Vestn. Samar. Gos. Tekhn. Univ., Ser. Fiz.-Mat. Nauki
[J. Samara State Tech. Univ., Ser. Phys. Math. Sci.], 2019, vol. 23, no. 3, pp. 407–416
ISSN: 2310-7081 (online), 1991-8615 (print) https://doi.org/10.14498/vsgtu1723

Differential Equations and Mathematical Physics

MSC: 76B47, 76D17

Closed vortex lines in fluid and gas

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Abstract

Continuous fluid and gas flows with closed vortex tubes are investigated. The circulation along the vortex line of the ratio of the density of the resultant of all forces (applied to the fluid or gas) to the density of the fluid or gas is considered. It coincides with the circulation (along the same vortex line) of the partial derivative of the velocity vector with respect to time and, therefore, for stationary flows, it is equal to zero on any closed vortex line. For non-stationary flows, vortex tubes are considered, which remain closed for at least a certain time interval. A previously unknown regularity has been discovered, consisting in the fact that at, each fixed moment of time, such circulation is the same for all closed vortex lines that make up the vortex tube. This regularity is true for compressible and incompressible, viscous (various rheologies) and non-viscous fluids in a field of potential and non-potential external mass forces. Since this regularity is not embedded in modern numerical algorithms, it can be used to verify the numerical calculations of unsteady flows with closed vortex tubes by checking the equality of circulations on different closed vortex lines (in a tube).

The expression for the distribution density of the resultant of all forces applied to fluid or gas may contain higher-order derivatives. At the same time, the expression for the partial derivative of the velocity vector with respect to time and the expression for the vector of vorticity (which is necessary for constructing the vortex line) contain only the first derivatives; which makes it possible to use new regularity for verifying the calculations made by methods of high and low orders simaltaniously.

Research Article

Please cite this article in press as:
Sizykh G. B. Closed vortex lines in fluid and gas, *Vestn. Samar. Gos. Tekhn. Univ., Ser. Fiz.-Mat. Nauki* [J. Samara State Tech. Univ., Ser. Phys. Math. Sci.], 2019, vol. 23, no. 3, pp. 407–416. doi: 10.14498/vsgtu1723.

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Keywords: closed vortex tubes, verification of calculations of fluid and gas flows, vortex theorems, Zorawski's criterion.

Received: 17th July, 2019 / Revised: 26th August, 2019 /
Accepted: 16th September, 2019 / First online: 19th September, 2019

Introduction. The classical Helmholtz theorems on the motion of vortices in barotropic fluid were summarized in Zoravski's criterion [1], which is also referred to the Friedmann theorem [2]. This criterion deals with the possibility of considering the evolution of an arbitrary vector field (not only the vorticity field) as the movement of vector lines and vector tubes of this field with the fluid particles. Later, other patterns of vortex movements were discovered. One group of laws is associated with the conservation of certain quantities (which depend on the flow parameters) along streamlines or vortex lines in the general 3D case in a non-barotropic gas [3–6]. These laws deal with "generalized" circulation or "generalized" velocity and vorticity fields (which makes it difficult to use them to verify numerical calculations). The regularities have a simpler form, especially for plane and axisymmetric flows with an additional assumption on the isoenergeticity of the vortex gas flow (which is valid, for example, in the flows behind the detached shock wave). These regularities, in the first place, include the result of Crocco [7]. It consists in the fact that, in the plane case, along the streamlines, the ratio of vorticity to pressure $I_1 = \Omega/p$ is maintained, and in the axisymmetric case the invariant is $I_2 = \Omega/(pr)$, where r is the distance from the axis of symmetry. Another group of regularities includes the results discussed in [8–11], where formulas are obtained for calculating vorticity at some points of the flow behind a detached shock wave through the free stream parameters and through parameters determined by the shape of the shock wave (slope, curvature). We can also mention studies dealing with the existence of vortex flows of incompressible fluid with certain properties [12–16] and the principles of maximum in vortex flows [17–21].

