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On a boundary value problem for a third-order parabolic-hyperbolic type equation with a displacement boundary condition in its hyperbolicity domain

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Abstract

In the article, we investigate a boundary-value problem with a third-order inhomogeneous parabolic-hyperbolic equation with a wave operator in a hyperbolicity domain. A linear combination with variable coefficients in terms of derivatives of the sought function on independent characteristics, as well as on the line of type and order changing is specified as a boundary condition. We have established necessary and sufficient conditions that guarantee existence and uniqueness of a regular solution to the problem under study. In some cases, a solution representation is written out explicitly.

Keywords: degenerate hyperbolic equation, equation with multiple characteristics, parabolic-hyperbolic equation of third order, problems with a shift.

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Statement of the problem. A summary of the outcomes

Consider the next equation in the Euclidean plane points x and y

$$f = \begin{cases} u_{xx} - u_{yy}, & y < 0, \\ u_{xxx} - u_y, & y > 0, \end{cases} \tag{1}$$

where

$$f = f(x, y) = \begin{cases} f_1(x, y), & y < 0, \\ f_2(x, y), & y > 0 \end{cases}$$

is a given function, $u = u(x, y)$ is the desired function.

Equation (1) as $y > 0$ coincides with the equation

$$u_{xxx} - u_y = f_2(x, y), \tag{2}$$

which belongs to the class of third order equations with multiple characteristics [1, p. 9] of parabolic type [2, p. 69] and as $y < 0$ equation (1) coincides with the inhomogeneous wave equation

$$u_{xx} - u_{yy} = f_1(x, y). \tag{3}$$

Thus equation (1) is parabolic-hyperbolic equation with type and order degeneration along the line $y = 0$ and, as stated in [3], study of boundary value problems for the above equations brings a new aspect to the mixed type equations theory.

Equation (1) is considered in the domain Ω bounded by $AC : x + y = 0$ and $CB : x - y = r$ of equation (3) as $y < 0$ leaving the point $C = (r/2, -r/2)$ and passing through the points $A = (0, 0)$ and $B = (r, 0)$ respectively, and also a rectangle with vertices A, B, $A_0 = (0, h)$, $B_0 = (r, h)$, $h > 0$, as $y > 0$. Denote $\Omega_1 = \Omega \cap \{y < 0\}$, $\Omega_2 = \Omega \cap \{y > 0\}$, $J = \{(x, 0) : 0 < x < r\}$, $\Omega = \Omega_1 \cup \Omega_2 \cup J$ and assume that $f_i \in C(\overline{\Omega_i})$, $i = 1, 2$.

The function $u = u(x, y)$ in the class $C(\overline{\Omega}) \cap C^1(\Omega) \cap C^2(\Omega_1) \cap C^3_x(\Omega_2)$, u_x, $u_y \in L_1(J)$ satisfying equation (1) is *regular solution of equation* (1) in Ω.

The study problem is as follows.

Problem 1. *Find a regular solution to equation* (1) *in the domain* Ω *satisfying the conditions*

$$u(0, y) = \varphi_1(y), \quad u_x(0, y) = \varphi_2(y), \quad u(r, y) = \varphi_3(y), \quad 0 \leqslant y < h, \tag{4}$$

$$\alpha(x) \frac{d}{dx} u\left[\theta_0(x)\right] + \beta(x) \frac{d}{dx} u\left[\theta_r(x)\right] + \gamma(x)\, u_x\,(x, 0) + \\ + \delta(x)\, u_y\,(x, 0) = \psi(x), \quad 0 < x < r, \tag{5}$$

where $\theta_0(x) = \left(\frac{x}{2}; -\frac{x}{2}\right)$, $\theta_r(x) = \left(\frac{x+r}{2}; \frac{x-r}{2}\right)$ *are affixes of intersection of the characteristics of the equation* (3) *leaving the point* $(x, 0)$ *with* AC *and* BC *respectively;* $\varphi_1(y)$, $\varphi_2(y)$, $\varphi_3(y)$*;* $\alpha(x)$, $\beta(x)$, $\gamma(x)$, $\delta(x)$, $\psi(x)$ *are smooth enough given functions.*

Formulated problem (1), (4), (5) belongs to the class of the Nakhushev nonlocal boundary value problems with displacement [4].

The problem with boundary conditions connecting values of a sought solution on the characteristics of the both families for Lavrent'ev–Bitsadze equation was first posed and investigated in [5].

In [6,7] the displacement boundary value problem was introduced, and a number of nonlocal boundary value problems with various types of displacements for hyperbolic, degenerate hyperbolic, and mixed-type equations have been investigated since. In particular, in [6] the existence of an unique solution to the nonlocal problem for equation (3) with the conditions

$$u(x,0)=\tau(x), \quad 0\leqslant x\leqslant r, \tag{6}$$

and (5) as $\gamma(x)=\delta(x)\equiv 0$, $\alpha(x)\neq\beta(x)\ \forall\, x\in[0,r]$ has been proved.

