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A common fixed-point result via a supplemental function with an application

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Abstract

In this paper, we prove a novel common fixed-point theorem for two commuting mappings. This assertion is proved using the measure of non-compactness in Banach spaces. Moreover, an application is given to demonstrate the usability of the obtained results.

Keywords: fixed point, measure of non-compactness, $\mathcal{T}$-contraction.

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1. Introduction. One of the most widely used techniques for proving that a certain system of equations has a solution is to reformulate the problem as a common fixed point problem and see if the latter can be solved using this approach. Measures of non-compactness play an important role in fixed point theory and have many applications in various branches of nonlinear analysis, including differential equations, optimization, variational inequalities, etc. We refer the reader to [1–10].

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As a very important result in fixed point theory, the Darbo fixed-point theorem [11] has novel applications in both linear and nonlinear models and generalizes both the classical Schauder fixed point principle [12] and a special type of Banach contraction principle [13].

In 1998, Jungck [14] introduced the concept of weakly compatible pairs of mappings, which are mappings that commute at their coincidence points. In a recent paper [15], the authors proved a common fixed point result for this type of mappings in a bounded metric space (X, d), without assuming compactness, and satisfying the condition:

$$\inf_{x\neq y\in X}\{d(fx, fy) - d(gx, gy)\} > 0.$$

For more information, see [16–21]. Similarly, the authors in [22] showed a result for a class of condensing mappings without using the regularity of the measure μ, under the condition:

$$\inf\big\{\mu(\Omega) - \mu(T(\Omega)) : \Omega \subset X, \mu(\Omega) > 0\big\} > 0. \tag{1}$$

Our main purpose in this work is to extend condition (1) to a pair of mappings S and T under the condition:

$$\inf\big\{\mu(\Omega) - \mu(ST(\Omega)) + \beta(\Omega) : \Omega \subset X, \mu(\Omega) > 0\big\} > 0, \tag{2}$$

without using the regularity ofμ, where β is an auxiliary function to ensure the extension in the same direction (i.e., without regularity).

In this paper, we introduce the concept of a $\mathcal{T}$-contractive mapping in Banach spaces and thus a new common fixed point theorem for the new type mentioned in (2).

Finally, in the last section, we provide an existence result for a class of systems of the type:

$$\begin{cases} x(t) = k(t, x(t)), \\ x(t) = k(t, Tx(t)), \end{cases}$$

where $x \in X = \mathcal{C}([0, \tau], \mathbb{R})$ is the space of all continuous functions from $[0, \tau]$ into $\mathbb{R}$, with $\tau > 0$.

2. Preliminary. Here, we recall some facts that will be used in our main result. Let X be a real Banach space. Furthermore, we suppose that $\overline{B}$ and $\mathrm{Co}(B)$ denote the closure and convex hull of B, respectively. Moreover, let us denote by $\mathcal{M}_X$ the family of all nonempty and bounded subsets of X, and by $\mathcal{N}_X$ its subfamily consisting of all relatively compact sets.

Definition 1 [23]. A map $\mu : \mathcal{M}_X \to [0, +\infty)$ is called a measure of non-compactness defined on X if it satisfies the following properties:

(i) The family $\ker\mu = \{B \in \mathcal{M}_X : \mu(B) = 0\}$ is nonempty and $\ker\mu \subset \mathcal{N}_X$;
(ii) $A \subset B \Rightarrow \mu(A) \leqslant \mu(B)$;
(iii) $\mu(B) = \mu(\overline{B}) = \mu(\mathrm{Co}(B))$;
(iv) $\mu(\lambda A + (1-\lambda)B) \leqslant \lambda\mu(A) + (1-\lambda)\mu(B)$ for all $\lambda \in [0, 1]$ and A, $B \in \mathcal{M}_X$;
(v) If $\{B_n\}$ is a decreasing sequence of nonempty, closed, and bounded subsets of X with $\lim \mu(B_n) = 0$, then $B_\infty = \cap_n B_n \neq \emptyset$.

We distinguish important classes of measures of non-compactness.

Definition 2 [23]. Let μ be a measure of non-compactness in a Banach space X. The measure μ is homogeneous if $\mu(\lambda A) = |\lambda|\mu(A)$ for all $\lambda \in \mathbb{R}$. If the measure μ satisfied the condition $\mu(A + B) \leqslant \mu(A) + \mu(B)$, it is called subadditive. A measure μ that is both homogeneous and subadditive is said to be sublinear.

