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ON CONSTRUCTION OF QUANTUM LOGICAL GATE BASED ON ESR

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A quantum computer is a computation device operated by means of quantum mechanical phenomena. There are many candidates that are being pursued for physically implementing the quantum computer. The quantum logical gate based on the electron spin resonance (ESR) was studied in ref. [3]. In this paper, we discuss a construction of Controlled-Controlled-NOT (CCNOT) gate by using the nonrelativistic formulation of ESR.

Key words: *Electron Spin Resonance (ESR), quantum computer, quantum logical gates, Feynman gates, Controlled–Controlled NOT (CCNOT).*

1. Introduction. In classical computer, there exist inevitable demerits for discussing logical gates. One of the demerits is an irreversibility of logical gates, that is the AND and the OR gates. This property causes to the restriction of computational speed for the classical computer. There are several kinds of approaches for avoiding these demerits. One of these approaches is proposed by Feynman [1]. He proved that every logical gates can be constructed by combining with only two reversible gates, i.e., the NOT and the Controlled-NOT (CNOT) gates.

There are several approaches for realizing quantum logical gates. One of those approaches is the study by means of nuclear magnetic resonance (NMR). Quantum logical gate based on NMR is performed by controlling the nuclear spin under the additve magnetic fields from the environments. However, it might be difficulty to make the logical gate of NMR using a large number of quantum bits (qubits) because of the weakness of the spin-spin interactions among the nuclears. Our study uses ESR to construct Feynman gates that has NOT gate and CNOT gate, CCNOT gate. As quantum logical gate based on NMR, quantum gate based on ESR is performed by controlling the electron spin under the additive magnetic

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fields from the environments. By employing Ising model, Ohya, Volovich and Watanabe constructed in [3] both NOT and CNOT gates based on ESR.

In this paper, we construct the CCNOT gate in order to complete Feynman gates and universal quantum gates based on ESR. In general, any unitary operation on n qubits can be described by composing single qubit and CNOT gates. Unfortunately, no straightforward method is known to implement all these gates resisting errors. On the other hand, a discrete set of gates can be used to perform quantum computation in an error-resistant fashion. To perform fault-tolerant quantum computation, we consider discrete set of gates which are Feynman gates.

2. NOT gate based on ESR. In this section, we explain the NOT gate based on ESR. It is one of Feynman gates, which includes CNOT and CCNOT gate, and has been constructed [2]. First of all, let us consider one particle case. Let $\mathcal{H}$ be $\mathbb{C}^2$ with its canonical basis $u_+ = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, u_- = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\mathbf{B}(\mathcal{H})$ be the set of all bounded operators on $\mathcal{H}$ and $\mathbf{B}(\mathcal{H})_{sa} \equiv \{A \in \mathbf{B}(\mathcal{H}); A = A^*\}$, where A^* is the adjoint of A defined by

$$\langle A^* u, v\rangle = \langle u, Av\rangle \text{ for any } u, v \in \mathcal{H}.$$

$\mathbf{B}(\mathcal{H})_{sa}$ has the basis $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, which are called Pauli spin matrices and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is an identity matrix on $\mathcal{H}$. That is, $\sigma = \{\sigma_x, \sigma_y, \sigma_z\}$ is an orthogonal basis of $\mathbf{B}(\mathcal{H})_{sa}$ with the scalar product

$$\langle \sigma_i, \sigma_j\rangle = \frac{1}{2} \operatorname{tr} \sigma_i \sigma_j, \quad j \in \{x, y, z\}.$$