In [7], the way of posing non-local boundary value problems with displacement for a degenerating hyperbolic equation of the form

$$(-y)^m u_{xx}-u_{yy}=0, \quad m=\text{const}>0 \tag{7}$$

is offered employing D^{ε}_{cx} the Riemann–Liouville fractional derivatives. The criteria for the unique solvability with conditions (6) and

$$\alpha(x)D^{1-\varepsilon}_{0x}u\left[\theta_0(x)\right]+\beta(x)D^{1-\varepsilon}_{rx}u\left[\theta_r(x)\right]=\psi(x), \quad 0<x<r$$

for equation (7) are determined, where $[\theta_0(x)]$, $[\theta_r(x)]$, are the affixes of the intersection points of the characteristics of equation (7), as above, and what is more $2(m+2)\varepsilon=m$.

Specific cases for displacement related problems include such nonlocal problems as the Bitsadze–Samarsky problem [8–10], Dezin problem [11–13; 14, p. 174], Carleman problem [15], Steklov problem [16, p. 67], Frankl problem [17; 18, p. 339; 19–24], etc. In case of displacement problems, for mixed equations a nonlocal condition is imposed connecting values of desired solution or derivative of a certain order at two, three or more points lying on the boundary characteristics of different families and on the line of degeneracy or the type change line. If one or several coefficients of the displacement problem for mixed type equations is zero it becomes an ordinary Tricomi problem.

The displacement problems are well applicable in mathematical problems modeling in biology (synergetics), transonic gas dynamics. Similar nonlocal boundary conditions arise in the study of heat and mass transfer in capillary-porous media, in mathematical modeling of problems of gas dynamics and nonlocal physical processes, in the study of cell propagation, in the theory of electromagnetic wave propagation within an inhomogeneous media [2,18,25]. A bibliography of research papers devoted to the displacement boundary value problems is presented quite completely in monographs [4,14,26–34].

In [35], the displacement boundary value problem is studied under condition (5) for mixed type equations of second order and a heat equation in the parabolicity domain; a necessary and sufficient condition for the existence of an unique solution is obtained. In this paper, we study the displacement boundary value problem for inhomogeneous parabolic-hyperbolic equation of the third order (1) and a third-order parabolic and wave equations in the hyperbolicity domain. One of the boundary conditions is a linear combination of the sought functions and their derivatives with variable coefficients in AC and BC, as well as in $J=AB$ lines of type and order change. Necessary and sufficient conditions for the existence and uniqueness of a regular solution to the problem under study have been obtained. The solution to the studied problem under certain conditions have been written out explicitly. We have shown that violation of the necessary conditions imposed on the specified functions leads to non-uniqueness of the studied problem. That is, the corresponding homogeneous problem has an infinite number of linear independent solutions. In addition, solutions to a non-homogeneous problem could exist only with additional requirements for the given functions.

We would also like to mention research-related papers [36–42].

Theorem on the existence and uniqueness of a solution

The following theorem holds true.

THEOREM 1. *Assume the given functions $\varphi_1(y)$, $\varphi_2(y)$, $\varphi_3(y)$; $\alpha(x)$, $\beta(x)$, $\gamma(x)$, $\delta(x)$, $\psi(x)$ have the following properties:*

$$\varphi_1(y),\ \varphi_2(y),\ \varphi_3(y) \in C\,[0,h]\,;\ \ \alpha(x),\ \beta(x),\ \psi(x) \in C^1\,[0,r] \cap C^2\,]0,r[\,; \tag{8}$$

$$[\beta(x)+\alpha(x)+\gamma(x)]^2 + [\beta(x)-\alpha(x)+\delta(x)]^2 \neq 0 \quad \forall\, x \in [0,r] \tag{9}$$

and one of the below conditions is satisfied:

$$\alpha(x)+\beta(x)+\gamma(x) \equiv 0 \quad \forall\, x \in [0,r]\,; \tag{10}$$

$$\alpha(x)-\beta(x)-\delta(x) \equiv 0 \quad \forall\, x \in [0,r]\,; \tag{11}$$

$$2\beta(x)+\gamma(x)+\delta(x) \equiv 0 \quad \forall\, x \in [0,r]\,; \tag{12}$$

$$2\alpha(x)+\gamma(x)-\delta(x) \equiv 0 \quad \forall\, x \in [0,r] \ \text{ and } \ r \neq 2\pi n,\ n \in \mathbb{N}; \tag{13}$$

$$\alpha(x)=\alpha, \quad \beta(x)=\beta, \quad \gamma(x)=\gamma, \quad \delta(x)=\delta$$

and

$$r \neq 2\pi n\sqrt{\frac{\beta-\alpha+\delta}{\beta+\alpha+\gamma}}, \quad n \in \mathbb{N} \quad (\alpha,\beta,\gamma,\delta = const); \tag{14}$$

$$[\alpha(x)+\beta(x)+\gamma(x)][\alpha(x)-\beta(x)-\delta(x)] \neq 0,$$

and

$$\Big[\frac{\alpha(x)+\beta(x)+\gamma(x)}{\alpha(x)-\beta(x)-\delta(x)}\Big]' < 0 \quad \forall\, x \in [0,r]. \tag{15}$$

Therefore there is the unique regular solution for problem 1 in the domain Ω.

