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A note on common fixed point theorems in a bounded metric space

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Abstract

In this paper, we introduce the concept of T_β-contraction for a pair of commuting self-mappings and prove a common fixed point theorem for this type. Our results improve and extend many existing results in the literature. The paper also contains an application for non-linear integral equations.

Keywords: fixed point, T_β-contraction, $T-\alpha$-admissible, τ-distance.

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1. Introduction. The importance of fixed point theories lies in finding solutions for many problems in applied sciences such as physics, variational inequality, optimization, and many other problems in non-linear analysis.

In 1998, Jungck [1] introduced the concept of weakly compatible pairs of mappings, that is, the class of mappings such that they commute at their coincidence points. In recent years, several authors have obtained common fixed point results for different classes of mappings on various metric spaces, such as complete metric spaces.

In 2012, Samet *et al.* [2] introduced the notion of α-admissible mappings. By using this concept, the authors defined α-ψ-contractive mappings and proved

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a nice result for such mappings in the setting of metric spaces. Then Abdeljawad [3] expanded the notion of α-admissibility to a pair of functions.

Recently, the authors in [4] established a common fixed point theorem without using any additional condition on the space. Namely, we assert the following theorem.

Theorem 1.1 [4]. *Let (X, d) be a bounded complete metric space. Let f and g be two weakly compatible self-mappings of X satisfying the following conditions*

i) $g(X) \subset f(X)$,

ii) $\inf\limits_{x \neq y \in X} \{d(fx, fy) - d(gx, gy)\} > 0$.

If the range of f or g is a S-complete subspace of X, then f and g have a unique common fixed point.

Recent works in this direction can be found in [5–11].

Note that we can find a class of weakly compatible mappings satisfying

$$\inf_{x \neq y \in X} \{d(fx, fy) - d(gx, gy)\} = 0,$$

which have common fixed points, in this case, Theorem 1.1 does not work.

Inspired by the above facts, we prove a common fixed point theorem satisfying a new condition, named T_β-contraction, which improves Theorem 1.1, and present an example to illustrate the usability of our result.

Finally, on the basis of our main result, we study the existence of solutions for a system of differential equations.

2. Preliminaries. The purpose of this section is to explain some notions and results utilized in the paper.

Let (X, τ) be a topological space and $p : X \times X \to [0, \infty)$ be a function. For any $\varepsilon > 0$ and any $x \in X$, let $B_p(x, \varepsilon) = \{y \in X : p(x, y) < \varepsilon\}$.

Definition 2.1 [12]. The function p is said to be τ-distance if for each $x \in X$ and any neighborhood V of x, there exists $\varepsilon > 0$ such that $B_p(x, \varepsilon) \subset V$.

Definition 2.2 [12]. A sequence $\{x_n\}$ in a Hausdorff topological space (X, τ) is a p-Cauchy if it satisfies the usual metric condition with respect to p, in other words, if $\lim\limits_{n,m\to\infty} p(x_n, x_m) = 0$.

Definition 2.3 [12]. Let (X, τ) be a topological space with a τ-distance p.

1) X is S-complete if for every p-Cauchy sequence $\{x_n\}$, there exists x in X with $\lim p(x, x_n) = 0$.
2) X is p-Cauchy complete if for every p-Cauchy sequence $\{x_n\}$, there exists x in X such that $\lim x_n = x$ with respect to τ.
3) X is said to be p-bounded if $\sup\{p(x, y) : x, y \in X\} < \infty$.

Lemma 2.1 [12]. *Let (x_n) be a sequence in a Hausdorff topological space (X, τ) with a τ-distance p and $x, y \in X$, then*

1) *if $\{\alpha_n\} \subset \mathbb{R}^+$ a sequence converging to 0 such that $p(x, x_n) \leqslant \alpha_n$ for all $n \in \mathbb{N}$, then $\{x_n\}$ converges to x with respect to the topology τ;*
2) *$p(x, y) = 0$ implies $x = y$;*
3) *if $\lim\limits_{n\to\infty} p(x, x_n) = 0$ and $\lim\limits_{n\to\infty} p(y, x_n) = 0$, then $x = y$.*

Definition 2.4 [12]. Ψ is the class of all functions $\psi : [0, +\infty) \to [0, +\infty)$ satisfying:

i) ψ is nondecreasing;
ii) $\lim \psi^n(t) = 0$, for all $t \in [0, \infty)$.