Let $\vec{S} = (S_x, S_y, S_z)$ be a spin (angular momentum) operator of electron, where $S_i = \frac{1}{2}\sigma_i$ is a component of spin operator of electron in the direction of i-axis ($i = x, y, z$). We denote unit vectors of x, y, z axis by $\vec{e_x}, \vec{e_y}, \vec{e_z}$ and $\vec{S}$ is the spin vector given by

$$\vec{S} = (S_x, S_y, S_z) = S_x \vec{e_x} + S_y \vec{e_y} + S_z \vec{e_z}.$$

Let us consider two magnetic fields $\vec{B_0}$ and $\vec{B_1}$. $\vec{B_0}$ is a static magnetic field given by

$$\vec{B_0} = B_0 \vec{e_z}$$

in the z direction and $\vec{B_1}$ is a rotating magnetic field given by

$$\vec{B_1}(t) = B_1(\vec{e_x} \cos\omega t + \vec{e_y} \sin\omega t)$$

with frequency ω in the xy plain, where B_0 and B_1 are certain constants due to the magnetic fields. If $\vec{B}(t)$ is a magnetic vector defined by

$$\vec{B}(t) = \vec{B_1}(t) + \vec{B_0},$$

then one has

$$\frac{d\vec{S}}{dt} = \vec{S} \times \vec{B}(t) = B_1(S_x \cos\omega t + S_y \sin\omega t) + B_0 S_z.$$

Let $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ be spin vectors related to spin up and spin down, respectively. Let us take an initial state $\psi(0) = a_0|\uparrow\rangle + b_0|\downarrow\rangle = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$, then state vector at time t is denoted by

$$\psi(t) = a(t)|\uparrow\rangle + b(t)|\downarrow\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix},$$

where $a(t)$, $b(t) \in \mathbb{C}$ are satisfying $|a(t)|^2 + |b(t)|^2 = 1$.

Let Schrödinger equation in one particle be

$$i\frac{\partial\psi(t)}{\partial t} = -\vec{S} \times \vec{B}(t)\psi(t) = -[B_1(S_x \cos\omega t + S_y \sin\omega t) + B_0 S_z]\psi(t)$$

where B_0, B_1, ω are arbitrary constants, A solution of the Schrödinger equation is given by

$$\psi(t) = e^{-i\omega t S_z} e^{it((\omega+B_0)S_z + B_1 S_x)}\psi(0),$$

which means time evolution. In particular, we see the resonance condition

$$\omega + B_0 = 0,$$

that is, $\psi(t) = e^{iB_0 t S_z} e^{itB_1 S_x}\psi(0)$. Based on the above results, we reconstruct the Not gate based on ESR. If we take $t = t_1$ such that

$$\frac{B_0 t_1}{2} = \frac{B_1 t_1}{2} = \frac{\pi}{2},$$

then

$$\psi(t_1) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\psi(0) = b_0 u_+ + a_0 u_-.$$

It means that this gate is performed as the NOT gate based on ESR. Let $U_{NOT}(t) \equiv e^{iB_0 t S_z} e^{iB_1 t S_x}$ be a unitary operator expressing the NOT gate based on ESR. Quantum channel denoting the NOT gate based on ESR is defined by

$$\Lambda^*_{NOT(t_1)}(\,\cdot\,) \equiv U_{NOT}(t_1)(\,\cdot\,)U^*_{NOT}(t_1).$$

For the initial state $|\psi(0)\rangle\langle\psi(0)|$ at time 0, the output state of $\Lambda^*_{NOT(t_1)}$ is obtained by

$$\Lambda^*_{NOT(t_1)}(|\psi(0)\rangle\langle\psi(0)|) = |\psi(t_1)\rangle\langle\psi(t_1)|.$$

3. CNOT gate based on ESR. In this section, we introduce the CNOT gate based on ESR. Let us consider N particle systems to treat the Controlled Not gate. Let $\vec{e_1}, \vec{e_2}, \vec{e_3}$ be unit vectors of x, y, z axis, respectively, and let $\vec{S^{(1)}}, \dots, \vec{S^{(N)}}$ be spin vectors of N electrons such as

$$\vec{S^{(i)}} = (S_1^{(i)}, S_2^{(i)}, S_3^{(i)}) = S_1^{(i)}\vec{e_1} + S_2^{(i)}\vec{e_2} + S_3^{(i)}\vec{e_3}.$$

