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TWO-DIMENSIONAL CONVECTION OF AN INCOMPRESSIBLE VISCOUS FLUID WITH THE HEAT EXCHANGE ON THE FREE BORDER

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Abstract

The exact stationary solution of the boundary-value problem that describes the convective motion of an incompressible viscous fluid in the two-dimensional layer with the square heating of a free surface in Stokes's approach is found. The linearization of the Oberbeck–Boussinesq equations allows one to describe the flow of fluid in extreme points of pressure and temperature. The condition under which the counter-current flows (two counter flows) in the fluid can be observed, is introduced. If the stagnant point in the fluid exists, six non-closed whirlwinds can be observed.

Keywords: exact solution, Newton–Rikhsman law, thermal convection, Oberbeck–Boussinesq equations, counter-current flow.

Introduction. The research of the convective flows of an incompressible viscous fluid is caused by a considerable drop of temperature in a wide range of processes connected with the deformation of dissipative environments. The convection induced by the non-uniform heating of incompressible and compressed substances is the most widespread kind of gas dynamics and fluid flows in the Universe. The convective motion of the fluid heated from below in a two-dimensional horizontal layer is one of the most popular subjects of studying. The first example of self-organization of the nonlinear phenomena is Rayleigh–Benard convection. The choice of the two-dimensional layer as the abstract mathematical object is mainly connected with the fact that this geometry can be quite easily realized in an experiment and provides certain conveniences for taking thermal and optical measurements. The two-dimensional horizontal layer is a matter of great

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importance in connection with the theory of convective stability applications in meteorology, geophysics and astrophysics [1, 2].

The first exact solution of the natural convection for a two-dimensional layer of fluid with a constant gradient of the temperature on the borders of the layer is described in the article [3], where two cases of boundary conditions for the velocity are considered. A brief survey of the articles and reviews, in which the possibilities of finding exact solutions to free convection equations and the research of stability of these solutions were studied [4–10], is given in the papers [1, 2] and the reference lists.

In the work [5], a method for the description of heat exchange with the localized parabolic heating of the border is offered. This method is offered in the context of the exact solutions class in which velocities linearly depend on horizontal coordinates, and fields of the pressure and temperature are distributed under the square law [2, 6, 11]. In this case, finding the exact solution is reduced to solving the nonlinear system of one-dimensional evolution heat conduction equations like heat conductivity, and stationary equations of the gradient type. When finding the exact solutions, which describe the convective fluid flow [1–10], an ideal heat transfer on the borders is assumed. The aim of the present work is to investigate the motion of a viscous incompressible fluid with heat exchange on the upper boundary.

1. Mathematical model and main equations. The plane layer stationary convection of a viscous incompressible fluid (Fig. 1) can be presented by the system of Oberbeck–Boussinesq equations:

$$\begin{aligned}
 V_x \frac{\partial V_x}{\partial x} + V_z \frac{\partial V_x}{\partial z} &= -\frac{\partial P}{\partial x} + \nu \left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial z^2} \right), \\
 V_x \frac{\partial V_z}{\partial x} + V_z \frac{\partial V_z}{\partial z} &= -\frac{\partial P}{\partial z} + \nu \Delta V_z + g\beta T, \\
 \frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} &= 0, \\
 V_x \frac{\partial T}{\partial x} + V_z \frac{\partial T}{\partial z} &= \chi \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right).
 \end{aligned}
 \tag{1}$$

In the system of equations (1), the following designations are introduced: V_x , V_z are the velocities that are parallel to x axis and z axis, respectively; P is the pressure deviation from hydrostatic pressure divided by the constant average fluid density ρ ; T is a deviation from the average temperature; ν , χ , β are the dissipative coefficients of the kinematic viscosity, heat diffusivity and thermal

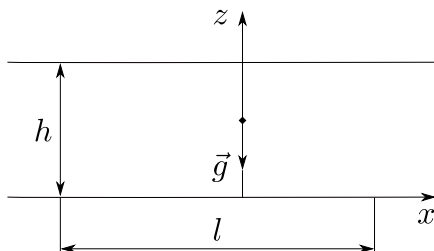


Figure 1. Schematic view of the model problem

expansion of fluid, respectively; g is the acceleration of gravity;

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

is the two-dimensional Laplace operator written down in the Cartesian orthogonal coordinate system.