Proof. Let there be a solution to problem *1* and assume that

$$u(x,0)=\tau(x)\ (0 \leqslant x \leqslant r); \quad u_y(x,0)=\nu(x),\ (0<x<r). \tag{16}$$

Passing to the limit as $y \to +0$ in equation (1) in view of notation used in (16) we obtain fundamental relation for the functions $\tau(x)$ and $\nu(x)$ moved from the parabolic part Ω_2 of the domain Ω to the line $y=0$:

$$\nu(x) = \tau'''(x) + f_2(x,0), \tag{17}$$

and with boundary conditions (4) obtain

$$\tau(0)=\varphi_1(0), \quad \tau'(0)=\varphi_2(0), \quad \tau(r)=\varphi_3(0). \tag{18}$$

Now find fundamental relation for the functions $\tau(x)$ and $\nu(x)$ moved from the hyperbolic part Ω_1 of the domain Ω to the line $y=0$ of the type changing.

A solution to problem (16) for equation (3) in Ω_1 is obtained by d'Alembert's formula [43, p. 59]:

$$u(x,y)=\frac{\tau(x+y)+\tau(x-y)}{2}+\frac{1}{2}\int_{x-y}^{x+y}\nu(s)\,ds+\int_0^y\int_{x-y+t}^{x+y-t}f_1(s,t)\,dsdt. \quad (19)$$

By formula (19) we find

$$\frac{d}{dx}u\left[\theta_0(x)\right]=\frac{1}{2}\left(\tau'(x)-\nu(x)+\int_{-x/2}^{0}f_1(x+t,t)\,dt\right),$$

$$\frac{d}{dx}u\left[\theta_r(x)\right]=\frac{1}{2}\left(\tau'(x)+\nu(x)-\int_{(x-r)/2}^{0}f_1\ (x-t,t)\,dt\right).$$

Substituting values $\frac{d}{dx}u\left[\theta_0(x)\right]$ and $\frac{d}{dx}u\left[\theta_r(x)\right]$ into equation (5) find

$$[\alpha(x)+\beta(x)+\gamma(x)]\tau'(x)-[\alpha(x)-\beta(x)-\delta(x)]\nu(x)=$$
$$=2\psi(x)-\alpha(x)\int_{-x/2}^{0}f_1(x+t,t)\,dt+\beta(x)\int_{(x-r)/2}^{0}f_1(x-t,t)\,dt. \quad (20)$$

Formula (20) is the fundamental relation for the functions $\tau(x)$ and $\nu(x)$ moved from the hyperbolic part Ω_1 of the domain Ω to the line $y=0$ of the type changing.

Assume that initially conditions (8), (9) and (10) are satisfied, i.e. $\alpha(x)+\beta(x)+\gamma(x)\equiv 0$ and hence $2\beta(x)+\gamma(x)+\delta(x)\neq 0\quad\forall\ x\in[0,r]$. In this case by (20) we find

$$\nu(x)=\frac{2\psi(x)}{2\beta(x)+\gamma(x)+\delta(x)}+\frac{\beta(x)+\gamma(x)}{2\beta(x)+\gamma(x)+\delta(x)}\int_{-x/2}^{0}f_1(x+t,t)\,dt+$$
$$+\frac{\beta(x)}{2\beta(x)+\gamma(x)+\delta(x)}\int_{(x-r)/2}^{0}f_1(x-t,t)\,dt. \quad (21)$$

Therefore by equations (17), (18), we easily find the function $\tau(x)$:

$$\tau(x)=\left[1-\frac{x^2}{r^2}\right]\varphi_1(0)+\left[x-\frac{x^2}{r}\right]\varphi_2(0)+\frac{x^2}{r^2}\varphi_3(0)+$$
$$+\frac{1}{2}\int_0^x(x-t)^2\nu(t)\,dt-\frac{x^2}{2r^2}\int_0^r(r-t)^2\nu(t)dt+$$
$$+\frac{1}{2}\int_0^x(x-t)^2f_2(t,0)\,dt-\frac{x^2}{2r^2}\int_0^r(r-t)^2f_2(t,0)\,dt. \quad (22)$$

Under assumption of (8), (9), (11), i.e. for $\alpha(x)-\beta(x)-\delta(x)\equiv 0$ and $2\beta(x)+\gamma(x)+\delta(x)\neq 0\ \ \forall\, x\in[0,r]$ by (20), we arrive at the identity

$$\tau'(x)=\frac{2\psi(x)}{2\beta(x)+\gamma(x)+\delta(x)}-\frac{\beta(x)+\delta(x)}{2\beta(x)+\gamma(x)+\delta(x)}\int_{-x/2}^{0}f_1(x+t,t)\,dt+$$

$$+\frac{\beta(x)}{2\beta(x)+\gamma(x)+\delta(x)}\int_{(x-r)/2}^{0} f_1(x-t,t)\,dt,$$