DEFINITION 2.5 [2]. Let (X, d) be a metric space, $T : X \to X$ and $\alpha : X \times X \to \mathbb{R}^+$ be two given mappings. Then, T is called an α-admissible mapping if

$$\alpha(x, y) \geqslant 1 \implies \alpha(Tx, Ty) \geqslant 1 \text{ for all } x, y \in X.$$

LEMMA 2.2 [4]. *Let (X, d) be a metric space and $p : X \times X \to \mathbb{R}^+$ be a function defined by*

$$p(x, y) = e^{d(x,y)} - 1.$$

Then p is a τ_d-distance on X, where τ_d is the metric topology.

LEMMA 2.3 [4]. *Let (X, d) be a bounded metric space. Then the function p defined in Lemma 2.2 is a bounded τ-distance.*

LEMMA 2.4 [4]. *Let (X, d) be a complete metric space. Then the function p defined in Lemma 2.2 is a S-complete $\tau-$distance.*

3. Main results. We start our work by introducing the notion of T-α-admissible for a pair of self-mappings f and g on a metric space X.

DEFINITION 3.1. Let f, g be two self-mappings of a bounded metric space (X, d) and $\alpha : X \times X \to \mathbb{R}^+$ be a function. (f, g) is said to be a pair of T-α-admissibility if $fg = gf$ and

$$\alpha(x, y) \geqslant 1 \implies \alpha(gx, gy) \geqslant 1 \text{ and } \alpha(fx, fy) \geqslant 1,$$

for all $x, y \in X$.

THEOREM 3.1. *Let (X, τ) be a Hausdorff topological space with a τ-distance p. Suppose that X is p-bounded and $f(X)$ is S-complete. Let f and g be two self-mappings of X such that*

i) $g(X) \subset f(X)$;
ii) (f, g) is a pair of T-α-admissibility;
iii) $\alpha(x, gx) \geqslant 1$ for all $x \in X$;
iv) there exists $x_0 \in X$ such that $\alpha(x_0, g^n x_0) \geqslant 1$ and $\alpha(x, g^n x_0) \neq 0$, for all $x \in X$ and $n \in \mathbb{N}$;
v) $\alpha(x, y)p(gx, gy) \leqslant \psi(p(fx, fy))$, for all $x, y \in X$, where $\psi \in \Psi$.

Then f and g have a common fixed point.

Proof. Let $x_0 \in X$ such that $\alpha(x_0, g^n x_0) \geqslant 1$. Since $g(X) \subset f(X)$, then there exist x_1, $x_2 \in X$ such that $g(x_0) = f(x_1) = x_2$, continuing this process, we can choose $x_n \in X$ such that $x_{2n+2} = fx_{2n+1} = gx_{2n}$ for any $n \in \mathbb{N}$.

Now, consider the sequences $\{y_n\} = \{x_{2n}\}$, $\{z_n\} = \{x_{2n+1}\}$ and $\{t_n\} = \{x_{3n}\}$.

Let n, $m \in \mathbb{N}$, since (f, g) is T-α-admissible, we obtain $\alpha(x_{2n}, x_{2n+2m}) \geqslant 1$ and we have

$$\begin{aligned} p(fz_n, fz_{n+m}) &= p(fx_{2n+1}, fx_{2n+2m+1}) = p(gx_{2n}, gx_{2n+2m}) \leqslant \\ &\leqslant \alpha(x_{2n}, x_{2n+2m})p(gx_{2n}, gx_{2n+2m}) \leqslant \\ &\leqslant \psi(p(fx_{2n}, fx_{2n+2m})) \leqslant \\ &\ \ \vdots \\ &\leqslant \psi^{2n}(p(f^{2n}x_0, f^{2n}x_{2m})) \leqslant \psi^{2n}(M), \end{aligned}$$

where $M = \sup\{p(x,y) : x, y \in X\}$. As $\lim \psi^n(M) = 0$, so the sequence $\{fz_n\}$ is a p-Cauchy sequence. Since $f(X)$ is S-complete, there exists $u \in X$ such that $\lim p(fu, fz_n) = 0$.