We search the stationary solution of the system (1) in the form of [2, 6, 11]:

$$\begin{aligned} V_x &= xu(z), & V_z &= w(z), \\ T &= T_0(z) + T_{11}(z)\frac{x^2}{2}, & P &= P_0(z) + P_{11}(z)\frac{x^2}{2}. \end{aligned} \quad (2)$$

Note that if the temperature T is formally substituted with the concentration function C in the system (1) the solutions (2) are also true for the concentration convection. β is the coefficient of concentration fluid extension, in this case.

We substitute the class of solutions (2) in the system of equations (1) and receive the following nonlinear system defining unknown functions u , w , T_0 , T_{11} , P_0 :

$$\begin{aligned} \nu \frac{d^2 u}{dz^2} &= u^2 + w \frac{du}{dz} + P_{11}, & \frac{dw}{dz} &= -u, \\ \frac{dP_0}{dz} &= \nu \frac{d^2 w}{dz^2} + g\beta T_0 - w \frac{dw}{dz}, \\ \frac{dP_{11}}{dz} &= g\beta T_{11}, & w \frac{dT_0}{dz} &= \chi \left(T_{11} + \frac{d^2 T_0}{dz^2} \right), \\ 2uT_{11} + w \frac{dT_{11}}{dz} &= \chi \frac{d^2 T_{11}}{dz^2}. \end{aligned} \quad (3)$$

We reduce the system of ordinary ninth-order differential equations (3) to the dimensionless form. We introduce the following characteristic geometric scale values as basis [2, 6]: h is the transverse characteristic size, l is the axial characteristic size. The units of measurements are: Θ is for the temperature, V_z and V_x are for the velocities $\frac{g\beta\Theta h^4}{\nu l^2}$ and $\frac{g\beta\Theta h^3}{\nu l}$ respectively. Thus, the accounting of geometrical anisotropy of the task results in the formation of a flow and availability of two scales for the velocities, since

$$\frac{g\beta\Theta h^4}{\nu l^2} = \delta \frac{g\beta\Theta h^3}{\nu l};$$

the pressure for an incompressible fluid divided by the constant density is $g\beta\Theta h$; $\delta = h/l$.

We write down the system (3) in the dimensionless form:

$$\begin{aligned} \frac{dP_{11}}{dz} &= T_{11}, & \frac{dw}{dz} &= -u, & \text{Gr } \delta^2 \left(2uT_{11} + w \frac{dT_{11}}{dz} \right) &= \frac{1}{\text{Pr}} \frac{d^2 T_{11}}{dz^2} \\ \text{Gr } \delta^2 w \frac{dT_0}{dz} &= \frac{1}{\text{Pr}} \left(\delta^2 + \frac{d^2 T_0}{dz^2} \right), & \text{Gr } \delta^2 w \frac{dw}{dz} &= -\frac{dP_0}{dz} + \delta^2 \frac{d^2 w}{dz^2} + T_0, \\ \text{Gr } \delta^2 \left(u^2 + \delta \frac{du}{dz} \right) &= -P_{11} + \delta^2 \frac{d^2 u}{dz^2}. \end{aligned} \quad (4)$$

Both dimensionless and dimensional (3) variables and functions in the system (4) are designated by the same symbols. Only dimensionless variables are used further on. In this work

$$\text{Gr} = \frac{g\beta\Theta h^5}{\nu^2 l^2}$$

is the modified Grashof number.

We find the solution of the system (4) in the extremum points of the temperature. For this purpose we linearize the equations (3). The linearization of the nonlinear system (4) is possible when $\text{Gr} \delta^2 \ll 1$. In this case, the received exact solutions are fair with any Grashof number they satisfy the assessed value $\text{Gr} \ll 1/\delta^2$ where $\text{Gr} \in (0; 1/\delta^2)$. It should be noted that with an isotropic geometry ($\delta = 1$), we receive the classical criterion of reducing the equations (1) to Stokes equation [11].