The spin operators satisfy the following commutation relations

$$[S_\alpha^{(p)}, S_\beta^{(q)}] = i\delta_{pq}\sum_{\gamma=1}^{3}\epsilon_{\alpha\beta\gamma}S_\gamma^{(q)},$$

where $\epsilon_{\alpha\beta\gamma} = \begin{cases} +1 \\ -1 \end{cases}$ and δ_{pq} is a certain constant. Let us consider a Hamiltonian operator for N particle systems given by

$$\mathrm{H}_{(N)} \equiv B_3\Big(\sum_{i=1}^{N} S_3^{(i)}\Big) + B_1\Big(\sum_{i=1}^{N} S_1^{(i)}\Big)f(t) + \sum_{i,j=1}^{N} J_{ij}S_3^{(i)} \otimes \overline{S}_3^{(j)},$$

where $f(t)$ is a certain function, for example $f(t) = \cos\omega t$ and J_{ij} is a coupling constant with respect to i-th spin and j-th spin. $S_k^{(i)}$ is embedding S_k into i-th position of N tensor product.

$$S_k^{(i)} = I \otimes \cdots \otimes S_k \otimes \cdots \otimes I \quad (k = 1, 2, 3).$$

Let us take a Hamiltonian $\mathrm{H}_{(N)}$ as a Ising type interaction, that is

$$\mathrm{H}_{(N)} \equiv B_3\Big(\sum_{i=1}^{N} S_3^{(i)}\Big) + \sum_{i,j=1}^{N} J_{ij}S_3^{(i)} \otimes \overline{S}_3^{(j)}.$$

If $N = 2$ then one can denote

$$\mathrm{H}_{(2)} = B_3(S_3 \otimes I + I \otimes \tilde{S}_3) + J(S_3 \otimes \overline{S}_3) + B_0(I \otimes I),$$

where B_0, B_3 and J are determined by a certain phase parameter ω.

Let us take u_+, u_-, v_+, v_- as

$$u_+\otimes v_+ = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u_+\otimes v_- = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad u_-\otimes v_+ = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad u_-\otimes v_- = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Let $\psi(0)$ be an initial state vector given by

$$\begin{aligned} \psi(0) &= a_0 u_+ \otimes v_+ + b_0 u_+ \otimes v_- + c_0 u_- \otimes v_+ + d_0 u_- \otimes v_- \\ &= \begin{pmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \end{pmatrix} \qquad (a_0, b_0, c_0, d_0 \in \mathbb{C}). \end{aligned}$$

For the initial state vector $\psi(0)$, if $J=2\omega, B_3=-\omega \text{and} B_0=\frac{1}{2}\omega$ are hold, then the state vector at time t is expressed by

$$\begin{aligned}\psi(t) &= e^{-itH_{(2)}}\psi(0)\\ &= e^{-it(B_3(S_3\otimes I+I\otimes\tilde{S}_3)+J(S_3\otimes\overline{S}_3)+B_0(I\otimes I))}\psi(0)\\ &= e^{i\omega t(S_3\otimes I)}e^{i\omega t(I\otimes\tilde{S}_3)}e^{-2\omega t(S_3\otimes\tilde{S}_3)}e^{-\frac{1}{2}\omega t(I\otimes I)}\psi(0).\end{aligned}$$

If we take $t=t_1$ such that $2\omega t=\pi$ ($\frac{\pi}{2}$pulse) then one can denote the matrix form $U_\Phi(t_1)$ of $e^{i\omega t(S_3\otimes I)}e^{i\omega t(I\otimes\tilde{S}_3)}e^{-2\omega t(S_3\otimes\tilde{S}_3)}e^{-\frac{1}{2}\omega t(I\otimes I)}$ by

$$U_\Phi(t_1)=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}.$$

Next we construct a unitary operator $U_H(t)$ related to a Hadamard transformation based on ESR. Let us define $U_H(t)$ by

$$U_H(t)=e^{-i\omega_2 t(I\otimes\tilde{S}_2)},$$

where ω_2 is a certain phase parameter. Then we have

$$e^{-i\omega_2 t(I\otimes\tilde{S}_2)}=\cos(\frac{\omega_2 t}{2})(I\otimes I)-2i\sin(\frac{\omega_2 t}{2})(I\otimes\tilde{S}_2).$$