hence

$$\tau(x)=\int\limits_0^x \frac{2\,\psi(t)}{2\beta(t)+\gamma(t)+\delta(t)}dt-\int\limits_0^x \frac{\beta(t)+\delta(t)}{2\beta(t)+\gamma(t)+\delta(t)}\int\limits_{-t/2}^{0} f_1\,(t+s,\ s)\,dsdt+$$

$$+\int\limits_0^x \frac{\beta(t)}{2\beta(t)+\gamma(t)+\delta(t)}\int\limits_{(t-r)/2}^{0} f_1\,(t-s,\ s)\,dsdt+\varphi_1(0), \qquad (23)$$

moreover, the following conditions should be satisfied:

$$\int_0^r \frac{2\psi(t)}{2\beta(t)+\gamma(t)+\delta(t)}\,dt-\int_0^r \frac{\beta(t)+\delta(t)}{2\beta(t)+\gamma(t)+\delta(t)}\int_{-t/2}^{0} f_1(t+s,s)\,dsdt+$$
$$+\int_0^r \frac{\beta(t)}{2\beta(t)+\gamma(t)+\delta(t)}\int_{(t-r)/2}^{0} f_1(t-s,s)\,dsdt=\varphi_3(0)-\varphi_1(0),$$

$$\frac{2\psi(0)}{2\beta(0)+\gamma(0)+\delta(0)}+\frac{\beta(0)}{2\beta(0)+\gamma(0)+\delta(0)}\int_{-r/2}^{0} f_1(-t,t)\,dt=\varphi_2(0).$$

At that the second sought function $\nu(x)$ in view of conditions (8) and (23) can be obtained by relation (17).

Next, assume that theorem conditions (8), (9), (12) are satisfied. Then by inequality (20) we arrive at the identity

$$\nu(x)=\tau'(x)-\frac{2\psi(x)}{\alpha(x)+\beta(x)+\gamma(x)}+\frac{\alpha(x)}{\alpha(x)+\beta(x)+\gamma(x)}\int_{-x/2}^{0} f_1(x+t,t)\,dt-$$
$$-\frac{\beta(x)}{\alpha(x)+\beta(x)+\gamma(x)}\int_{(x-r)/2}^{0} f_1(x-t,t)\,dt. \quad (24)$$

Exclude from relations (17) and (24) the function $\nu(x)$. Then for the function $\tau(x)$ we arrive at the boundary problem for the equation

$$\tau'''(x)-\tau'(x)=-\frac{2\psi(x)}{\alpha(x)+\beta(x)+\gamma(x)}+\frac{\alpha(x)}{\alpha(x)+\beta(x)+\gamma(x)}\int_{-x/2}^{0} f_1(x+t,t)\,dt-$$
$$-\frac{\beta(x)}{\alpha(x)+\beta(x)+\gamma(x)}\int_{(x-r)/2}^{0} f_1(x-t,t)dt-f_2(x,0) \quad (25)$$

with conditions (18).

The solution of problem (25), (18) is written out explicitly by the formula

$$\tau(x)=\frac{(\operatorname{ch}x-\operatorname{ch}r)\varphi_1(0)+\left[\operatorname{sh}x-\operatorname{sh}r-\operatorname{sh}(x-r)\right]\varphi_2(0)+(1-\operatorname{ch}x)\varphi_3(0)}{1-\operatorname{ch}r}-$$

$$-\int_0^r \frac{2G(x,t)\psi(t)}{\alpha(t)+\beta(t)+\gamma(t)}dt+\int_0^r \frac{\alpha(t)G(x,t)}{\alpha(t)+\beta(t)+\gamma(t)}\int_{-t/2}^0 f_1(t+s,s)\,dsdt-$$
$$-\int_0^r \frac{\beta(t)G(x,t)}{\alpha(t)+\beta(t)+\gamma(t)}\int_{(t-r)/2}^0 f_1(t-s,s)\,dsdt-\int_0^r G(x,t)f_2(t,0)\,dt,$$

where

$$G(x,t)=\begin{cases} a(1-\operatorname{ch}x), & 0\leqslant x<t,\\ a(1-\operatorname{ch}x)+\operatorname{ch}(x-t)-1, & t<x\leqslant r,\end{cases}\qquad a=\frac{1-\operatorname{ch}(r-t)}{1-\operatorname{ch}r}$$

is the Green function of the differential operator $L[\tau]=\tau'''(x)-\tau'(x)$ involving conditions (18).