The linearized system (3) in the dimensionless form is written as:

$$\begin{aligned} \frac{d^2 T_{11}}{dz^2} = 0, \quad \frac{dP_{11}}{dz} = T_{11}, \quad \frac{d^2 u}{dz^2} = P_{11}, \quad \frac{dw}{dz} = -u, \\ \frac{d^2 T_0}{dz^2} = -\delta^2 T_{11}, \quad \frac{dP_0}{dz} = T_0 + \delta^2 \frac{d^2 w}{dz^2}. \end{aligned} \quad (5)$$

The equations in the system (5) are written out in the order, the integration of equations is effected.

2. Boundary conditions and exact solutions. The system of the linear differential equations (5) has the exact polynomial solution:

$$\begin{aligned} T_{11} = C_1 z + C_2, \quad P_{11} = C_1 \frac{z^2}{2!} + C_2 z + C_3, \\ u = C_1 \frac{z^4}{4!} + C_2 \frac{z^3}{3!} + C_3 \frac{z^2}{2} + C_4 z + C_5, \\ w = -C_1 \frac{z^5}{5!} - C_2 \frac{z^4}{4!} - C_3 \frac{z^3}{3!} - C_4 \frac{z^2}{2!} - C_5 z + C_6, \\ T_0 = -C_1 \delta^2 \frac{z^3}{3!} - C_2 \delta^2 \frac{z^2}{2!} + C_7 z + C_8, \\ P_0 = -2C_1 \delta^2 \frac{z^4}{4!} - 2C_2 \delta^2 \frac{z^3}{3!} + (C_7 - \delta^2 C_3) \frac{z^2}{2} + (C_8 - \delta^2 C_4) z + C_9. \end{aligned} \quad (6)$$

We formulate boundary conditions for finding the constants of integration—the coefficients of polynomials (6). The heat source is introduced on the lower solid bound:

$$T = A + Bx^2 = \frac{\Theta}{2}(1 - x^2)$$

with dimensionless variables, it is written as:

$$T = \frac{1}{2}(1 - x^2).$$

The boundary conditions for defining the constants of integration of dimensionless solutions (5) can be written as:

$$z = -1 : w = u = 0, \quad T_{11} = -1, \quad T_0 = \frac{1}{2}; \quad (7)$$

$$z = 0 : w = 0, \quad P_{11} = 0, \quad P_0 = 1, \quad \frac{dT_0}{dz} = -\text{Bi} T_0, \quad \frac{dT_{11}}{dz} = -\text{Bi} T_{11}. \quad (8)$$

The no-slip condition is set on the lower bound ($z = -1$), and the heat exchange under Newton–Rikhman law is set on the free plane layer bound [12]. Bi is Biot number [12]. It is obvious that the integration constants defining the structure of the exact solution (6) by boundary conditions (7) and (8) can be as follows:

$$C_3 = C_6 = 0, \quad C_9 = 1.$$

$$C_1 = \frac{\text{Bi}}{\text{Bi} + 1}, \quad C_2 = -\frac{1}{\text{Bi} + 1}, \quad C_4 = \frac{2(4\text{Bi} + 15)}{5!(\text{Bi} + 1)}, \quad C_5 = \frac{3\text{Bi} + 10}{5!(\text{Bi} + 1)},$$

$$C_7 = \frac{\delta^2 \text{Bi}(\text{Bi} + 3) - 3\text{Bi}(\text{Bi} + 1)}{3!(\text{Bi} + 1)^2}, \quad C_8 = \frac{3(\text{Bi} + 1) - \delta^2(\text{Bi} + 3)}{3!(\text{Bi} + 1)^2}.$$

Let's substitute (6) with the boundary conditions (7) and (8) into the equations (2), then the expressions of hydrodynamic fields are written in the following form:

$$V_x = x \left(\frac{\text{Bi} z^4}{4!(\text{Bi} + 1)} - \frac{z^3}{3!(\text{Bi} + 1)} + \frac{2(4\text{Bi} + 15)z}{5!(\text{Bi} + 1)} + \frac{3\text{Bi} + 10}{5!(\text{Bi} + 1)} \right),$$

$$V_z = - \left(\frac{\text{Bi} z^5}{5!(\text{Bi} + 1)} + \frac{z^4}{4!(\text{Bi} + 1)} - \frac{(4\text{Bi} + 15)z^2}{5!(\text{Bi} + 1)} - \frac{(3\text{Bi} + 10)z}{5!(\text{Bi} + 1)} \right),$$