By the same argument, it is easy to prove that $\{fy_n\}$, $\{ft_n\}$ are p-Cauchy sequences, which leads to $\lim p(fu, fz_n) = \lim p(fu, fy_n) = \lim p(fu, ft_n)$.

On the other hand we have

$$\begin{aligned}\alpha(u, y_n)p(gu, fz_n) &= \alpha(u, y_n)p(gu, fx_{2n+1}) = \\ &= \alpha(u, x_{2n})p(gu, gx_{2n}) < \\ &< p(fu, fx_{2n}) = p(fu, fy_n).\end{aligned}$$

Since $\alpha(u, y_n) = \alpha(u, g^n x_0) \neq 0$, we have

$$\lim p(gu, fz_n) = \lim p(fu, fy_n) = \lim p(fu, fz_n) = 0.$$

By Lemma 2.1, we conclude that $fu = gu$.

Suppose that $p(gu, ggu) \neq 0$, the assumption that $\alpha(u, gu) \geqslant 1$ implies

$$\begin{aligned}p(gu, ggu) &\leqslant \alpha(u, gu)p(gu, ggu) \leqslant \\ &\leqslant \psi(p(fu, ggu)) < p(gu, ggu)\end{aligned}$$

which is a contradiction.

Hence $ggu = gu$ and $fgu = gfu = ggu = gu$, it follows that that gu is a common fixed point of f and g. □

For $f = Id_X$, $\alpha(x,y) = 1$ and $\psi(t) = kt$, where $k \in [0,1)$ in Theorem 3.1, we get.

Corollary 3.1 [12]. *Let (X, τ) be a Hausdorff topological space with a τ-distance p. Suppose that X is p-bounded and S-complete. Let g be a self-mapping of X, if there exist $k \in [0,1)$ such that*

$$p(gx, gy) \leqslant kp(x,y),$$

for all $x, y \in X$. Then g has a fixed point.

Now, we introduce the notion of pair of T_β-contraction.

Definition 3.2. Let f, g be two self-mappings of a bounded metric space (X, d), (f, g) is said to be a pair of T_β-contraction if $fg = gf$, $\beta(x, gx) \leqslant 0$ and

$$\inf_{x \neq y \in X} \big\{d(fx, fy) - d(gx, gy) + \beta(x,y)\big\} > 0,$$

where $\beta : X \times X \to \mathbb{R}$ is a function satisfying

$$\beta(x,y) \leqslant 0 \implies \beta(gx, gy) \leqslant 0 \text{ and } \beta(fx, fy) \leqslant 0,$$

for all $x, y \in X$.

Our first result is the following.

Theorem 3.2. *Let (f, g) be a pair of T_β-contraction of a bounded complete metric space (X, d) such that*

i) $g(X) \subset f(X)$;
ii) *there exists* $x_0 \in X$ *such that* $\beta(x_0, g^n x_0) \leqslant 0$, *for all* $n \in \mathbb{N}$;
iii) $\beta(a,b) \leqslant \inf\limits_{x \neq y \in X} \{d(fx,fy) - d(gx,gy) + \beta(x,y)\}$, *for all* $a, b \in X$.

Then f *and* g *have a common fixed point.*

Proof. Since (f, g) is a pair of T_β-contraction, so there exists a function $\beta : X \times X \to \mathbb{R}$ such that

$$\inf_{x \neq y \in X} \{d(fx,fy) - d(gx,gy) + \beta(x,y)\} > 0.$$

We put

$$\gamma = \inf_{x \neq y \in X} \{d(fx,fy) - d(gx,gy) + \beta(x,y)\},$$

which implies that for all $x \neq y \in X$, we have

$$d(gx,gy) - \beta(x,y) \leqslant d(fx,fy) - \gamma.$$

Thus

$$\alpha(x,y) e^{d(gx,gy)} \leqslant k e^{d(fx,fy)},$$

where $k = e^{-\gamma} < 1$ and $\alpha(x,y) = e^{-\beta(x,y)}$. Then, it follows from (iii) that

$$\alpha(x,y) p(gx,gy) \leqslant k p(fx,fy),$$

for all $x, y \in X$, with $p(x,y) = e^{d(x,y)} - 1$ is the τ-distance defined in Lemma 2.2.