Under conditions (8), (9), (13) we get

$$\nu(x)=-\tau'(x)+\frac{2\psi(x)}{\alpha(x)+\beta(x)+\gamma(x)}-\frac{\alpha(x)}{\alpha(x)+\beta(x)+\gamma(x)}\int_{-x/2}^0 f_1(x+t,t)\,dt+$$
$$+\frac{\beta(x)}{\alpha(x)+\beta(x)+\gamma(x)}\int_{(x-r)/2}^0 f_1(x-t,t)\,dt. \quad (26)$$

$$\tau(x)=\frac{\cos x-\cos r}{1-\cos r}\varphi_1(0)+\left[\sin x-\frac{\sin r\,(1-\cos x)}{1-\cos r}\right]\varphi_2(0)+\frac{1-\cos x}{1-\cos r}\varphi_3(0)+$$
$$+\int_0^r \frac{2G(x,t)\psi(t)}{\alpha(t)+\beta(t)+\gamma(t)}dt-\int_0^r \frac{\alpha(t)G(x,t)}{\alpha(t)+\beta(t)+\gamma(t)}\int_{-t/2}^0 f_1(t+s,s)\,dsdt+$$
$$+\int_0^r \frac{\beta(t)G(x,t)}{\alpha(t)+\beta(t)+\gamma(t)}\int_{(t-r)/2}^0 f_1(t-s,s)\,dsdt-\int_0^r G(x,t)f_2(t,0)\,dt,$$

where

$$G(x,t)=\begin{cases} b(1-\cos x), & 0\leqslant x<t,\\ b(1-\cos x)+1-\cos(x-t), & t<x\leqslant r,\end{cases}\qquad b=-\frac{1-\cos(r-t)}{1-\cos r}$$

is the Green function of the operator $L[\tau]=\tau'''(x)+\tau'(x)$ involving conditions (18).

Further we consider the case when conditions (8), (9), (14) are satisfied, i.e. $\alpha(x)=\alpha$, $\beta(x)=\beta$, $\gamma(x)=\gamma$, $\delta(x)=\delta$ (α, β, γ, $\delta=$ const). If $\alpha+\beta+\gamma\equiv 0$ then as well as for $\alpha=\alpha(x)$, $\beta=\beta(x)$, $\gamma=\gamma(x)$ and $\alpha(x)+\beta(x)+\gamma(x)\equiv 0$ $\forall\, x\in[0,r]$ the functions $\tau(x)$ and $\nu(x)$ are uniquely determined by (21) and (22). But if $\alpha-\beta-\delta=0$ then $\tau(x)$ and $\nu(x)$ are found by formulae (23) and (17) respectively.

Now let $(\alpha+\beta+\gamma)\,(\alpha-\beta-\delta)\neq 0$ and condition (14) be satisfied. By relations (17) and (20) for $\tau(x)$ we arrive at the boundary problem for the equation

$$\tau'''(x)-\frac{\alpha+\beta+\gamma}{\alpha-\beta-\delta}\tau'(x)=\frac{\alpha}{\alpha-\beta-\delta}\int_{-x/2}^0 f_1(x+t,t)\,dt-$$

$$-\frac{\beta}{\alpha-\beta-\delta}\int_{(x-r)/2}^{0} f_1(x-t,t)\,dt-\frac{2\psi(x)}{\alpha-\beta-\delta}-f_2(x,0) \quad (27)$$

subject to conditions (18).

The solution to the problem (18) for equation (27) is written out with respect to the sign of the value of $(\alpha+\beta+\gamma)(\alpha-\beta-\delta)$ using one of the following formulae:

$$\tau(x)=\frac{\alpha}{\alpha-\beta-\delta}\int_0^r G(x,t)\int_{-t/2}^{0} f_1(t+s,s)\,dsdt-$$
$$-\frac{\beta}{\alpha-\beta-\delta}\int_0^r G(x,t)\int_{(t-r)/2}^{0} f_1(t-s,s)\,ds-\frac{2}{\alpha-\beta-\delta}\int_0^r G(x,t)\psi(t)\,dt-$$
$$-\int_0^r G(x,t)f_2(t,0)dt+\frac{\operatorname{ch}(\sqrt{p}x)-\operatorname{ch}(\sqrt{p}r)}{1-\operatorname{ch}(\sqrt{p}r)}\varphi_1(0)+$$
$$+\frac{\operatorname{sh}(\sqrt{p}x)-\operatorname{sh}(\sqrt{p}r)+\operatorname{sh}\left[\sqrt{p}(r-x)\right]}{\sqrt{p}\left[1-\operatorname{ch}(\sqrt{p}r)\right]}\varphi_2(0)+\frac{1-\operatorname{ch}(\sqrt{p}x)}{1-\operatorname{ch}(\sqrt{p}r)}\varphi_3(0),$$

if $p=\frac{\alpha+\beta+\gamma}{\alpha-\beta-\delta}>0$. Here

$$G(x,t)=\frac{1}{p}\begin{cases} a\left[1-\operatorname{ch}(\sqrt{p}x)\right], & 0\leqslant x<t,\\ a\left[1-\operatorname{ch}(\sqrt{p}x)\right]+\operatorname{ch}\left[\sqrt{p}(x-t)\right]-1, & t<x\leqslant r,\end{cases}$$

$$a=\frac{1-\operatorname{ch}\left[\sqrt{p}(r-t)\right]}{1-\operatorname{ch}(\sqrt{p}r)},$$