$$T = -\frac{\text{Bi} \delta^2 z^3}{3!(\text{Bi} + 1)} + \frac{\delta^2 z^2}{2!(\text{Bi} + 1)} + \frac{\delta^2 \text{Bi}(\text{Bi} + 3) - 3\text{Bi}(\text{Bi} + 1)}{3!(\text{Bi} + 1)^2} z +$$

$$+ \frac{3(\text{Bi} + 1) - \delta^2(\text{Bi} + 3)}{3!(\text{Bi} + 1)^2} + \frac{x^2(\text{Bi} z - 1)}{2!(\text{Bi} + 1)},$$

$$P = -\frac{2\text{Bi} \delta^2 z^4}{4!(\text{Bi} + 1)} + \frac{2\delta^2 z^3}{3!(\text{Bi} + 1)} + \frac{2(\delta^2 \text{Bi}(\text{Bi} + 3) - 3\text{Bi}(\text{Bi} + 1))z^2}{4!(\text{Bi} + 1)^2} +$$

$$+ \left(\frac{3(\text{Bi} + 1) - \delta^2(\text{Bi} + 3)}{3!(\text{Bi} + 1)^2} - \frac{2(4\text{Bi} + 15)\delta^2}{5!(\text{Bi} + 1)} \right) z +$$

$$+ 1 + \left(\frac{\text{Bi} z^2}{2!(\text{Bi} + 1)} - \frac{z}{\text{Bi} + 1} \right) \frac{x^2}{2}.$$

3. Analysis of the plane convective motion of the fluid. Since the solutions (6) of the system of equations (5) are polynomial, the analysis of flows can always be converted to the solution of the generalized Raus–Gurvits problem. We consider the characteristic properties of the velocities depending on the value of Biot number. Considering the boundary conditions (6) and (7), we find out that the variety of the function's values can be presented in the following form:

$$u = (z + 1)f(z) = (z + 1) \left(\frac{\text{Bi} z^3}{4!(\text{Bi} + 1)} - \frac{(\text{Bi} + 4)z^2}{4!(\text{Bi} + 1)} + \right.$$

$$+ \frac{(\text{Bi} + 4)z}{4!(\text{Bi} + 1)} + \frac{3 \text{Bi} + 10}{5!(\text{Bi} + 1)} \Bigg).$$

The function f is a cubic polynomial with the coefficients depending on number Bi under the linear-fractional law. We investigate the spectral properties of a polynomial f on the domain of the definition. It is known that the function contains an odd number of zero values (the quantity of stagnant points of the flow) when an inequality is being solved:

$$f(-1)f(0) < 0$$

and an even number when the opposite inequality is being solved. Thus, the function f has the only one solution in the interval with

$$\text{Bi} \in (-\infty; -10/3) \cup (-5/2; -1) \cup (-1; +\infty).$$

When $\text{Bi} = 0$, the assumed cubic polynomial degenerates into a linear function. Two solutions are possible in the one case with $\text{Bi} \in (-10/3; -5/2)$. Let's consider graphs of the stream function and vorticity when $\text{Bi} = -2.7$ (Fig. 2) and $\text{Bi} = -2$ (Fig. 3). The expressions are assumed for the stream function of the velocity:

$$\psi = -xz(z + 1)^2 \left(-\frac{\text{Bi} z^2}{5!(\text{Bi} + 1)} + \frac{(2\text{Bi} + 5)z}{5!(\text{Bi} + 1)} - \frac{3\text{Bi} + 10}{5!(\text{Bi} + 1)} \right)$$

and vorticity:

$$\Omega_y = x \left(\frac{\text{Bi} z^3}{3!(\text{Bi} + 1)} - \frac{z^2}{2!(\text{Bi} + 1)} + \frac{2(4\text{Bi} + 15)}{5!(\text{Bi} + 1)} \right).$$

Investigating the localization of the polynomial f roots with the values of numbers $\text{Bi} = -10/3$ and $\text{Bi} = -5/2$, we find out that the function takes the zero