For the initial state vector $\psi(0)$, the state vector at time t is expressed by

$$\psi(t)=U_H(t)\psi(0)=e^{-i\omega_2 t(I\otimes\tilde{S}_2)}\psi(0).$$

If we take $t=t_2$ such that

$$\frac{\omega_2 t}{2}=\frac{\pi}{4}\quad\left(\frac{\pi}{2}\text{pulse}\right),$$

then one can denote the matrix form $U_H(t_2)$ of $e^{-i\omega_2 t_2(I\otimes\tilde{S}_2)}$

$$U_H(t_2)=\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}.$$

Thus unitary operator $U_{CNOT}(t_1+2t_2)$ related to the CNOT gate can be reconstructed by the combination of $U_\Phi(t_1)$ and $U_H(t_2)$ as

$$\begin{aligned}&U_{CNOT}(t_1+2t_2)\\ =\;&U_H(t_2)^*U_\Phi(t_1)U_H(t_2)\\ =\;&e^{i\omega_2 t_2(I\otimes\tilde{S}_2)}e^{i\omega t_1(S_3\otimes I)}e^{i\omega t_1(I\otimes\tilde{S}_3)}e^{-2i\omega t_1(S_3\otimes\tilde{S}_3)}e^{-\frac{1}{2}i\omega t_1(I\otimes I)}e^{-i\omega_2 t_2(I\otimes\tilde{S}_2)}.\end{aligned}$$

It means that this unitary operator $U_{CNOT}(t_1+2t_2)$ is performed as CNOT (Controlled-NOT) gate based on ESR. Quantum channel denoting the CNOT gate based on ESR is defined by

$$\Lambda^*_{CNOT(t_1+2t_2)}(\,\cdot\,)\equiv U_{CNOT}(t_1+2t_2)(\,\cdot\,)U^*_{CNOT}(t_1+2t_2).$$

For the initial state $|\psi(0)\rangle\langle\psi(0)|$ at time 0 , the output state of $\Lambda^*_{CNOT(t_1+2t_2)}$ is obtained by

$$\Lambda^*_{CNOT(t_1+2t_2)}(|\psi(0)\rangle\langle\psi(0)|) = |\psi(t_1+2t_2)\rangle\langle\psi(t_1+2t_2)|.$$

4. CCNOT gate based on ESR. This section shows how to construct CCNOT gate, which is our main result, based on ESR. Let us consider some gates to treat the CCNOT gate. First of all, we construct Controlled-phase gate based on ESR. If Hamiltonian $H_{(N)}$ has $N=3$ then one can denote

$$\begin{aligned} H_{(3)} &= B_3(S_3^{(1)}+S_3^{(2)}+S_3^{(3)}) + J_{(1,2)}(S_3\otimes\dot{S}_3\otimes I) + J_{(2,3)}(I\otimes\dot{S}_3\otimes\ddot{S}_3) \\ &\quad +B_0(I\otimes I\otimes I), \end{aligned}$$

where $B_0, B_3, \text{and} J_{(1,2)}, J_{(2,3)}$ are determined by a certain phase parameter ω. Let $\psi(0)$ be an initial state vector given by

$$\begin{aligned} \psi(0) &= a_0u_+\otimes u_+\otimes u_+ + b_0u_-\otimes u_+\otimes u_+ + c_0u_+\otimes u_-\otimes u_+ \\ &\quad +d_0u_-\otimes u_-\otimes u_+ + \\ &\quad e_0u_+\otimes u_+\otimes u_- + f_0u_-\otimes u_+\otimes u_- + g_0u_+\otimes u_-\otimes u_- \\ &\quad +h_0u_-\otimes u_- + \otimes u_-. \end{aligned}$$