In this paper, we study flows with closed vortex tubes. In some cases, it is possible a priori (before making calculations) to state that vortex lines and tubes will be closed (for example, a flow behind a detached shock wave [22]) and checking the closure of vortex lines can serve to verify the calculations. In other cases, closed vortex tubes are detected as a result of a calculation (for example, [23–28]), and checking the regularities specific to vortex tubes would allow verification of the calculation. This article is devoted to the search for such regularities in the general 3D case for continuous flows of fluid and gas, hereinafter called as fluid flows.

1. Basic notation and Zorawski's criterion. In what follows, we consider the fluid flows whose velocity field $\mathbf{V} = \mathbf{V}(x, y, z, t)$ is described by the equation

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{\Omega} \times \mathbf{V} + \nabla\Big(\frac{V^2}{2}\Big) = \mathbf{F}, \tag{1}$$

where $V = |\mathbf{V}|$, $\mathbf{\Omega} = \mathbf{rot}\,\mathbf{V}$ is vorticity, $\mathbf{F} = \mathbf{F}(x, y, z, t)$ is the ratio of the density of the resultant of all forces (applied to the fluid) to the density of the fluid. Such flows include stationary and non-stationary flows of compressible and incompressible, viscous (various rheologies) and non-viscous fluids.

Let the spatial domain G be located inside a fluid with a velocity field $\mathbf{V} = \mathbf{V}(x, y, z, t)$ and inside this field is vortex for some open time interval ($\boldsymbol{\Omega} \neq 0$). In the domain G, we also consider the flow of an imaginary fluid whose particles move with a velocity $\mathbf{q} = \mathbf{q}(x, y, z, t)$. The particles of an imaginary fluid do not interact with the particles of a real fluid and do not affect its movement. Suppose that, in the domain G, within the time interval (t_1, t_2), the vorticity of a real fluid $\boldsymbol{\Omega}$ and the velocity of an imaginary fluid $\mathbf{q}$ are related by the equation

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} + \mathbf{rot}(\boldsymbol{\Omega} \times \mathbf{q}) = 0. \tag{2}$$

In this case, Zoravski's criterion [1] states that, in the interval (t_1, t_2), the evolution of vorticity $\boldsymbol{\Omega}$ can be viewed as the movement of vortex lines (and vortex tubes with preservation of their intensity), together with the particles of an imaginary medium moving with a velocity $\mathbf{q}$ as long as these particles are inside G. This conclusion from Zoravski's criterion will be used below. For brevity, the particles of an imaginary fluid will be termed q-particles.

All the parameters of the flows discussed in this paper are considered sufficiently smooth.

2. The velocity q in a closed vortex tube. Consider the region of vortex motion of a fluid ($\boldsymbol{\Omega} \neq 0$) in which there is a fixed flat region (surface) σ such that during a nonzero time interval (t_1, t_2) each vortex line intersects this surface at an acute angle to the normal, and only once.

Denote by G_σ a fragment of space, the points of which can be reached if we fix the time and move from the flat region σ along the vortex lines starting at σ (due to the closedness of the vortex lines you can move to either side of σ). In other words, the fragment G_σ is the union of all vortex lines passing through σ. Although the surface σ is fixed in space, but, due to the change in the shape of the vortex lines with time, the shape of the G_σ fragment may change with time, i.e. $G_\sigma = G_\sigma(t)$. The fragment $G_\sigma(t)$ consisting only of closed vortex lines at each moment of time is a closed vortex tube.

We construct the function $f(x, y, z, t)$ as follows. At first we define it at points of the surface σ. Let for each time $t \subset (t_1, t_2)$ at the points of the surface σ the function f be equal to zero. The surface σ has two sides. Let one of the sides be called the first, the other being called the second. We continue the function f in the rest part of the vortex tube $G_\sigma(t)$ by integrating along the vortex lines (from the first side of σ to the second side of σ), so that the following equality holds (the dot denotes the scalar product):

$$\boldsymbol{\Omega} \cdot \nabla f = \boldsymbol{\Omega} \cdot \mathbf{F}. \tag{3}$$

It follows from the theory of ordinary differential equations [29] that the field f constructed in this way will be uniquely determined for each time instant $t \subset (t_1, t_2)$ at all points of the vortex tube $G_\sigma(t)$. In this case, at points of the surface σ, the field f will be continuous from the first side of σ (from which integration began), and it may prove to be discontinuous from the second side of σ.

