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Convective layered flows of a vertically whirling viscous incompressible fluid. Temperature field investigation

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Abstract

The paper discusses a class of exact solutions of the Oberbeck–Boussinesq equations suitable for describing three-dimensional nonlinear layered flows of a vertically swirling viscous incompressible fluid. An inhomogeneous distribution of the velocity field (there is a dependence of the field components on the horizontal coordinates) generates a vertical swirl in the fluid without external rotation (excluding Coriolis acceleration). Setting the linearly distributed heat field and the field of shear stresses at the boundaries of the flow region is one of the reasons inducing convection in a viscous incompressible fluid. The main attention is paid to the study of the temperature field properties. The effect of vertical twist on the distribution of isolines of this field is studied. It is shown that the homogeneous component of the temperature field can be stratified into several zones relative to the reference value, and the number of such zones does not exceed nine. The inclusion of inhomogeneous components of the temperature field can only decrease this number. It is also demonstrated that the class discussed in the paper allows one to generalize the previously obtained results on modeling convective flows of viscous incompressible fluids.

Research Article

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Introduction

One of the main factors causing convection in a viscous fluid is the uneven heating of this fluid. The reasons for the heterogeneity of the temperature field distribution are different, e.g. the presence of a heat source inside the volume occupied by the fluid, heating/cooling of the boundaries of this volume, etc.[1, 2].

The energy equation (or the heat equation as one of its simplest versions) is known to depend, besides the physical characteristics of the liquid, on the value of the flow velocity $\mathbf{V}$ in view of mixing. Thus, it is necessary to take into account the mutual influence of hydrodynamic fields. Another important case illustrating this dependence is a model of viscous fluid flow based on the Boussinesq hypothesis [2]. This hypothesis suggests a linear relationship between fluid density ρ and temperature T. As a result, the specific gravity ρg appearing in the Navier–Stokes vector equation is substituted by the term $g\beta T$, where β is the volume expansion coefficient, and it is neglected in the inertia forces. In this case, the fluid is considered incompressible. Thus, the relationship between the flow velocity field determined by the velocity vector $\mathbf{V}$ and the temperature field T becomes mutual in the sense that both the equation of motion and the heat equation include the components of both fields: the temperature field and the velocity field.

In addition to the Navier–Stokes equation and the heat equation, the constitutive equations for constructing models of viscous fluid mechanics include the law of mass conservation [1–11]. In the case of incompressible fluids, this law is written in the divergent form of the incompressibility equation $\nabla\cdot\mathbf{V} = 0$ [1, 2]. The resulting system consists of five scalar equations with respect to five unknowns, namely the components V_x, V_y, V_z of the velocity vector $\mathbf{V}$, pressure P, and temperature T. When considering a number of practically important flows belonging to the class of layered and shear (unidirectional and non-one-dimensional) flows, a problem arises related to the overdetermination of the Oberbeck–Boussinesq system since $V_z \equiv 0$ for these flows [12–25].

One can resolve such an overdetermined system if, for example, one selects the projections of the velocity vector from a certain generalized class of exact solutions which allows one to satisfy the "unnecessary" equations [12–14, 16–19, 26, 27]. The families of such classes differ, among other things, in that some of them can describe only flows of vertically unvortexed fluids, while others are suitable for modeling flows of fluids with nonzero vertical swirl [12–19, 28–37]. Moreover, taking into account the vertical twist is certain to complicate the structure of the solution to the boundary value problem under study.

The velocity field of convective flows of a vertically swirling fluid was studied in [13, 18, 19]. It was shown that the vertical vorticity component can exist when the fluid does not rotate. Isothermal flows of this kind were studied in [38, 39]. When considering thermal factors, it is important not only to study their influence on the velocity field, but also to evaluate the contribution of the velocity field to the stratification of the temperature field.

This paper considers the exact solution of a boundary value problem describing the convective flow of a viscous fluid under the action of a given field of shear stresses. The effect of a constant vertical swirl on the temperature field is studied, as well as the features of the temperature field distribution depending on the given shear and normal stresses at the boundaries of a horizontal infinite fluid layer.