For the initial state vector $\psi(0)$, if $B_0=\frac{\omega}{4}$, $B_3=\frac{-\omega}{2}$ and $J_{(1,2)}=J_{(2,3)}=\omega$ are hold, then operator U_Φ related to the state vector at time t is expressed by

$$U_\Phi(t) = e^{i\frac{\omega t}{2}(S_3^{(1)}+S_3^{(2)}+S_3^{(3)})-i\omega t(S_3\otimes\dot{S}_3\otimes I+I\otimes\dot{S}_3\otimes\ddot{S}_3)-i\frac{\omega t}{4}(I\otimes I\otimes I)}\ .$$

If we take t_1, t_2 such that $\omega t_1=-\frac{\pi}{2}$, $\omega_2 t_2=\frac{\pi}{2}$ then operator U_S of Controlled-phase is denoted by

$$\begin{aligned} U_S(t_1+2t_2) &= U_\Phi(t_1)e^{-i\omega_2t_2(S_3\otimes\dot{S}_3\otimes I)}e^{i2\omega_2t_2(S_3\otimes I\otimes I)} \\ &= I\otimes\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&i\end{pmatrix}. \end{aligned}$$

Second, we reconstruct a CNOT gate of three particles, employing Ising model. We denote unitary operator U^3_{CNOT} related to a CNOT gate of three particles by use of operator $U_\Phi(t)$ of time evolution. Let us define U^3_{CNOT} by

$$U^3_{CNOT}(t) = e^{-i\omega_3tS_2^{(2)}}U_\Phi(t)e^{i2\omega_5t(S_3^{(2)}\cdot S_3^{(3)})}e^{-i\omega_5tS_3^{(3)}}e^{i\omega_3tS_2^{(2)}}.$$

If we take t_3, t_4, t_5 such that $\omega_3t_3=\frac{\pi}{2}$, $\omega t_4=\pi$, $\omega_5t_5=\frac{\pi}{2}$, then one has

$$U^3_{CNOT}(2t_3+t_4+2t_5) = e^{-i\omega_3t_3S_2^{(2)}}U_\Phi(t_4)e^{i2\omega_5t_5(S_3^{(2)}\cdot S_3^{(3)})}e^{-i\omega_5t_5S_3^{(3)}}e^{i\omega_3t_3S_2^{(2)}}.$$

Then one can denote the matrix form $U^3_{CNOT}(2t_3+t_4+2t_5)$ by

$$U^3_{CNOT}(2t_3+t_4+2t_5) = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\otimes I.$$

Next we denote operator $\overline{U^3_{CNOT}(t)}$ of CNOT gate that the role of control and target are changed. Let us define $\overline{U^3_{CNOT}(t)}$ by

$$\overline{U^3_{CNOT}(t)} = e^{-i\omega_3 t S_2^{(1)}} U_\Phi(t) e^{i2\omega_5 t (S_3^{(2)} \cdot S_3^{(3)})} e^{-i\omega_5 t S_3^{(3)}} e^{i\omega_3 t S_2^{(1)}}.$$

If we take t_3, t_4, t_5 such that $\omega_3 t_3 = \frac{\pi}{2}$, $\omega t_4 = \pi$, $\omega_5 t_5 = \frac{\pi}{2}$, then one has

$$\begin{aligned} & \overline{U^3_{CNOT}(2t_3 + t_4 + 2t_5)} \\ = \; & e^{-i\omega_3 t_3 S_2^{(1)}} U_\Phi(t_4) e^{i2\omega_5 t_5 (S_3^{(2)} \cdot S_3^{(3)})} e^{-i\omega_5 t_5 S_3^{(3)}} e^{i\omega_3 t_3 S_2^{(1)}}. \end{aligned}$$

and the matrix form of $\overline{U^3_{CNOT}(2t_3 + t_4 + 2t_5)}$ is obtained by

$$\overline{U^3_{CNOT}(2t_3 + t_4 + 2t_5)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \otimes I.$$

It means that this unitary operator $\overline{U^3_{CNOT}(2t_3 + t_4 + 2t_5)}$ is performed as CNOT gate of three particles.