We use the well-known formula for a double vector product

$$\mathbf{\Omega} \times [\mathbf{\Omega} \times \{\mathbf{F} - \nabla f\}] = \mathbf{\Omega}(\mathbf{\Omega} \cdot \{\mathbf{F} - \nabla f\}) - \{\mathbf{F} - \nabla f\}(\mathbf{\Omega} \cdot \mathbf{\Omega}).$$

The first term in the right-hand side is zero by virtue of (3). Therefore,

$$\mathbf{F} = \nabla f - \frac{[\mathbf{\Omega} \times \{\mathbf{F} - \nabla f\}]}{\mathbf{\Omega}^2}.$$

Substitute this expression for the vector $\mathbf{F}$ into equation (1):

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{\Omega} \times \Big(\mathbf{V} + \frac{[\mathbf{\Omega} \times \{\mathbf{F} - \nabla f\}]}{\mathbf{\Omega}^2}\Big) = \nabla\Big(f - \frac{V^2}{2}\Big). \tag{4}$$

Let $\mathbf{n}_\sigma$ denotes a unit vector normal to the flat surface σ on the side from which integration began when constructing the function f; $(\nabla f)_1$ and $(\nabla f)_2$ denote the limiting values of the gradient of the function f from the first side σ and from the second side σ, respectively.

The surface σ is chosen so that $\mathbf{\Omega} \cdot \mathbf{n}_\sigma \neq 0$ everywhere on this surface. Therefore, there are an infinite number of functions $g(x, y, z, t)$ defined (and smooth) at all the points of the tube $G_\sigma(t)$, except for the points on the surface σ, such that the limiting values of g_1 and g_2 on different sides of the surface σ satisfy the conditions

$$\Big(\mathbf{V} + \frac{[\mathbf{\Omega} \times \{\mathbf{F} - (\nabla f)_i\}]}{\mathbf{\Omega}^2} + g_i \mathbf{\Omega}\Big) \cdot \mathbf{n}_\sigma = 0, \quad i = 1, 2.$$

(These conditions mean that on both the first and second sides of the surface σ, the vectors in round brackets are parallel to σ.) Let $g(x, y, z, t)$ be one of such functions. This function is defined in the tube $G'_\sigma(t) = G_\sigma(t) \backslash \sigma$ (the tube $G'_\sigma(t)$ is obtained by cutting out the surface σ from the tube $G_\sigma(t)$). Inside the tube $G'_\sigma(t)$, we consider the vector field

$$\mathbf{q} = \mathbf{V} + \mathbf{\Omega} \times \frac{[\mathbf{\Omega} \times \{\mathbf{F} - \nabla f\}]}{\mathbf{\Omega}^2} + g\mathbf{\Omega}.$$

Since $\mathbf{\Omega} \times \mathbf{\Omega} = 0$, equation (4) is equivalent to

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{\Omega} \times \mathbf{q} = \nabla\Big(f - \frac{V^2}{2}\Big).$$

Applying the **rot** operator to both sides of the latter equality leads to the equation

$$\frac{\partial \mathbf{\Omega}}{\partial t} + \mathbf{rot}\,[\mathbf{\Omega} \times \mathbf{q}] = 0.$$

Comparing this equation with (2) and using Zoravski's criterion, we arrive at the following conclusion.

The evolution of the vorticity in the tube $G'_\sigma(t)$ can be considered as moving the vortex lines (and vortex tubes with preserving their intensity) together with q-particles moving with velocity $\mathbf{q}$.