Finally, we deduce from Lemmas 2.2, 2.3, 2.4 and Theorem 3.1 that f and g have a common fixed point. □

Corollary 3.2 [5]. *Let* $g : X \to X$ *be a mapping of a bounded complete metric space* (X, d) *such that*

$$\inf_{x \neq y \in X} \{d(x,y) - d(gx,gy)\} > 0.$$

Then g *has a fixed point.*

Example. Let $X = \{0, 1, 2\}$ endowed with the discrete metric

$$d(x,y) = \begin{cases} 0, & \text{if } x = y, \\ 1, & \text{if } x \neq y. \end{cases}$$

Define self-mappings g and f on X by

$$g0 = g1 = g2 = 2, \quad f0 = f1 = 0, \quad f2 = 2,$$

and a function $\beta : X \times X \to \mathbb{R}$ by

$$\beta(x,y) = \begin{cases} 1, & \text{if } x, y \in \{0,1\}, \\ 0, & \text{otherwise.} \end{cases}$$

It is clear that

$$\beta(x,y) \leqslant 0 \implies \beta(gx,gy) \leqslant 0 \text{ and } \beta(fx,fy) \leqslant 0,$$

and

$$fgx = gfx = 2,$$

for all $x, y \in X$.

Also,

$$g(X) = \{2\} \subset f(X) = \{0, 2\},$$

$$\beta(x, gx) \leqslant 0, \quad \beta(2, g^n 2) \leqslant 0,$$

for all $x \in X$ and $n \in \mathbb{N}$,

$$d(fx, fy) - d(gx, gy) + \beta(x, y) = 1,$$

for all $x \neq y \in X$.

Then g and f satisfy all conditions of Theorem 3.2 and have the common fixed point 2.

Remark. Note that, in the class of commuting mappings, Theorem 3.2 is a real extension of Theorem 1.1, indeed:

$$d(f0, f1) - d(g0, g1) = 0.$$

4. Application. In this section, we will prove the existence of a common solution for two nonlinear integral equations

$$x(t) = \int_0^t k\Big(s, \int_0^s k(u, x(u)\Big)du)ds, \quad t \in [0, \tau], \tag{1}$$

$$x(t) = \int_0^t k(s, x(s))ds, \quad t \in [0, \tau], \tag{2}$$

where $x \in \mathcal{C}[0, \tau]$, the space of all continuous functions from $[0, \tau]$ into $\mathbb{R}$, with $\tau > 0$. $K : [0, \tau] \times \mathbb{R} \to \mathbb{R}$ is a continuous mapping.

Let $X = \mathcal{C}[0, \tau]$ be endowed by the metric

$$d(x, y) = \sup_{t \in [0, \tau]} |x(t) - y(t)|.$$

Define the mappings $f, g : X \to X$ as follows

$$gx(t) = \int_0^t k\Big(s, \int_0^s k(u, x(u))du\Big)ds, \quad t \in [0, \tau], \tag{3}$$

$$fx(t) = \int_0^t k(s, x(s))ds, \quad t \in [0, \tau]. \tag{4}$$

Hence, equations (1) and (2) have a common solution if and only if the mappings f and g have a common fixed point.