Definition 3 [23]. We say that a measure of non-compactness μ has the maximum property if $\mu(A \cup B) = \max\{\mu(A), \mu(B)\}$.

Definition 4 [23]. A sublinear measure of non-compactness μ that has the maximum property and $\ker \mu = \mathcal{N}_X$ is called a regular measure.

Theorem 1 [12]. Let C be a closed, convex subset of a Banach space X. Then every compact, continuous map $T : C \to C$ has at least one fixed point.

Theorem 2 (Darbo's Theorem) [11]. Let C be a nonempty, bounded, closed and convex subset of a Banach space X, and let $T : C \to C$ be a continuous mapping. Assume that there exists a constant $k \in [0, 1)$ such that:

$$\mu(T(\Omega)) \leqslant k\mu(\Omega)$$

for any subset Ω of C. Then, T has at least one fixed point. Here, μ is an arbitrary measure of non-compactness.

Lemma 1 [22]. If μ is a measure of non-compactness, then $\nu = e^{\mu} - 1$ is a measure of non-compactness.

We present generalizations of Darbo's theorem.

Theorem 3 [24]. Suppose that C is a nonempty, bounded, closed, and convex subset of a Banach space X and $T : C \to C$ a continuous mapping. If for any nonempty subset Ω of C with $\mu(\Omega) > 0$ we have:

$$\mu(T(\Omega)) < \mu(\Omega).$$

Then, T has at least one fixed point in C. Here, μ is a regular measure of non-compactness in X.

Theorem 4 [25]. Let X be a Hausdorff complete and locally convex space, whose topology is defined by a family of semi-norms $\mathcal{P}$. Let C be a convex closed bounded subset of X, let I be a set of indices, and let $\{T_i\}_{i\in I}$ and S be two continuous functions from C into C such that:

(i) For any $i \in I$, T_i commutes with S;
(ii) For any $\Omega \subset C$ and $i \in I$, we have $T_i(\mathrm{Co}(\Omega)) \subset \mathrm{Co}(T(\Omega))$;
(iii) There exists $k \in (0, 1)$ such that for any $\Omega \subset C$

$$\mu(S(\Omega))(p) \leqslant k \sup_{i\in I} \mu(T_i(\Omega))(p), \quad p \in \mathcal{P};$$

(iv) For any $i \in I$, T_i is a commuting family.

Then, T_i and S have a common fixed point.

3. The main theorem. In this section, we prove our main theorem. To this end, we introduce a definition and establish a lemma.

Definition 5. Let X be a bounded Banach space and $T : X \to X$ be a mapping. T is called $\mathcal{T}$-contractive if:

$$\inf\bigl\{\mu(\Omega) - \mu(T(\Omega)) + \beta(\Omega) : \Omega \subset X, \mu(\Omega) > 0\bigr\} > 0.$$

Where $\beta : \mathcal{M}_X \to \mathbb{R}$ is an arbitrary function.

LEMMA 2. Let X be a bounded Banach space and S, $T : X \to X$ be two continuous commuting mappings. Suppose that there exists a function $\alpha : \mathcal{M}_X \to [0, +\infty)$ such that:

(i) There exists a nonempty set $\Omega_0 \subset X$ such that $\alpha(\Omega_0) = 1$;
(ii) T is a linear mapping;
(iii) There exists $k \in (0, 1)$ such that for any $\Omega \subset X$ we have

$$\alpha(\Omega)\mu(ST(\Omega)) \leqslant k\mu(\Omega).$$

Then, S and T have at least one common fixed point. Where μ is a sublinear measure of non-compactness.