Third, we construct a SWAP gate based on ESR. The SWAP gate swaps two qubits. Let us define a unitary operator $U_{SWAP}(t)$ by

$$\begin{aligned} U_{SWAP}(6t_3 + 3t_4 + 6t_5) &= U^3_{CNOT}(2t_3 + t_4 + 2t_5)\overline{U^3_{CNOT}(2t_3 + t_4 + 2t_5)} \\ &\quad \times U^3_{CNOT}(2t_3 + t_4 + 2t_5) \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned}$$

Thus the unitary operator $U_{SWAP}(6t_3 + 3t_4 + 6t_5)$ related to the SWAP gate can be reconstructed by the combination of U^3_{CNOT}. It means that this unitary operator is performed as SWAP gate based on ESR.

We reconstruct the CCNOT gate based on ESR using by Controlled-phase and SWAP gate.

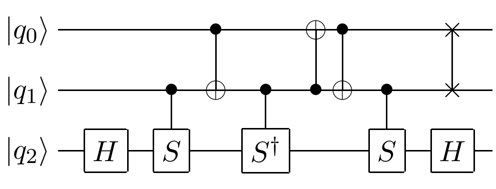

Fig. CCNOT

If U^3_H is operator related to a Hadamard gate of three particles then it is denoted by $e^{-i\omega_3 t_3 S_2^{(3)}}$. The unitary operator U_{CCNOT} related to the CCNOT gate can be reconstructed by the combination of U^3_{CNOT} ,U_S , U^3_H and U_{SWAP} as

$$U_{CCNOT}(3t_1 + 6t_2 + 14t_3 + 6t_4 + 12t_5)$$

$$
\begin{aligned}
= \quad & U_{SWAP}(6t_3+3t_4+6t_5)U_H^3(t_3)(I\otimes U_S(t_1+2t_2)) \\
& U_{CNOT}(2t_3+t_4+2t_5)^3\overline{U_{CNOT}^3(2t_3+t_4+2t_5)} \\
& (I\otimes U_S^*(t_1+2t_2))U_{CNOT}(2t_3+t_4+2t_5)^3 \\
& (I\otimes U_S(t_1+2t_2))U_H^3(t_3)
\end{aligned}
$$

and the matrix form of $U_{CCNOT(3t_1+6t_2+14t_3+6t_4+12t_5)}$ is obtained by

$$
U_{CCNOT}(3t_1+6t_2+14t_3+6t_4+12t_5)=\begin{pmatrix}
1&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&1&0
\end{pmatrix}.
$$

This unitary operator U_{CCNOT} is performed as CCNOT gate based on ESR. Quantum channel denoting the CCNOT gate based on ESR is defined by

$$
\begin{aligned}
& \Lambda^*_{CCNOT}(\,\cdot\,) \\
\equiv\quad & U_{CCNOT}(3t_1{+}6t_2{+}14t_3{+}6t_4{+}12t_5)(\,\cdot\,) \\
& \times U^*_{CCNOT}(3t_1{+}6t_2{+}14t_3{+}6t_4{+}12t_5).
\end{aligned}
$$

For the initial state $|\psi(0)\rangle\langle\psi(0)|$ at time 0, the output state of $\Lambda^*_{CCNOT(3t_1+6t_2+14t_3+6t_4+12t_5)}$ is obtained by

$$
\begin{aligned}
& \Lambda^*_{CCNOT}(|\psi(0)\rangle\langle\psi(0)|) \\
= \quad & |\psi(3t_1+6t_2+14t_3+6t_4+12t_5)\rangle\langle\psi(3t_1+6t_2+14t_3+6t_4+12t_5)|.
\end{aligned}
$$

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