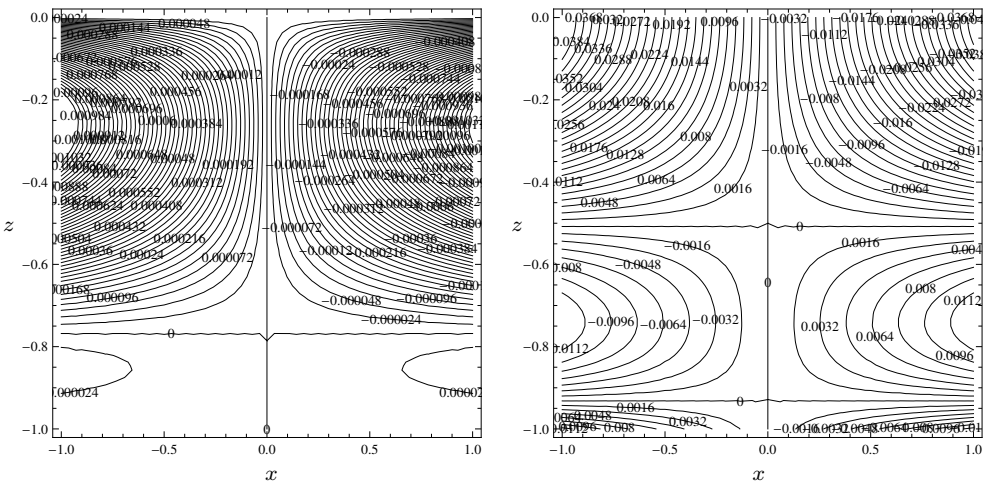


Figure 2. Isolines of the current function (left) and the function of vorticity (right) with $\text{Bi} = -2.7$

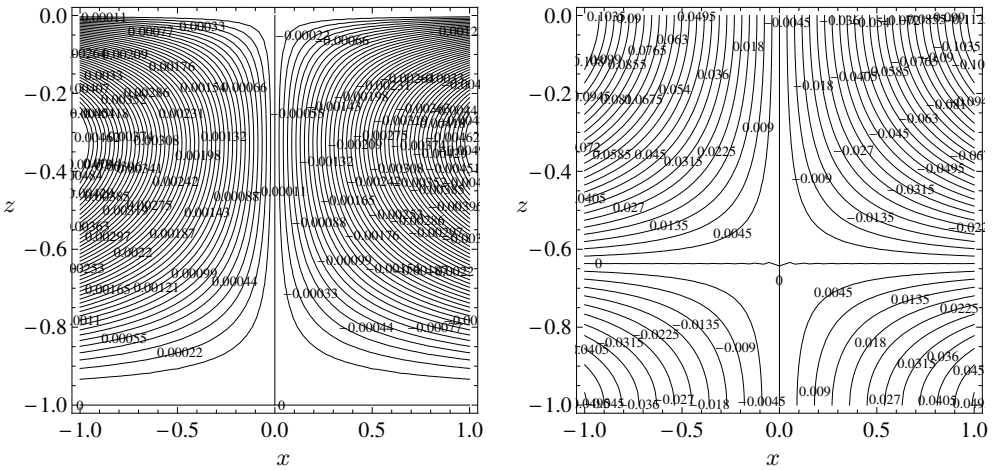


Figure 3. Isolines of the current function (left) and the function of vorticity (right) with $Bi = -2$

values in the intervals of definition $z = 0$, $z = -(1 + \sqrt{21})/10$ and $z = -1$, $z = (1 - \sqrt{6})/5$ respectively.

Let's note that for the velocity there is a point different from $z = -1$ in which the velocity does not depend on the values of Biot number. To prove this statement, we take two Biot numbers not equal among one another and substitute them in the expression for a gradient u . Subtracting one polynomial function from another, we receive the equation for the definition of this point within the interval $[-1; 0]$:

$$5z^4 + 20z^3 - 22z - 7 = 0.$$

Calculating the roots of this equation, we receive the point in which the velocity does not depend on Biot number, and the value of velocity is:

$$z = -0.370734, \quad u(-0.370734) = 0.01929.$$

Now we consider the characteristic properties of the velocity parallel to an axis of z -coordinates. Its value as well as the value of velocity V_x does not depend on parameter δ . If we consider the boundary conditions (7) and (8), we find out that the multiplicity of the function V_z values can be presented in the following form:

$$w = z(z + 1)^2 f(z) = z(z + 1)^2 \left(-\frac{Bi z^2}{5!(Bi + 1)} + \frac{(2Bi + 5)z}{5!(Bi + 1)} - \frac{3Bi + 10}{5!(Bi + 1)} \right).$$

