the network elements' preventive maintenance.

Determination of the system stationary characteristics

Herein, we identify the availability factor of the element of the *i*-th level $K_i(\tau_i)$, the average specific income of the element of the *i*-th level per unit of calendar time $S_i(\tau_i)$, and the average specific expenses of the link of the *i*-th level per time unit of operation $C_i(\tau_i)$. These properties re known to be determined using the following formulae [3, 5, 6]:

$$K_i(\tau_i) = \frac{T_i^{(1)}(\tau_i)}{T_i^{(1)}(\tau_i) + T_i^{(0)}(\tau_i) + T_i^{(2)}(\tau_i)};$$

$$S_i(\tau_i) = \frac{c_i\, T_i^{(1)}(\tau_i) - c_i^0\, T_i^{(0)}(\tau_i) - c_i^p T_i^{(2)}(\tau_i)}{T_i^{(1)}(\tau_i) + T_i^{(0)}(\tau_i) + T_i^{(2)}(\tau_i)}; \qquad (1)$$

$$C_i(\tau_i) = \frac{c_i^0\, T_i^{(0)}(\tau_i) + c_i^p T_i^{(2)}(\tau_i)}{T_i^{(1)}(\tau_i)},$$

where $T_i^{(1)}(\tau_i) = \int \overline{F}_i(t)\mathrm{d}t$ is the average operation time,
$T_i^{(0)}(\tau_i) = F_i(\tau_i)\mathrm{E}\beta_i$ is the average restoration time, and
$T_i^{(2)}(\tau_i) = \overline{F}_i(\tau_i)\mathrm{E}\beta_i^p$ is the average preventive maintenance time of the element of the *i*-th level for the regeneration period $i=\overline{0,n}$, i.e., between two adjacent moments of the elements operation after the completion of restoration or maintenance.

Let us find stationary characteristics of the network by means of recurrent formulae obtained by applying the structure to the calculation formulae obtained in [6]. Ultimately, we obtain the following. The characteristics of the elements' family of the n–th level, which contains a_n outlet elements controlled by one element of the $(n-1)$–th level, are determined by the formulae

$$K^{(n)}(\tau_n) = 1 - [1 - K_n(\tau_n)]^{a_n};$$

$$S^{(n)}(\tau_n) = a_n\, S_n(\tau_n); \qquad (2)$$

$$C^{(n)}(\tau_n) = \frac{a_n\, C_n(\tau_n)\, K_n(\tau_n)}{K^{(n)}(\tau_n)}.$$

For one family of the elements of the m–th level, which contains a_m links controlled by one link of the $(m-1)$–th level, the calculation formulae are as follows ($m=\overline{1, n-1}$):

$$K^{(m)}(\tau_m,...,\tau_n) = 1 - \left[1 - \frac{K_m(\tau_m)K^{(m+1)}(\tau_{m+1},...,\tau_n)}{K_m(\tau_m)+K^{(m+1)}(\tau_{m+1},...,\tau_n)-K_m(\tau_m)K^{(m+1)}(\tau_{m+1},...,\tau_n)}\right]^{a_m};$$

$$S^{(m)}(\tau_m,...,\tau_n) = \frac{a_m\left[S_m(\tau_m)K^{(m+1)}(\tau_{m+1},...,\tau_n)+K_m(\tau_m)S^{(m+1)}(\tau_{m+1},...,\tau_n)\right]}{K_m(\tau_m)+K^{(m+1)}(\tau_{m+1},...,\tau_n)-K_m(\tau_m)K^{(m+1)}(\tau_{m+1},...,\tau_n)};$$

(3)

$$C^{(m)}(\tau_m,...,\tau_n) = \frac{a_m\left[C_m+C^{(m+1)}\right]\left[K_m+K^{(m+1)}-K_mK^{(m+1)}\right]^{a_m-1}}{\sum_{i=0}^{a_m-1}\left[K_m+K^{(m+1)}-2K_mK^{(m+1)}\right]^{i}\left[K_m+K^{(m+1)}-K_mK^{(m+1)}\right]^{a_m-i-1}}.$$