1. Problem statement. The exact solution of the Oberbeck–Boussinesq system

A system of equations of thermal shear convection in the Boussinesq approximation is considered. For shear flows (the component V_z of the velocity vector $\mathbf{V}$ is assumed to be identically equal to zero), this system takes the form [12–15, 18, 19]:

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = -\frac{\partial P}{\partial x} + \nu \Delta V_x; \quad V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} = -\frac{\partial P}{\partial y} + \nu \Delta V_y;$$
$$\frac{\partial P}{\partial z} = g\beta T; \quad V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = \chi \Delta T; \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0. \tag{1}$$

Here, P is the deviation of the pressure from hydrostatic, divided by the constant mean density ρ of the fluid; T is the deviation from the average temperature; ν, χ are the coefficients of kinematic viscosity and thermal diffusivity of the fluid, respectively; $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

The system of equations (1) is overdetermined. It was shown in [13, 18, 19] that, if we consider the flow velocity field of the form

$$V_x = U(z) + u(z)y, \quad V_y = V(z), \tag{2}$$

the incompressibility equation in system (1) is satisfied identically. In this case, the temperature field T and the pressure field P are be described by linear functions of the longitudinal (horizontal) coordinates as

$$P = P_0(z) + P_1(z)x + P_2(z)y; \quad T = T_0(z) + T_1(z)x + T_2(z)y. \tag{3}$$

It was also shown in [13, 18, 19] that, substituting the families of generalized solutions (2), (3) we can reduce system (1) to the ordinary differential equations system of the following form:

$$u'' = 0, \quad T_1'' = 0, \quad P_1' = g\beta T_1, \quad \chi T_2'' = uT_1, \quad P_2' = g\beta T_2,$$
$$\nu V'' = P_2, \quad \nu U'' = Vu + P_1, \quad \chi T_0'' = UT_1 + VT_2, \quad P_0' = g\beta T_0. \tag{4}$$

Moreover, system (4) is integrated uniquely, and it can have a solution different from the trivial one. Here, the prime denotes derivation with respect to the vertical coordinate z. In what follows, we consider the case of constant vertical twist by setting $u = \Omega = \text{const}$.

2. Boundary value problem

We choose the conditions described in [12, 14, 18] as the boundary conditions for the horizontal temperature gradients T_1, T_2, the horizontal pressure gradients P_1, P_2, the background temperature T_0, the background pressure P_0, and the velocities U and V. We assume that the fluid flows in a horizontal infinite layer, the lower surface of which $z = 0$ is absolutely solid and selected by the reference level of temperature measurement. Without loss of generality, we assume that the reference temperature is zero,

$$T(x, y, 0) = 0.$$

The velocity of the lower boundary $z = 0$ is given as

$$V_x(0) = \Omega y, \quad V_y(0) = 0.$$

At the undeformed (free) upper boundary $z = h$, constant atmospheric pressure is set and, by analogy with the temperature setting, it is counted from zero,

$$P(x, y, h) = 0.$$

We also assume that the field of shear stresses is set at the upper boundary as

$$\eta \frac{\partial V_x}{\partial z} = \eta \frac{\partial U}{\partial z} = \xi_1, \quad \eta \frac{\partial V_y}{\partial z} = \eta \frac{\partial V}{\partial z} = \xi_2.$$

Here, η is the dynamic viscosity coefficient. Note that, due to the structure of the velocity field $\mathbf{V}$, the resulting shear stress field is homogeneous, as in [12, 14, 18]. In addition, thermal sources are set at both boundaries of the fluid layer,

$$T(x, y, 0) = Ax + By, \quad T(x, y, h) = \vartheta + Cx + Dy.$$

In view of the class of generalized solutions (2), (3), the selected boundary conditions are written as follows:

$$\begin{gathered} U(0) = V(0) = 0, \quad \eta U'(h) = \xi_1, \quad \eta V'(h) = \xi_2, \\ T_0(0) = 0, \quad T_1(0) = A, \quad T_2(0) = B, \\ T_0(h) = \vartheta, \quad T_1(h) = C, \quad T_2(h) = D, \\ P_0(h) = P_1(h) = P_2(h) = 0. \end{gathered} \tag{5}$$

The exact polynomial solution of the boundary value problem (4), (5) for the velocity field components for the special case $B = D = 0$ was given and analyzed in [18]; therefore, we restrict ourselves to the exact solution for the temperature field and the pressure field, which has the form

$$\begin{gathered} T_1 = A + (C - A)Z; \\ P_1 = \frac{g\beta h}{2}\big((C - A)Z^2 + 2AZ - (C + A)\big); \\ T_2 = -\frac{\Omega h^2}{6\chi}(1 - Z)Z\big((C + 2A) + (C - A)Z\big); \\ P_2 = \frac{g\beta\Omega h^3}{24\chi}(1 - Z)^2\big((C + A) + 2(C + A)Z + (C - A)Z^2\big); \end{gathered} \tag{6}$$