3. The theorem on closed vortex tubes. At time $t'_1 \subset (t_1, t_2)$, we choose any closed vortex line in the tube $G_\sigma(t'_1)$. Consider a set of q-particles that make up a part of this line lying in $G'_\sigma(t'_1)$, i.e. the part of the closed line without a point on the surface σ (where the velocity $\mathbf{q}$ is not defined and where there are no q-particles). The function g is chosen in such a way that the limit value of the velocity $\mathbf{q}$ on both sides of surface σ is parallel to this surface. Therefore, the q-particles cannot "get out" of the tube $G'_\sigma(t)$ through the surface σ, and new q-particles cannot "get in". Consequently, the considered set of q-particles inside $G_\sigma(t)$ will constitute a closed vortex line (except for one point on the surface σ) for some non-zero time interval $(t'_1, t'_2) \subset (t_1, t_2)$. Since the vortex lines are closed, this means that, during the time interval $(t'_1, t'_2) \subset (t_1, t_2)$, the velocity limits $\mathbf{q}$ on different sides of the surface σ must coincide,

$$\mathbf{V} + \frac{[\mathbf{\Omega} \times \{\mathbf{F} - (\nabla f)_1\}]}{\mathbf{\Omega}^2} + g_1\mathbf{\Omega} = \mathbf{V} + \frac{[\mathbf{\Omega} \times \{\mathbf{F} - (\nabla f)_2\}]}{\mathbf{\Omega}^2} + g_2\mathbf{\Omega}.$$

After rearrangements of the terms, we obtain

$$\mathbf{\Omega} \times \{(\nabla f)_2 - (\nabla f)_1\} = (g_2 - g_1)\mathbf{\Omega}. \tag{5}$$

By the properties of the vector product, the left and right sides of equality (5) are perpendicular to each other. This is only possible if both parts are zero. Therefore,

$$\mathbf{\Omega} \times \{(\nabla f)_2 - (\nabla f)_1\} = 0,$$

i.e. the vectors $\{(\nabla f)_2 - (\nabla f)_1\}$ and $\mathbf{\Omega} \neq 0$ are parallel. On the other hand, according to (3),

$$\mathbf{\Omega} \cdot \{(\nabla f)_2 - (\nabla f)_1\} = 0,$$

i.e. these vectors are orthogonal. This is only possible if $(\nabla f)_2 = (\nabla f)_1$.

By construction, the projection of $(\nabla f)_1$ onto the surface σ is zero. Therefore, the projection of $(\nabla f)_2$ onto the surface σ is also zero. The selected line and time $t'_1 \subset (t_1, t_2)$ were chosen arbitrarily (see the beginning of this section). Consequently, the projection $(\nabla f)_2$ on the surface σ is zero at all points σ during the entire time interval (t_1, t_2). This means that, at each instant of time, the limiting values of f at the ends of all lines are equal to one constant. This constant may be different for different points in time. According to equation (3), the aforementioned constant is the circulation of the vector $\mathbf{F}$ along a closed vortex line, i.e. the circulation of the right-hand side of equation (1) over a closed vortex line is the same for different vortex lines crossing the surface σ. Thus, the following theorem is proved.

A necessary condition for the existence in time of vortex tubes, consisting of closed vortex lines. *Let the vortex flow of a fluid* ($\mathbf{\Omega} \neq 0$) *be described by equation* (1). *And let, during the nonzero period of time* (t_1, t_2), *the entire (stationary) flat surface* σ *intersect with the closed vortex lines of this flow* *at an acute angle to the normal and, at the same time, each vortex line intersect with σ only once. Then, for any fixed moment of time $t \subset (t_1, t_2)$, the circulation of the right side of equation (1) along a closed vortex line is the same for different vortex lines crossing the surface σ.*

Note that the theorem leaves open the question of the time dependence on the circulation of the vector $\mathbf{F}$.