Proof. Let $\emptyset \neq \Omega \subset X$. Consider the operator A_Ω defined on X by

$$A_\Omega(x) = k\alpha(\Omega)ST(x) + (1 - k)T(x).$$

It is clear that the operator A_Ω commutes with T. By using sublinearity of μ and hypothesis (iii), we obtain:

$$\begin{aligned}\mu\big(A_\Omega(\Omega)\big) &\leqslant k\alpha(\Omega)\mu(ST(\Omega)) + (1 - k)\mu(T(\Omega)) \leqslant \\ &\leqslant k^2\mu(\Omega) + (1 - k)\mu(T(\Omega)) \leqslant \\ &\leqslant (k^2 + 1 - k)\sup\{\mu(\Omega), \mu(T(\Omega))\}.\end{aligned}$$

Since $k \in (0, 1)$, we have $k^2 < k \Rightarrow k^2 + 1 - k < k + 1 - k = 1$. It follows by applying Theorem 4 that $\mathcal{F} = \{x \in X : A_\Omega(x) = T(x) = x\} \neq \emptyset$ for all Ω.

Now, let $x \in \mathcal{F}$, we have $A_\Omega(x) = k\alpha(\Omega)ST(x) + (1 - k)T(x) = T(x) = x$, which leads to $\alpha(\Omega)S(x) = x = T(x)$ for all Ω. Since $\alpha(\Omega_0) = 1$, we deduce that S and T have at least one common fixed point. □

REMARKS. We present some remarks about Lemma 2:

– For $\alpha(\Omega) = 1$ for all Ω and $T = Id_X$, we obtain a new extension of Darbo's theorem (Theorem 2).
– It is well known that if the operator ST has a fixed point, then S and T do not necessarily have a fixed point or a common fixed point. In comparison, our lemma ensures the existence of common fixed points of S and T whenever ST has a fixed point.

Now, we are ready to prove the main theorem of this paper, which can be considered as a real extension of [22, Theorem 3.2].

THEOREM 5 (MAIN THEOREM). Let X be a bounded Banach space, and let S, $T : X \to X$ be two continuous commuting mappings such that:

(i) There exists a nonempty set $\Omega_0 \subset X$ such that $\beta(\Omega_0) = 0$;
(ii) T is a linear mapping;
(iii) ST is a $\mathcal{T}$-contractive operator;
(iv) $\beta(B) \leqslant \inf\big\{\mu(\Omega) - \mu(ST(\Omega)) + \beta(\Omega) : \Omega \subset X, \mu(\Omega) > 0\big\}$ for all $B \subset X$.

Then, S and T have at least one common fixed point. Here, μ is a sublinear measure of non-compactness.

Proof. Letting:

$$M = \inf\big\{\mu(\Omega) - \mu(ST(\Omega)) + \beta(\Omega) : \Omega \subset X, \mu(\Omega) > 0\big\}.$$

So, we have:

$$\mu(ST(\Omega)) - \beta(\Omega) \leqslant \mu(\Omega) - M,$$

for all $\Omega \subset X$, with $\mu(\Omega) > 0$.

Hence,

$$\alpha(\Omega)e^{\mu(ST(\Omega))} \leqslant ke^{\mu(\Omega)},$$

where

$$k = e^{-M} < 1 \quad \text{and} \quad \alpha(\Omega) = e^{-\beta(\Omega)}.$$

Thus, by using (iv), we get:

$$\alpha(\Omega)\nu(ST(\Omega)) \leqslant k\nu(\Omega),$$

where ν is the measure of non-compactness proved in Lemma 1.

According to Lemma 2, we deduce that S and T have at least one common fixed point. □

Example. Let $X = \mathbb{R}$ and $C = [-1, 1] \subset X$ endowed with the usual metric $d(x, y) = |x - y|$. Define two self-mappings S, T on C by:

$$Sx = x/3 \quad \text{and} \quad Tx = x/2.$$

We have:

$$STx = x/6 = TSx.$$

Consider the measure of non-compactness of the norm [23], defined on X by:

$$\mu(\Omega) = \sup_{x\in\Omega} \|x\|.$$

It is clear that μ is a sublinear measure of non-compactness and not regular, because:

$$\ker \mu = \{0\} \neq \mathcal{N}_X.$$

Let β be the function defined by:

$$\beta(\Omega) = \begin{cases} 0, & \text{if } \Omega = \{0\}, \\ 1 - 5\|\Omega\|/6, & \text{otherwise.} \end{cases}$$

Let $\Omega \subset X$ such that $\mu(\Omega) > 0$. We have:

$$\mu(\Omega) - \mu(ST(\Omega)) + \beta(\Omega) = 1.$$

Then,

$$\inf\big\{\mu(\Omega) - \mu(ST(\Omega)) + \beta(\Omega) : \Omega \subset X, \mu(\Omega) > 0\big\} \geqslant \beta(B),$$

for all $B \subset X$.