Characteristics of the entire network with branching structure are determined by the following formulae:

$$K(\tau_0,\tau_1,...\tau_n) = \frac{K_0(\tau_0)K^{(1)}(\tau_1,...,\tau_n)}{K_0(\tau_0)+K^{(1)}(\tau_1,...,\tau_n)-K_0(\tau_0)K^{(1)}(\tau_1,...,\tau_n)};$$

$$S(\tau_0,\tau_1,...,\tau_n) = \frac{S_0(\tau_0)K^{(1)}(\tau_1,...,\tau_n)+K_0(\tau_0)S^{(1)}(\tau_1,...,\tau_n)}{K_0(\tau_0)+K^{(1)}(\tau_1,...,\tau_n)-K_0(\tau_0)K^{(1)}(\tau_1,...,\tau_n)}; \quad (4)$$

$$C(\tau_0,\tau_1,...,\tau_n)=C_0(\tau_0) + \frac{a_1\left[C_1+C^{(2)}\right]\left[K_1+K^{(2)}-K_1K^{(2)}\right]^{a_1-1}}{\sum_{i=0}^{a_1-1}\left[K_1+K^{(2)}-2K_1K^{(2)}\right]^{i}\left[K_1+K^{(2)}-K_1K^{(2)}\right]^{a_1-i-1}}.$$

Optimization of the network elements preventive maintenance frequency

The stationary characteristics of the network are the functions of the age of τ_i links. Therefore, to determine the optimum frequency of preventive maintenance of the system elements, it is necessary to find the absolute extreme points of the selected criterion functions:

$$K(\tau_0,\tau_1,...\tau_n) \to \max_{\tau_i\in(0,\infty);\, i=\overline{0,n}};$$

$$S(\tau_0,\tau_1,...\tau_n) \to \max_{\tau_i\in(0,\infty);\, i=\overline{0,n}};$$

$$C(\tau_0,\tau_1,...\tau_n) \to \min_{\tau_i\in(0,\infty);\, i=\overline{0,n}}.$$

It should be noted that the link preventive maintenance does not always result in the improvement of the network stationary properties. Improvements of these characteristics takes place when the preventive maintenance average time and its costs are less than the similar indicators of elements restoration.

Examples of application of the optimization results for the operation of specific resource networks are provided.

Example 1. Electric power supply networks

Electricity is supplied from power plants, i.e., the companies supplying resource (hydro-, heat, nuclear, solar, wind, and geothermal plants) to the users' homes. Invariably, users are located at large distances from the electricity source, as well as from each other. Therefore, the electric power transmission requires a branching network of electric power supply that includes elements controlling the resource technical properties, i.e., transformers stepping-up the voltage to 1150 kV (depending on the distance). Subsequently, the overhead transmission lines transmit electric power to the central distribution substations, which are located close to the city or in its suburbs. central distribution substations step down the voltage to 220 or 110 kV, and then electric power is transmitted to the substations [7]. There, the voltage is stepped down once more (to 6–10 kV), and electric power is distributed to the transformer stations. Electricity can be transmitted to the transformer stations by underground cables rather than overhead transmission lines, which is more acceptable in the environs of a city. The voltage is stepped down one more time for the consumer (0.4 kV) (network of 380 V) at the transformer stations. Fig. 2 shows the path of electric power transmission from the source to users [8].