The classical Thomson (Kelvin) theorem [2] is also true for all types of fluids. It claims that the time derivative of the velocity circulation along a contour moving together with the fluid particles is equal to the circulation of the right-hand side (1). In this theorem, no mention is made of the relation between the values of the circulation on different contours. In the theorem proved above, such a connection is found (equality of circulations) for contours of a special form (closed vortex lines). These contours are different from the contours referred to in the Thomson theorem since it cannot be stated in the general case that the vortex lines move with the particles of the fluid. Therefore, the Thomson theorem and the theorem proved above are incomparable and complementary.

4. Equivalent formulation of the theorem and some special cases. The circulation of the second term in the left-hand side of equation (1) is zero, since the vector $\mathbf{\Omega} \times \mathbf{V}$ is orthogonal to the vortex line. The circulation of the third term is zero due to the closedness of the vortex line. Therefore, the circulation of $\mathbf{F}$ coincides with the circulation of $\frac{\partial \mathbf{V}}{\partial t}$. Consequently,

a) *under the conditions of the theorem proved above, for any fixed moment of time $t \subset (t_1, t_2)$, the circulation of $\frac{\partial \mathbf{V}}{\partial t}$ along a closed vortex line is the same for different vortex lines intersecting the surface σ;*

b) *for stationary flows, the circulation of the right-hand side of equation (1) along any closed vortex line is zero.*

For a non-stationary flow, it is also possible to specify a situation where the circulation of the right-hand side of equation (1) along any closed vortex line is zero. In particular, if there is a sequence of closed vortex lines in the vortex tube, the lengths of which tend to zero, then this circulation will be equal to zero.

Conclusion. For all types of fluids (from an ideal fluid to a viscous gas), a previously unknown property of closed vortex lines has been obtained. It is formulated as a theorem, and it can be used both for qualitative analysis and for verification of numerical calculations of flows in which there are closed vortex tubes.

Competing interests. None declared.

Author's Responsibilities. I take full responsibility for submitting the final manuscript in print. I approved the final version of the manuscript.

Funding. None declared.