The function f is a square polynomial with the coefficients depending on Bi under the fractional-linear function. The function f has the only one root within the interval $[-1; 0]$ with $Bi \in (-10/3; -5/2)$. If $Bi = 0$, the initial square polynomial degenerates into a linear function. When an inequality $f(-1)f(0) > 0$ is being solved in the interval $[-1; 0]$ no roots are found. Analyzing the arrangement of square polynomial roots f with the values of numbers $Bi = -10/3$ and $Bi = -5/2$, we find out that the function takes these values in the intervals of definition $z = 0$ and $z = -1$, respectively.

Let's consider the additive components of the temperature. The square additive component T_{11} is distributed under the linear law. The variety of the function T_{11} values for $z \in [-1; 0]$ takes negative values with Biot $\text{Bi} \in [-1; 0)$. The received T_{11} solution takes up the constant value when $\text{Bi} = 0$: $T_{11} = -1$. The additive component is monotonously increasing with $\text{Bi} \in (-\infty; -1) \cup (0; +\infty)$, otherwise it is monotonously decreasing. Function T_{11} vanishes in the interval of the definition in the point $z = 1/\text{Bi}$ with $\text{Bi} \in [-1; 0)$.

Now let's consider the properties of the background temperature and pressure. The background temperature can be presented in the following form:

$$T_0 = \text{Bi} C_2 \delta^2 \frac{z^3}{3!} - C_2 \delta^2 \frac{z^2}{2!} - \text{Bi} C_8 z + C_8.$$

The existence of an odd number of solutions is equivalent to an inequality being solved:

$$T_0(-1)T_0(0) = C_8 \left(C_8(\text{Bi} + 1) - \frac{C_2 \delta^2}{3!}(\text{Bi} + 3) \right) < 0.$$

Due to the boundary conditions (7) and (8), the temperature can be presented in another form:

$$\begin{aligned} T_0 &= \frac{1}{2} + \frac{(z + 1)}{3!(\text{Bi} + 1)^2} (-z^2 \text{Bi} \delta^2 (\text{Bi} + 1) + z \delta^2 (\text{Bi} + 1)(\text{Bi} + 3) - \\ &\quad - 3 \text{Bi}(\text{Bi} + 1) - \delta^2 (\text{Bi} + 3)) = \\ &= \frac{1}{2} + \frac{(z + 1)}{3!(\text{Bi} + 1)^2} g(z). \end{aligned}$$

Let's investigate the function $g(z)$. The function $g(z)$ is a square polynomial function depending on two parameters: δ and Bi . The function $g(z)$ takes a zero value in the interval of the definition when an inequality is being solved:

$$g(-1)g(0) = (3\text{Bi} + 3\text{Bi}^2 + 3\delta^2 + \text{Bi}\delta^2)(3\text{Bi} + 3\text{Bi}^2 + 6\delta^2 + 6\text{Bi}\delta^2 + 2\text{Bi}^2\delta^2) < 0.$$

The function $g(z)$ has two solutions when the system of inequalities is being solved:

$$\begin{cases} D > 0, \\ g(0)g(-1) > 0. \end{cases}$$

D is the discriminant of a quadratic equation. If the minimum of the function T_0 is not more than 0.5, the number of zeros of the function T_0 is equal to the number of zeros.

Further on, additive components of the pressure are analyzed. We provide isolines of the temperature and pressure when $\text{Bi} = -2$ and $\delta = 1$ (Fig. 4) and when $\text{Bi} = -0.4$ and $\delta = 0.48$ (Fig. 5).

It is obvious that the square law change of P_{11} converts into a linear dependence $P_{11} = -z$ when $\text{Bi} = 0$. The function P_{11} reduces to zero within the interval $[-1; 0)$ in the point $z = 2/\text{Bi}$. Since the square curve turns into a linear dependence, the function P_{11} monotonously decreases.