Hence, all assumptions of Theorem 5 are satisfied, and $0 = S0 = T0$.

Remark. The operators S, T defined in the above example are such that $\mu(ST(\Omega)) \leqslant \mu(\Omega)$. Thus, our result guarantees the existence of a class of contractions of this type without using regularity.

COROLLARY [22]. Let X be a bounded Banach space, and let $T : X \to X$ be a continuous mapping such that:

$$\inf\{\mu(\Omega) - \mu(T(\Omega)) : \Omega \subset X, \mu(\Omega) > 0\} > 0.$$

Then, T has at least one fixed point.

4. Application. In this section, we investigate the existence of solutions for the system of equations:

$$\begin{cases} x(t) = k(t, x(t)), \\ x(t) = k(t, Tx(t)), \end{cases} \tag{3}$$

where $x \in \mathcal{C}([0, \tau], \mathbb{R})$ is the space of all continuous functions from $[0, \tau]$ into $\mathbb{R}$, with $\tau > 0$.

The function $k : [0, \tau] \times \mathbb{R} \to \mathbb{R}$ is continuous, and $T : X \to X$ is a linear continuous mapping.

Let $X = \mathcal{C}([0, \tau], \mathbb{R})$ be equipped with the norm $\|\cdot\| : X \to \mathbb{R}^+$ defined by:

$$\|x\| = \sup_{t\in[0,\tau]} |x(t)|.$$

Let us now consider the mapping $S : X \to X$ defined as follows:

$$Sx(t) = k(t, x(t)),$$

for all $x \in X$.

Therefore, (3) has a solution if and only if S and T have a common fixed point.

Under the above assumptions, we have the following theorem.

THEOREM 6. Assume that there exist $M > 0$ and a function $\theta : [0, \tau] \times \mathbb{R} \to \mathbb{R}$ such that for any $t \in [0, \tau]$ and $x \in X$, we have:

$$\theta(t, x(t)) \geqslant 0 \implies k(t, T(x(t))) \leqslant x(t) - M,$$
$$\theta(t, x(t)) < 0 \implies k(t, T(x(t))) \leqslant x(t),$$

and

$$T(k(t, x(t))) = k(t, T(x(t))), \tag{4}$$

for any $(t, x) \in [0, \tau] \times X$.

Then, S and T have at least one common fixed point.

Proof. Note that (4) implies that S and T are commuting mappings.

Now, let $t \in [0, \tau]$, $\Omega \subset X$, and $x \in \Omega$. We discuss two cases:

Case 1: If $\theta(t, x(t)) \geqslant 0$, we have:

$$ST(x)(t) = k(t, Tx(t)) \leqslant x(t) - M \leqslant \|x\| - M \leqslant \sup_{x\in\Omega} \|x\| - M.$$

Case 2: If $\theta(t, x(t)) < 0$, we have:

$$ST(x)(t) = k(t, Tx(t)) \leqslant x(t) \leqslant \|x\| \leqslant \sup_{x\in\Omega} \|x\|.$$

By considering:

$$\beta(\Omega) = \begin{cases} M, & \text{if } \theta(t, x(t)) < 0, \\ 0, & \text{if } \theta(t, x(t)) \geqslant 0, \end{cases}$$

we obtain:

$$\inf\big\{\|\Omega\| - \|ST(\Omega)\| + \beta(\Omega) : \Omega \subset X, \|\Omega\| > 0\big\} \geqslant \beta(B),$$

for all $B \subset X$, where $\|\cdot\|$ is the measure of non-compactness of the norm [23].

By applying Theorem 5, we deduce that S and T have a common fixed point. □

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Результат об общей неподвижной точке, полученный через вспомогательную функцию, и его применение

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Аннотация

Представлена новая теорема об общей неподвижной точке для двух отображений, обладающих свойством коммутативности. Для доказательства теоремы используется мера некомпактности в банаховых пространствах. В заключительной части приводится пример практического применения полученной теоремы.

Ключевые слова: неподвижная точка, мера некомпактности, $\mathcal{T}$-сжимающее отображение.

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