References

1. Prim R., Truesdell C. A derivation of Zorawski's criterion for permanent vector-lines, *Proc. Amer. Math. Soc.*, 1950, vol. 1, no. 1, pp. 32–34. doi: 10.1090/s0002-9939-1950-0035136-9.
2. Kochin N. E., Kibel I. A., Roze I. V. *Theoretical Hydromechanics.* New York, Wiley, 1964, v+577 pp.
3. Golubinskii A. I., Sychev V. V. Some conservation properties of turbulent gas flows, *Dokl. Akad. Nauk SSSR*, 1977, vol. 237, no. 4, pp. 798–799 (In Russian).
4. Mobbs S. Some vorticity theorems and conservation laws for non-barotropic fluids, *J. Fluid Mech.*, 1981, vol. 108, pp. 475–483. doi: 10.1017/S002211208100222X.
5. Golubinskii A. I., Golubkin V. N. On certain conservation properties in gas dynamics, *J. Appl. Math. Mech.*, 1985, vol. 49, no. 1, pp. 88–95. doi: 10.1016/0021-8928(85)90133-9.
6. Markov V. V., Sizykh G. B. Vorticity evolution in liquids and gases, *Fluid Dyn.*, 2015, vol. 50, no. 2, pp. 186–192. doi: 10.1134/S0015462815020027.
7. Krocco L. Eine neue Stromfunktion für die Erforschung der Bewegung der Gase mit Rotation, *Z. Angew. Math. Mech.*, 1937, vol. 17, no. 1, pp. 1–7 (In German). doi: 10.1002/zamm.19370170103.
8. Truesdell C. On curved shocks in steady plane flow of an ideal fluid, *J. Aeronaut. Sci.*, 1952, no. 19, pp. 826–828. doi: 10.2514/8.2495.
9. Hayes W. D. The vortycity jump across a gasdynamic discontinuities, *J. Fluid Mech.*, 1957, no. 2, pp. 595–600.
10. Levin V. A., Markov V. V., Sizykh G. B. Vorticity on the Surface of an Axially Symmetric Body behind a Detached Shock Wave, *Doklady Physics*, 2018, vol. 63, no. 12, pp. 530–532. doi: 10.1134/S1028335818120108.
11. Sizykh G. B. Entropy Value on the Surface of a Non-Symmetric Convex Bow Part at Supersonic Streamlining, *Fluid Dyn.*, 2019, vol. 54 (to appear).
12. Beltrami E. Considerazioni idrodinamiche, *Il Nuovo Cimento Series 3*, 1889, vol. 25, no. 1, pp. 212–222. doi: 10.1007/bf02719090.
13. Biushgens S. S. On Helical Flow, *Nauchn. Zapiski Mosk. Gidrom. Inst. (MGMI)*, 1948, vol. 17, pp. 73–90 (In Russian).
14. Sizykh G. B. Axisymmetric Helical Flows of Viscous Fluid, *Russian Mathematics*, 2019, vol. 63, no. 2, pp. 44–50. doi: 10.3103/S1066369X19020063.
15. Sizykh G. B. Helical Vortex Lines in Axisymmetric Viscous Incompressible Fluid Flows, *Fluid Dyn.*, 2019, vol. 54 (to appear).
16. Kotsur O. S. On the existence of local formulae of the transfer velocity of local tubes that conserve their strengths, *Proceedings of MIPT*, 2019, vol. 11, no. 1, pp. 76–85 (In Russian).
17. Rowland H. On the Motion of a Perfect Incompressible Fluid When no Solid Bodies are Present, *Am. J. Math*, 1880, vol. 3, no. 3, pp. 226–268. doi: 10.2307/2369424.
18. Lamb H. *Hydrodynamics.* Cambridge, Cambridge Univ. Press, 1895, xvii+604 pp. doi: 10.5962/bhl.title.18729.
19. Hamel G. Ein allgemeiner Satz über den Druck bei der Bewegung volumbeständiger Flüssigkeiten, *Monatsh. Math. Phys.*, 1936, vol. 43, no. 1, pp. 345–363 (In German). doi: 10.1007/bf01707614.
20. Truesdell C. Two measures of vorticity, *Indiana Univ. Math. J.*, 1953, vol. 2, no. 2, pp. 173–217. doi: 10.1512/iumj.1953.2.52009.
21. Vyshinsky V. V., Sizykh G. B. The verification of the calculation of stationary subsonic flows and the presentation of the results, *Mathematical Models and Computer Simulations*, 2019, vol. 11, no. 1, pp. 97–106. doi: 10.1134/S2070048219010162.
22. Golubkin V. N., Sizykh G. B. On the vorticity behind 3-D detached bow shock wave, *Advances in Aerodynamics*, 2019, vol. 1, no. 1, 15. doi: 10.1186/s42774-019-0016-5.
23. Troshin A., Shiryaeva A., Vlasenko V., Sabelnikov V. Large-Eddy Simulation of Helium and Argon Supersonic Jets in Supersonic Air Co-flow, In: *Progress in Turbulence*