Theorem 4.1. *Let $g, f : X \to X$ be the mappings defined by (3) and (4) and assume the following condition is satisfied.*

There exist $M > 0$ and a function $\theta : X\times X \to \mathbb{R}$ such that for all $x, y \in X$ with $x \neq y$, we have:

$$\begin{aligned} \theta(x,y) \geqslant 0 &\implies \big|k(t,x(t)) - k(t,y(t))\big| \leqslant \frac{1}{\tau}\big(|x(t)-y(t)| - M\big), \\ \theta(x,y) < 0 &\implies \big|k(t,x(t)) - k(t,y(t))\big| \leqslant \frac{1}{\tau}\big|x(t)-y(t)\big| \end{aligned} \tag{5}$$

and

- *for all x, $y \in X$, $\theta(x,y) \geqslant 0$ implies $\theta(gx,gy) \geqslant 0$ and $\theta(fx,fy) \geqslant 0$,*
- *there exists $x_0 \in X$ such that $\theta(x_0, g^n x_0) \geqslant 0$ for all $n \in \mathbb{N}$,*
- *$\theta(x,gx) \geqslant 0$ for all $x \in X$.*

Then the functional equations (1) and (2) have a common solution.

Proof. It is easy to see that $fg(x) = gf(x)$ for all $x \in X$ and $g(X) \subset f(X)$. Let $x \neq y \in X$ and $t \in [0,\tau]$. We discuss two cases.

Case 1. If $\theta(x,y) \geqslant 0$, we have

$$\begin{aligned} |gx(t) - gx(t)| &= \left|\int_0^t k\left(s, \int_0^s k(u,x(u))du\right)ds - \int_0^t k\left(s, \int_0^s k(u,y(u))du\right)ds\right| \leqslant \\ &\leqslant \frac{1}{\tau}\int_0^\tau \left(\left|\int_0^t k(u,x(u))du - \int_0^t k(u,y(u))du\right| - M\right)ds \leqslant \\ &\leqslant d(fx,fy) - M. \end{aligned}$$

Then $d(gx,gy) \leqslant d(fx,fy) - M$.

Case 2. If $\theta(x,y) < 0$, we have

$$\begin{aligned} |gx(t) - gx(t)| &= \left|\int_0^t k\left(s, \int_0^s k(u,x(u))du\right)ds - \int_0^t k\left(s, \int_0^s k(u,y(u))du\right)ds\right| \leqslant \\ &\leqslant \frac{1}{\tau}\int_0^\tau \left(\left|\int_0^t k(u,x(u))du - \int_0^t k(u,y(u))du\right|\right)ds \leqslant d(fx,fy). \end{aligned}$$

So $d(gx,gy) \leqslant d(fx,fy)$.

Now, define $\beta : X\times X \to \mathbb{R}$ by

$$\beta(x,y) = \begin{cases} 0, & \text{if } \theta(x,y) \geqslant 0, \\ M, & \text{otherwise.} \end{cases}$$

Then, by (5) we have

$$\inf_{x\neq y\in X}\big\{d(fx,fy) - d(gx,gy) + \beta(x,y)\big\} > 0.$$

If we have $\beta(x,y) \leqslant 0$, from the definition of β we obtain that $\beta(gx,gy) \leqslant 0$ and $\beta(fx,fy) \leqslant 0$, and $\beta(x_0, g^n x_0) \leqslant 0$ for all $n \in \mathbb{N}$.

Also, we have $\beta(x,gx) \leqslant 0$ for all $x \in X$, so the pair (f,g) is a T_β-contraction.

Moreover, we have for all a, $b \in X$

$$\beta(a,b) \leqslant \inf_{x\neq y\in X}\big\{d(fx,fy) - d(gx,gy) + \beta(x,y)\big\}.$$

Finally, we conclude by Theorem 3.2 that the functional equations (1) and (2) have a common solution. □

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Замечание об общих теоремах о неподвижной точке в ограниченном метрическом пространстве

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Аннотация

Вводится концепция T_β-сжатия для пары коммутирующих самопреобразований и доказывается общая теорема о неподвижной точке для этого типа. Полученные результаты улучшают и обобщают многие известные в литературе результаты. В качестве приложения полученных результатов приводится доказательство существования общего решения для двух нелинейных интегральных уравнений.

Ключевые слова: неподвижная точка, T_β-сжатие, $T-\alpha$-допустимость, τ-расстояние.

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Авторский вклад и ответственность. Все авторы принимали участие в разработке концепции статьи; все авторы сделали эквивалентный вклад в подготовку публикации. Авторы несут полную ответственность за предоставление окончательной рукописи в печать. Окончательная версия рукописи была одобрена всеми авторами.

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