or

$$\tau(x)=\frac{\alpha}{\alpha-\beta-\delta}\int_0^r G(x,t)\int_{-t/2}^{0} f_1(t+s,s)\,dsdt-\int_0^r G(x,t)f_2(t,0)\,dt-$$
$$-\frac{\beta}{\alpha-\beta-\delta}\int_0^r G(x,t)\int_{(t-r)/2}^{0} f_1(t-s,s)\,ds-\frac{2}{\alpha-\beta-\delta}\int_0^r G(x,t)\psi(t)\,dt+$$
$$+\frac{\cos(\sqrt{-p}x)-\cos(\sqrt{-p}r)}{1-\cos(\sqrt{-p}r)}\varphi_1(0)+\frac{1-\cos(\sqrt{-p}x)}{1-\cos(\sqrt{-p}r)}\varphi_3(0)+$$
$$+\frac{\sin(\sqrt{-p}x)-\sin(\sqrt{-p}r)+\sin\left[\sqrt{-p}(r-x)\right]}{\sqrt{-p}\left[1-\cos(\sqrt{-p}r)\right]}\varphi_2(0),$$

where

$$G(x,t)=\frac{1}{p}\begin{cases} b\left[1-\cos(\sqrt{-p}x)\right], & 0\leqslant x<t,\\ b\left[1-\cos(\sqrt{-p}x)\right]+\cos\left[\sqrt{-p}(x-t)\right]-1, & t<x\leqslant r,\end{cases}$$

$p=\frac{\alpha+\beta+\gamma}{\alpha-\beta-\delta}<0$, $b=\frac{1-\cos[\sqrt{-p}(r-t)]}{1-\cos(\sqrt{-p}r)}$, while $p\neq-\left(\frac{2\pi n}{r}\right)^2$, $n\in\mathbb{N}$.

Thus, under the assumption of (8), (9) and also under the assumption of at least one of the conditions of (10), (11), (12), (13) or (14) by the obtained above representations it immediately implies the existence and uniqueness of functions $u\,(x,0)=\tau(x)$ and $u_y\,(x,0)=\nu(x)$.

Now we can pass to studying the general case. Let (8), (9) and (15) be satisfied for the given functions $\varphi_1(y)$, $\varphi_2(y)$, $\varphi_3(y)$; $\alpha(x)$, $\beta(x)$, $\gamma(x)$, $\delta(x)$, $\psi(x)$. First we prove the uniqueness of a regular solution to the problem 1. Consider the homogeneous problem corresponding to problem 1, that is, we let $\varphi_i(y) \equiv 0$, $(i = 1, 2, 3)$ $\forall\, y \in [0, h]$; $\psi(x) \equiv 0$, $\forall\, x \in [0, r]$; $f(x, y) \equiv 0$ $\forall\, (x, y) \in \bar{\Omega}$.

Consider the integral

$$J^* = \int_0^r \tau(x)\nu(x)\, dx.$$

By relation (17) in view of (18) obtain

$$J^* = -\frac{1}{2}[\tau'(r)]^2 \leqslant 0, \tag{28}$$

while by (20) with (15) and (18) we have

$$J^* = \int_0^r \frac{\alpha(x) + \beta(x) + \gamma(x)}{\alpha(x) - \beta(x) - \delta(x)} \tau(x)\tau'(x)\, dx = \\ = -\frac{1}{2}\int_0^r \Big[\frac{\alpha(x) + \beta(x) + \gamma(x)}{\alpha(x) - \beta(x) - \delta(x)}\Big]' \tau^2(x)\, dx \geqslant 0. \tag{29}$$

(28) and (29) imply the identity $J^* = 0$, which by (15) can hold if and only if $\tau(x) \equiv 0$. At that by relations (17) and (20) we can see that $\nu(x) \equiv 0$.

Thus, let us show that under the assumption (15) of theorem 1 the functions $\tau(x)$ and $\nu(x)$ are identically zero for the homogeneous problem corresponding to problem 1. At the same time formula (19) implies immediately that $u(x, y) \equiv 0$ in $\bar{\Omega}_1$. In the domain Ω_2 we arrive at the problem on finding a solution to the homogeneous equation $u_{xxx} - u_y = 0$, $(x, y) \in \Omega_2$ satisfying the homogeneous initial $u(x, 0) = 0$ $(0 \leqslant x \leqslant r)$ and boundary $u(0, y) = 0$, $u_x(0, y) = 0$, $u(r, y) = 0$ $(0 \leqslant y \leqslant r)$ conditions. This problem as it stated in [1, p. 144] has only a trivial solution $u(x, y) \equiv 0$ $\forall\, (x, y) \in \bar{\Omega}_2$. Therefore the solution $u(x, y)$ to the homogeneous problem corresponding to the problem under problem 1 is identically zero in the whole domain $\bar{\Omega}$; this implies the uniqueness of a regular solution to problem (1), (4), (5).