VIII. iTi 2018, Springer Proceedings in Physics, 226, 2019, pp. 253–258. doi: 10.1007/978-3-030-22196-6_40.

24. Vyshinsky V. V., Sizykh G. B. Verification of the Calculation of Stationary Subsonic Flows and Presentation of Results, In: *Smart Modeling for Engineering Systems. GCM50 2018*, Smart Modeling for Engineering Systems, 133. Cham, Springer, 2019, pp. 530–532. doi: 10.1007/978-3-030-06228-6_19.
25. Afonina N. E., Gromov V. G., Levin V. A., Manuilovich I. S., Markov V. V., Smekhov G. D., Khmelevskii A. N. Investigation of the annular nozzle start in actual and virtual intermittent aerodynamic setups, *Fluid Dyn.*, 2016, vol. 51, no. 2, pp. 281–287. doi: 10.1134/S0015462816020150.
26. Dergachev S. A., Marchevsky I. K., Scheglov G. A. Flow simulation around 3D bodies by using Lagrangian vortex loops method with boundary condition satisfaction with respect to tangential velocity components, *Aerospace Science and Technology*, 2019, 105374 (to appear). doi: 10.1016/j.ast.2019.105374.
27. Borovoy V. Y., Egorov I. V., Skuratov A. S., Struminskaya I. V. Two-Dimensional Shock-Wave/Boundary-Layer Interaction in the Presence of Entropy Layer, *AIAA Journal*, 2013, vol. 51, no. 1, pp. 80–93. doi: 10.2514/1.J051496.
28. Egorov I. V., Novikov A. V. Direct numerical simulation of laminar–turbulent flow over a flat plate at hypersonic flow speeds, *Comput. Math. Math. Phys.*, 2016, vol. 56, no. 6, pp. 1048–1064. doi: 10.1134/S0965542516060129.
29. Pontryagin L. S. *Ordinary differential equations*, Adiwes International Series in Mathematics. London, Paris, Pergamon Press, 1962, vi+298 pp.

Вестн. Сам. гос. техн. ун-та. Сер. Физ.-мат. науки. 2019. Т. 23, № 3. С. 407–416
ISSN: 2310-7081 (online), 1991-8615 (print) https://doi.org/10.14498/vsgtu1723

УДК 532.5.032; 532.517.43

Замкнутые вихревые линии в жидкости и газе

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Аннотация

Исследуется непрерывное течение жидкости и газа с замкнутыми вихревыми трубками. Рассмотрена циркуляция вдоль вихревой линии отношения плотности равнодействующей всех сил (приложенных к жидкости или газу) к плотности жидкости или газа. Она совпадает с циркуляцией по той же вихревой линии частной производной вектора скорости по времени и поэтому для стационарных течений равна нулю на любой замкнутой вихревой линии. Для нестационарных течений рассмотрены вихревые трубки, которые остаются замкнутыми по крайней мере в течение некоторого интервала времени. Обнаружена неизвестная ранее закономерность, состоящая в том, что в каждый фиксированный момент времени такая циркуляция одинакова для всех замкнутых вихревых линий, составляющих вихревую трубку. Указанная закономерность верна для течений сжимаемых и несжимаемых, вязких (различных реологий) и невязких жидкостей в поле потенциальных и непотенциальных внешних массовых сил. Поскольку эта закономерность не заложена в современные численные алгоритмы, она может использоваться для верификации численных расчетов нестационарных течений с замкнутыми вихревыми трубками путем проверки равенства циркуляций на разных замкнутых вихревых линиях (в одной трубке).

Выражение для плотности распределения равнодействующей всех сил, приложенных к жидкости или газу, может содержать производные высших порядков. В то же время выражение для частной производной вектора скорости по времени и выражение для вектора завихренности, который необходим для построения вихревой линии, содержат только первые производные, что позволяет использовать обнаруженную закономерность для верификации расчетов, проведенных методами не только высокого, но и низкого порядков.

Ключевые слова: замкнутые вихревые трубки, верификация расчетов течений жидкости и газа, теоремы о вихрях, критерий Зоравского.

Научная статья

Образец для цитирования

Sizykh G. B. Closed vortex lines in fluid and gas, *Vestn. Samar. Gos. Tekhn. Univ., Ser. Fiz.-Mat. Nauki* [J. Samara State Tech. Univ., Ser. Phys. Math. Sci.], 2019, vol. 23, no. 3, pp. 407–416. doi: 10.14498/vsgtu1723.

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Получение: 17 июля 2019 г. / Исправление: 26 августа 2019 г. /
Принятие: 16 сентября 2019 г. / Публикация онлайн: 19 сентября 2019 г.

Конкурирующие интересы. Не указано.

Авторская ответственность. Я несу полную ответственность за предоставление окончательной версии рукописи в печать. Окончательная версия рукописи мною одобрена.

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