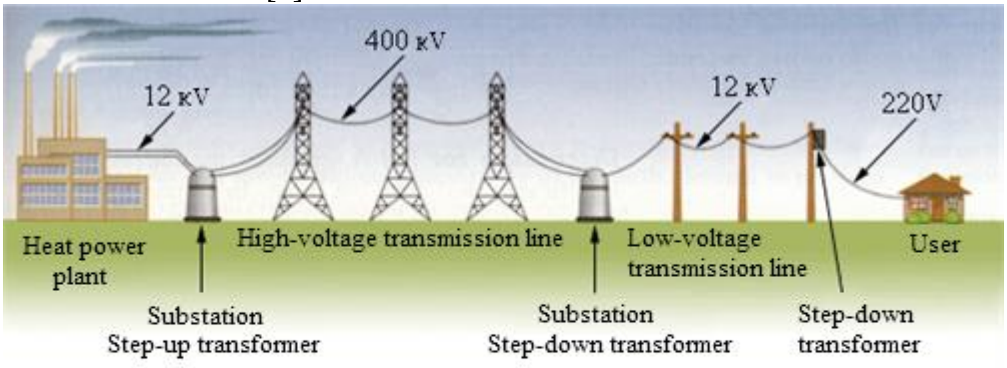

Fig. 2. The route of electric power transmission from the heat power plant to users

The major links of an electric power supply network are the following: two transformers stepping-up the voltage for electric power transmission (elements of level 1) and 36 transformers gradually stepping-down the voltage (six step-down transformers to 220 kV and 30 step-down transformers to 6–10 kV). According to the manual, maintenance of the network elements should be performed not less than $\tau_0 = 100$ day, $\tau_1 = 80$ day, $\tau_2 = 60$ day, $\tau_3 = 30$ day to ensure the required network reliability. Suppose that random values α_i, β_i, and β_i^p for the network elements have the Erlang distribution in accordance with the distribution formulae:

$$F_i(t) = 1 - e^{-\lambda_i t} \sum_{j=0}^{2} \frac{(\lambda_i t)^j}{j!}, \qquad G_i(t) = 1 - e^{-\mu_i t} \sum_{j=0}^{2} \frac{(\mu_i t)^j}{j!}, \qquad G_i^p(t) = 1 - e^{-\mu_i^p t} \sum_{j=0}^{3} \frac{(\mu_i^p t)^j}{j!}.$$

The basic characteristics of the elements are provided in Table 1.

Table 2, using S, C, marks the economic characteristics of the network operation when the recommended maintenance strategy is applied, i.e.,$\tau_0 = 100$ day, $\tau_1 = 80$ day, $\tau_2 = 60\ day, \tau_3 = 30\ day$.

Table 1

Baseline data of the network for Example 1

Level No.	Number of elements in the	Average time of fault-	Average restoration	Average preventive	Link rincome $c_i, m.u./mon$	Expenses for restoration	Preventive

	level family	free operation $M\alpha_i$, day	time $M\beta_i$, day	maintenance time $M\beta_i^p$, hour		$c_i^0, m.u./$ mon.	maintenance cost $c_i^p, m.u./$mon
0	$a_0 = 1$	200	8.6	16.0	1500	2000	500
1	$a_1 = 2$	75	6.0	14.5	1200	1600	300
2	$a_2 = 3$	54	4.6	14.1	1000	1200	200
3	$a_3 = 5$	46	3.8	12.9	1000	900	200

Table 2

Results of optimization of the network properties using different criteria for Example 1

Level No.	τ_i, day	τ_i^S, day	$S^{\max}$. $m.u./\mathrm{mon}.$	$S*$. $m.u./\mathrm{mon}.$	τ_i^C, day	$C^{\min}$. $m.u./\mathrm{mon}.$	$C*$. $m.u./\mathrm{mon}.$
0	100.0	79.6	34061.5	32882.1	44.2	623.5	1261.6
1	80.0	33.4			8.6		
2	60.0	26.2			9.8		
3	30.0	20.0			12.2		

Elements' preventive maintenance τ_i^S, τ_i^C, i=$\overline{0,3}$improves these indicators by 3.5% and 50.6%, respectively, at the point of fault-free operation reaching τ_i^S, τ_i^C, i=$\overline{0,3}$.

Example 2. Network equipment

An example of calculation of the characteristics and preventive maintenance frequency optimization for the commutational network equipment at the enterprise is as follows.