Now we prove the existence of a regular solution to problem 1 subjected to conditions (8), (9) and (15). By relations (17) and (20) we arrive at the problem of finding a solution to the equation

$$\tau'''(x) - \frac{\alpha(x) + \beta(x) + \gamma(x)}{\alpha(x) - \beta(x) - \delta(x)}\tau'(x) = \\ = -\frac{2\psi(x)}{\alpha(x) - \beta(x) - \delta(x)} \frac{\alpha(x)}{\alpha(x) - \beta(x) - \delta(x)} \int_{-x/2}^0 f_1(x + t, t)\, dt - \\ - \frac{\beta(x)}{\alpha(x) - \beta(x) - \delta(x)} \int_{(x-r)/2}^0 f_1(x - t, t)\, dt - f_2(x, 0) \tag{30}$$

satisfying condition (18).

By means of repeated integration 3 times of equation (30) within the limit from 0 to x in view of boundary conditions (18) the solution to problem (30) (18) is equivalently reduced to the solution of the integral equation

$$\tau(x) = \int_0^r L(x,t)\tau(t)dt + \left[1 - \frac{x^2}{r^2}\right]\varphi_1(0) + \left[x - \frac{x^2}{r}\right]\varphi_2(0) + \frac{x^2}{r^2}\varphi_3(0) - \\ - \frac{x^2}{2r^2}\int_0^r (r-t)^2 F(t)\, dt + \frac{1}{2}\int_0^x (x-t)^2 F(t)\, dt, \quad (31)$$

where

$$L(x,t) = \frac{1}{r^2}\begin{cases} x^2 K(r,t), & 0 \leqslant x \leqslant t, \\ x^2 K(r,t) - r^2 K(x,t), & t \leqslant x \leqslant r, \end{cases}$$

$$K(x,t) = (x-t)p(t) - \frac{(x-t)^2}{2}p'(t), \quad p(x) = \frac{\alpha(x) + \beta(x) + \gamma(x)}{\alpha(x) - \beta(x) - \delta(x)},$$

$$F(x) = \frac{\alpha(x)}{\alpha(x) - \beta(x) - \delta(x)}\int_{-x/2}^0 f_1(x+t,t)\, dt - \\ - \frac{\beta(x)}{\alpha(x) - \beta(x) - \delta(x)}\int_{(x-r)/2}^0 f_1(x-t,t)\, dt - \frac{2\psi(x)}{\alpha(x) - \beta(x) - \delta(x)} - f_2(x,0).$$

By properties (8) to the given functions $\alpha(x)$, $\beta(x)$, $\gamma(x)$, $\delta(x)$, $\psi(x)$, $\varphi_i(y)$, ($i = 1,2,3$) and $f(x,y)$ we can conclude that equation (31) is the Fredholm integral equation of the second kind with the kernel $L(x,t) \in C\big([0,r] \times [0,r]\big)$; the right hand $F(x)$ is of the class $C[0,r]$. Unconditionally unique solvability of equation (31) follows from the uniqueness of the solution to problem 1 and, what is more, the solution $\tau = \tau(x)$ belongs to the class $C[0,r] \cap C^3(0,r)$. Employing the obtained value $\tau = \tau(x)$ by (17) or (20) we can find function $\nu = \nu(x)$ as well.

Once the functions $\tau(x)$ and $\nu(x)$ are found the solution to problem 1 in Ω_1 is defined as a solution to problem (16) for equation (3) and is written out by formula (19) while in the domain Ω_2 we arrive at the problem of finding a regular solution to equation (2) satisfying the initial $u(x,0) = \tau(x)$ and boundary (4) conditions; the solution to the above problem is written out in [1]. Theorem 1 is proved. □

Assume that for the coefficients $\alpha(x)$, $\beta(x)$, $\gamma(x)$, $\delta(x)$ condition (9) is violated, i.e. the identity

$$[\alpha(x) + \beta(x) + \gamma(x)]^2 + [\beta(x) - \alpha(x) + \delta(x)]^2 = 0 \quad \forall\, x \in [0,r]. \quad (32)$$

holds true.

Identity (32) holds if for example

$$\alpha(x) = \frac{\delta(x) - \gamma(x)}{2} \quad \text{and} \quad \beta(x) = -\frac{\delta(x) + \gamma(x)}{2}.$$

In this case, the homogeneous problem corresponding to problem 1 has an infinite number of linearly independent solutions of the form

$$2u(x,y) = \begin{cases} g''(x+y) - g''(x-y) + g(x+y) + g(x-y), & y < 0, \\ \dfrac{2}{\pi}\displaystyle\int_0^r G(x,y;\xi,0)g(\xi)\, d\xi, & y > 0, \end{cases}$$

where $g(x)$ is an arbitrary function of the class $C^2[0,r]\cap C^4]0,r[$, and $G(x,y;\xi,\eta)$ is the Green function of the operator $Lu = u_{xxx} - u_y$ with boundary (4) and initial $u(x,0) = g(x)$ conditions [1, pp. 135–137]. The solution to the inhomogeneous problem 1 with condition (32) exists for the given functions $\alpha(x)$, $\beta(x)$, $\psi(x)$, $f(x,y)$ if and only if it satisfies the additional condition:

$$2\psi(x) = \alpha(x)\int_{-x/2}^{0} f_1(x+t,t)\,dt - \beta(x)\int_{(x-r)/2}^{0} f_1(x-t,t)\,dt. \tag{33}$$