$$
\begin{aligned}
T_0 = \vartheta Z + \frac{4Z(1-Z)}{12!\eta\nu^2\chi^3}\Big\{ &-23760 g\beta\eta\nu\chi^2 h^5 \times \\
&\times \big[5A^2(2-Z)(1+2Z-Z^2)(4-2Z+Z^2)+ \\
&+ 5AC(26+26Z-30Z^2+5Z^3+5Z^4-2Z^5)+ \\
&+ C^2(82+82Z+82Z^2-58Z^3+5Z^4+5Z^5)\big]- \\
&- 3g\beta\eta\nu\Omega^2 h^9\big[2A^2(2-Z)(912+1368Z+1596Z^2-3680Z^3+ \\
&+ 1998Z^4+294Z^5-833Z^6+336Z^7-42Z^8)+ \\
&+ AC(7851+7851Z+7851Z^2-27569Z^3+15397Z^4+ \\
&+ 921Z^5-3479Z^6-14Z^7+756Z^8-168Z^9)+ \\
&+ 2C^2(2088+2088Z+2088Z^2-4072Z^3-607Z^4+ \\
&+ 1857Z^5-343Z^6-343Z^7+42Z^8+42Z^9)\big]- \\
&- g\beta\eta\chi\Omega^2 h^9\big[A^2(2-Z)(19456+29184Z-9512Z^2-7080Z^3+ \\
&+ 3838Z^4-636Z^5-398Z^6+216Z^7-27Z^8)+ \\
&+ AC(82985+82985Z-23935Z^2-14035Z^3+8141Z^4- \\
&- 637Z^5-637Z^6-142Z^7+243Z^8-54Z^9)+ \\
&+ C^2(43268+43268Z+43268Z^2-10192Z^3-10192Z^4+ \\
&+ 4592Z^5-358Z^6-358Z^7+27Z^8+27Z^9)\big]- \\
&- 9979200h^3\nu^2\xi_1\chi^2\big[A(1+Z-Z^2)+C(1+Z+Z^2)\big]+ \\
&+ 332640h^5\nu^2\xi_2\chi\Omega\big[A(3+3Z+3Z^2-7Z^3+2Z^4)+ \\
&+ C(3+3Z+3Z^2-2Z^3-2Z^4)\big]+ \\
&+ 332640h^5\nu\xi_2\chi^2\Omega\big[A(14+14Z-16Z^2-Z^3+2Z^4)+ \\
&+ C(13+13Z+13Z^2-2Z^3-2Z^4)\big]\Big\}. \qquad (7)
\end{aligned}
$$

Here, $Z = z/h \in [0, 1]$ is the dimensionless vertical coordinate.

The expression for the background pressure P_0 is not given here since it is cumbersome; however, it can be easily obtained by integrating the corresponding equation of system (4) due to the exact solution (7).

Note that the condition $u = 0$, which determines the degeneracy of the class (2) to the class

$$V_x = U(z), \quad V_y = V(z),$$

considered in [12, 14, 16, 17], is equivalent to the condition $\Omega = 0$; therefore, the effect of the parameter Ω on the temperature field topology will be studied in more detail below.

3. Temperature field analysis

For further convenience, we introduce the functions $T_1^{\Omega=0}$, $T_2^{\Omega=0}$, $T_0^{\Omega=0}$, which are obtained from the exact solution (6), (7) when the vertical twist Ω proves to be zero. In this case we have

$$T_1^{\Omega=0} = A + (C - A)Z; \quad T_2^{\Omega=0} = 0;$$

$$T_0^{\Omega=0} = \vartheta Z + \frac{4h^3 Z(1-Z)}{12!\eta\nu\chi}\Big\{-23760 g\beta\eta h^2 \times \times \big[5A^2(2-Z)(1+2Z-Z^2)(4-2Z+Z^2) + + 5AC(26+26Z-30Z^2+5Z^3+5Z^4-2Z^5) + + C^2(82+82Z+82Z^2-58Z^3+5Z^4+5Z^5)\big] - - 9979200\nu\xi_1\big[A(1+Z-Z^2)+C(1+Z+Z^2)\big]\Big\}. \quad (8)$$

3.1. Analysis of the properties of the longitudinal gradients T_1, T_2. Note that, regardless of the magnitude of the twist Ω, the component $T_1 = T_1^{\Omega=0}$ is a monotonic function, and it can take a single zero value in the layer $[0,1]$ only if the inequality

$$T_1(0) \cdot T_1(1) < 0$$

is valid. This inequality is equivalent to the condition

$$AC < 0.$$

Thus, when the longitudinal temperature gradients A and C take values of different signs, the thermal field $T_1 x$ (and the field $T_1^{\Omega=0} x$, respectively) admits stratification at the point $Z = A/(A-C)$.