Let's consider the background pressure function $P_0(z)$. The function $P_0(z)$ is a quartic polynomial depending on two parameters: δ and Bi . The function

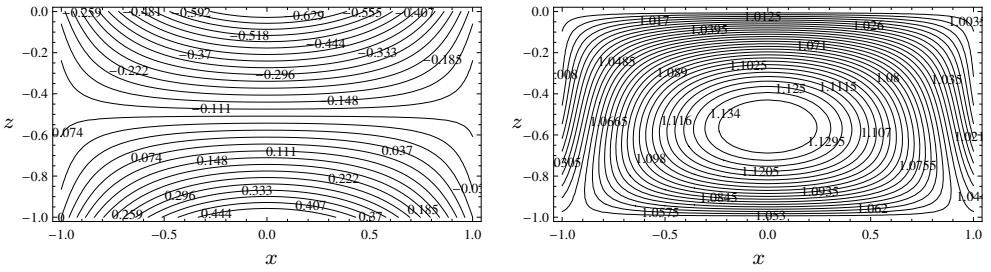


Figure 4. Isolines of the functions of temperature and pressure with $Bi = -2$ and $\delta = 1$

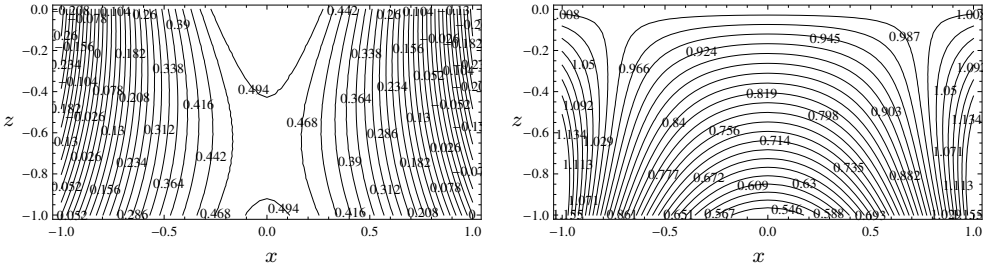


Figure 5. Isolines of the function of temperature pressure with $Bi = -0.4$ and $\delta = 0.48$

$P_0(z)$ takes one zero value in the interval of the definition $z \in [-1; 0]$ when the inequality is being solved:

$$P(-1)P(0) < 0.$$

The function of the pressure comes up to the maximum with some values of parameters δ and Bi in the considered interval. The existence of the extremum points in the interval of the definition $z \in [-1; 0]$ is confirmed by the inequality solution:

$$P'(-1)P'(0) < 0.$$

Thus, the structure of the function for the temperature shows that some locally hyperbolic level lines exist. In other words, they can not be closed with any Biot number and δ values. The local ellipticity isolines of the pressure can be observed in a rather wide range of the dimensionless complexes defining the topology of the fluid flow.

Conclusion. In this paper the convective motion of the two-dimensional flow of a viscous incompressible fluid under the Newton–Rikhman law on one of the borders of an infinite fluid layer in Stokes’s approach has been analyzed.

The assessed value of the function that makes the linearization of the Navier–Stokes equation in the Oberbeck–Boussinesq approach possible has been obtained. It is shown that the considered system of ordinary differential equations received within the announced class of exact solutions, exactly describes a fluid flow in extreme points of the temperature and pressure. The values when counter-current flows in the fluid can be observed, are found. It is shown that depending on similarity numbers in fluid, a different number of whirlwinds and the local ellipticity of pressure isolines can be observed.

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ПЛОСКАЯ КОНВЕКЦИЯ ВЯЗКОЙ НЕСЖИМАЕМОЙ ЖИДКОСТИ ПРИ ЗАДАННОЙ ТЕПЛОТДАЧЕ НА СВОБОДНОЙ ГРАНИЦЕ

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Аннотация

Найдено точное стационарное решение краевой задачи, описывающее конвективное движение вязкой несжимаемой жидкости в плоском слое при квадратичном нагреве свободной поверхности в приближение Стокса. Линеаризация уравнений Обербека–Буссинеска позволяет описать движение жидкости в точках экстремумов давления и температуры. Выведено условие, при котором наблюдается противотечение (два встречных потока) в жидкости. При наличии застойной точки в жидкости наблюдается шесть незамкнутых вихрей.

Ключевые слова: точное решение, закон Ньютона–Рихмана, тепловая конвекция, уравнения Обербека–Буссинеска, противотечение.

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