We consider the network consisting of the central exchange and six switching devices (level 1 elements). Each of them services 15 personal computers. According to the instruction, maintenance of the network elements should be performed not less than each $\tau_0 = 215$ day, $\tau_1 = 115$ day, $\tau_2 = 70$ day to ensure the required network reliability. Herein, we assume that the laws of distribution of random values α_i, β_i, and β_i^p describing the network elements have the following distribution functions:

$F_i(t) = 1 - e^{-\lambda_i t} \sum_{j=0}^{3} \frac{(\lambda_i t)^j}{j!}$, $G_i(t) = 1 - e^{-\mu_i t} \sum_{j=0}^{2} \frac{(\mu_i t)^j}{j!}$, $G_i^p(t) = 1 - e^{-\mu_i^p t} \sum_{j=0}^{4} \frac{(\mu_i^p t)^j}{j!}$.

The basic properties of the elements are provided in Table 3.

Table 4, using S, C, marks the economic characteristics of the system operation quality when the recommended maintenance strategy is used, i.e., $\tau_0 = 215$ day, $\tau_1 = 115$ day, $\tau_2 = 70$ day. The elements' preventive maintenance τ_i^S, τ_i^C, i=$\overline{0,3}$improves these indicators by 1.2% and 39.4%, respectively at the point where the time of the link fault-free operation is reached τ_i^S, τ_i^C, i=$\overline{0,3}$,.

Table 3

Baseline data of the system for Example 2

Level No.	Number of elements in the level family	Average time of fault-free operation $M\alpha_i$, day	Average restoration time $M\beta_i$, day	Average preventive maintenance time $M\beta_i^p$, hour	Link income $c_i, m.u.$mon	Expenses for restoration $c_i^0, m.u./$mon	Preventive maintenance cost $c_i^p, m.u./$mon
0	$a_0 = 1$	450	4.0	12.3	1500	2800	1300
1	$a_1 = 6$	225	3.5	11.6	1200	2400	1200
2	$a_2 = 15$	129	3.2	11.4	1000	2000	800

Table 4

Results of optimization of the network properties using different criteria for Example 2

Level No.	τ_i, day	τ_i^S, day	S^{max} $m.u./$ mon.	S $m.u./$ mon.	τ_i^C, day	C^{min} $m.u./$ mon.	$C*$ $m.u./$ mon.
0	300.0	214.3	94782.56	93574.47	159.7	1405.2	2320.26
1	200.0	107.8			80.6		
2	100.0	53.2			45.7		

Conclusions

In this article we construct iteration process to calculate the stationary reliability and economic characteristics of branching-structure networks with regard to preventive maintenance of the network elements. Examples of specific resource networks demonstrate that the most favorable selection of the element preventive maintenance frequency results in network optimization. Stationary characteristics of operation efficiency can be improved in comparison with the existing strategy: for an electric power network–the average specific income is increased by 3.5%, and the average specific expense is reduced by 50.6%; for the network equipment–the average specific income is increased by 1.2%, and the average specific expense is reduced by 39.4%.

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ОПТИМИЗАЦИЯ ПЕРИОДИЧНОСТИ ТЕХНИЧЕСКОГО ОБСЛУЖИВАНИЯ ЗВЕНЬЕВ РЕСУРСОСНАБЖАЮЩИХ СЕТЕЙ С ВЕТВЯЩЕЙСЯ СТРУКТУРОЙ*

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Рассматривается восстанавливаемая система с ветвящейся линейной структурой и конечной надежностью, в которой проводится предупредительное техническое обслуживание ее звеньев. Предполагается, что времена безотказной работы и восстановления звеньев являются случайными величинами общего вида. Строятся итерационные процессы расчета стационарных надежностных и экономических характеристик сети. Определяются оптимальные сроки проведения технического обслуживания звеньев сети в зависимости от времени их работы без отказа. На примерах работы электроснабжающей сети и сети коммутации сетевого оборудования на предприятии показано, что оптимальный выбор периодичности проведения технического обслуживания звеньев может приводить к улучшению экономических характеристик сети.

Ключевые слова: *ресурсоснабжающие сети, ветвящаяся структура, техническое обслуживание, стационарный коэффициент технического использования, средний удельный доход, средние удельные затраты, оптимизация сроков профилактики.*

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