In the domain Ω_1 a set of solutions to problem 1 subjected to condition (33) is written out by the formula

$$u(x,y) = \frac{g''(x+y) - g''(x-y)}{2} + \frac{g(x+y)+g(x-y)}{2} + \\ + \frac{1}{2}\int_{x-y}^{x+y} f_2(t,0)\,dt + \frac{1}{2}\int_0^y \int_{x-y+t}^{x+y-t} f_1(s,t)\,dsdt,$$

while in Ω_2 for the set of solutions to problem 1 holds the representation [1]

$$\pi u(x,y) = \int_0^y G_{\xi\xi}(x,y;0,\eta)\varphi_1(\eta)\,d\eta - \int_0^y G_{\xi}(x,y;0,\eta)\varphi_2(\eta)\,d\eta - \\ - \int_0^y G_{\xi\xi}(x,y;r,\eta)\varphi_3(\eta)\,d\eta + \int_0^r G(x,y;\xi,0)g(\xi)\,d\xi - \\ - \int_0^h \int_0^r G(x,y;\xi,\eta) f_2(\xi,\eta)\,d\xi d\eta.$$

If condition (13) of Theorem 1 is violated, i.e. as $2\alpha(x)+\gamma(x)-\delta(x)\equiv 0\ \forall\, x\in[0,r]$ and $r = 2\pi n$, $n\in\mathbb{N}$ homogeneous problem (18) for the system of equations (17), (26) has a nontrivial solutions of the form $\tau(x) = c(1-\cos x)$, $c = \text{const}$, that implies that the solutions to problem 1 are non-unique.

Similarly if $\alpha(x) = \alpha$, $\beta(x) = \beta$, $\gamma(x) = \gamma$, $\delta(x) = \delta$ and $r = \frac{2\pi n}{\sqrt{-p}}$, $n\in\mathbb{N}$, $(\alpha,\beta,\gamma,\delta = \text{const})$, $p = \frac{\alpha+\beta+\gamma}{\alpha-\beta-\delta}$, i.e. if condition (14) of Theorem 1 is violated a homogeneous problem corresponding to problem (27), (18) as well as in the previous case have nontrivial solutions of the form $\tau(x) = c\,[1-\cos(\sqrt{-p}x)]$, $c = \text{const}$, which also indicates that solutions to problem 1 are non-unique.

Therefore conditions

$$\varphi_1(y), \varphi_2(y), \varphi_3(y) \in C[0,h]; \quad \alpha(x),\beta(x),\psi(x)\in C^1[0,r]\cap C^2]0,r[;$$
$$[\alpha(x)+\beta(x)+\gamma(x)]^2 + [\beta(x)-\alpha(x)+\delta(x)]^2 \neq 0 \quad \forall\, x\in[0,r];$$
$$r\neq 2\pi n,\ \ n\in\mathbb{N}\ \text{ when }\ 2\alpha(x)+\gamma(x)-\delta(x)\equiv 0;$$
$$r \neq \frac{2\pi n}{\sqrt{-p}},\quad n\in\mathbb{N}\ \text{ when }\ \alpha,\ \beta,\ \gamma,\ \delta = \text{const}$$

are necessary for the existence of an unique regular solution to problem (1), (4), (5).

Conclusion. The paper studies a displacement boundary value problem for inhomogeneous parabolic-hyperbolic equation of the third order (1) with a third-order parabolic and wave equations in the hyperbolicity domain. A linear combination of the sought functions is given as one of the boundary conditions. Their derivatives with variable coefficients are in AC and BC, and in $J = AB$ lines of type and order change. A necessary and sufficient conditions for the existence and uniqueness of a regular solution to the problem under study are obtained. In some special cases, the representation of the solution to the studied problem is written out explicitly. We have shown that violation of the obtained necessary conditions imposed on the specified functions leads to non-uniqueness of the studied problem. That is, the corresponding homogeneous problem has an infinite number of linear independent solutions. In addition, solutions to a non-homogeneous problem could exist only with additional requirements for the given functions.

Thus, in contrast to the results obtained in [35], necessary and sufficient conditions (8), (9) for the functions specified become insufficient if in the parabolicity domain consider the third-order equation with multiple characteristics (2) instead of the heat equation.

Competing interests. I declare that I have no competing interests.

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Об одной краевой задаче для уравнения параболического-гиперболического типа третьего порядка с граничным условием смещения в области его гиперболичности

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Аннотация

Исследована краевая задача со смещением для неоднородного уравнения параболо-гиперболического типа третьего порядка с волновым оператором в области гиперболичности, когда в качестве одного из граничных условий задана линейная комбинация с переменными коэффициентами производных от значений искомой функции на независимых характеристиках, а также на линии изменения типа и порядка. Найдены необходимые и достаточные условия существования и единственности регулярного решения задачи. В некоторых частных случаях представление решения исследуемой задачи выписано в явном виде.

Ключевые слова: вырождающиеся уравнения, уравнения с кратными характеристиками, параболо-гиперболическое уравнение третьего порядка, задачи со смещением.

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