We now consider the second longitudinal gradient (T_2). Obviously, $T_2^{\Omega=0} y \equiv 0$; therefore, this thermal field does not admit stratifications. Let us now study the behavior of the field $T_2 y$ determined from the expression (6) when $\Omega \neq 0$. It is easy to see that, in the degenerate case $A = C$, the component T_2 takes values of the same sign; therefore, everywhere in the layer, the field $T_2 y$ is determined by either heating or cooling of the fluid.

Let now the horizontal temperature gradients be different ($A \neq C$); therefore, $C - A \neq 0$ and consequently, by virtue of (6), the longitudinal gradient T_2 can be represented as

$$T_2 = -\frac{\Omega h^2 (C-A)}{6\chi}(1-Z)Z(Z+a),$$

where $a = (C+2A)/(C-A)$. It is easy to verify that the function $(1-Z)Z(Z+a)$ can have a single zero inside the layer $[0,1]$ only when $-1 < a < 0$. Therefore, the thermal field $T_2 y$ can change its sign no more than once inside the studied fluid layer.

3.2. Analysis of the properties of background temperature T_0. We now study the features of the behavior of the background temperature. We begin with the case that the vertical vorticity component is zero, $\Omega = 0$. In this case, the background temperature is determined by the expression (8). The field $T_0^{\Omega=0}$ results from the interaction of several individual thermal fields induced by heating the boundaries of the layer under study and setting the shear stress field at the upper boundary. If both gradients A and C are simultaneously zero, the background temperature, according to (8), is determined only by a homogeneous term (with respect to the horizontal coordinates),

$$T_0^{\Omega=0} = \vartheta Z,$$

the temperature T being unaffected by the value of the components ξ_1, ξ_2 of the shear stress field.

Let us now consider the case that only one of the longitudinal temperature gradients is zero. Assume that, for definiteness, $A \neq 0$. In this case, the expression (8) can be represented as

$$T_0^{\Omega=0} = Z \cdot f(Z) = Z\Big[\vartheta - \frac{Ah^3(1-Z)}{1008\eta\nu\chi}\times \\ \times\Big\{g\beta\eta h^2(2-Z)(1+2Z-Z^2)(4-2Z+Z^2) + 84\nu\xi_1(1+Z-Z^2)\Big\}\Big]. \quad (9)$$

It is obvious from (9) that, if some point $Z_1 \in (0,1)$ is the zero of the auxiliary function $f(Z)$, the stratification of the thermal field $T_0^{\Omega=0}$ can occur at this point. Note that the polynomials

$$f_1(Z) = (1-Z)(2-Z)\left(1+2Z-Z^2\right)\left(4-2Z+Z^2\right),$$

$$f_2(Z) = (1-Z)\left(1+Z-Z^2\right),$$

included in the solution (9) are strictly monotonic inside the layer under study. Therefore, the background temperature $T_0^{\Omega=0}$ can have no more than two zero points in the layer $(0,1)$ (Fig. 1).

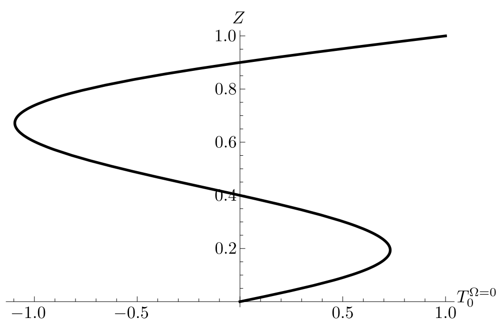

Figure 1. Profile of the temperature $T_0^{\Omega=0}$ defined by the expression (9) when $A \neq 0$, $C = 0$

Assume now that the horizontal gradient is nonzero, $C \neq 0$. We write the solution (8) as

$$T_0^{\Omega=0} = Z \cdot F(Z) = Z\Big[\vartheta - \frac{g\beta C^2 h^5}{7!\nu\chi}(1-Z)\times \\ \times\Big\{5c^2(2-Z)\left(1+2Z-Z^2\right)\left(4-2Z+Z^2\right) + \\ + 5c(26+26Z-30Z^2+5Z^3+5Z^4-2Z^5) + \\ + (82+82Z+82Z^2-58Z^3+5Z^4+5Z^5) + \\ + \frac{420\nu\xi_1}{g\beta\eta h^2 C}\left[c\left(1+Z-Z^2\right)+\left(1+Z+Z^2\right)\right]\Big\}\Big],$$

where $c = A/C$ is a dimensionless parameter. Obviously, all the zero points of the function $F(Z)$ will automatically be the zero points of the background temperature (8). The structure of the function $F(Z)$, in addition to the above polynomials f_1 and f_2, includes the polynomials

$$f_3(Z) = (1 - Z)\left(26 + 26Z - 30Z^2 + 5Z^3 + 5Z^4 - 2Z^5\right),$$

$$f_4(Z) = (1 - Z)\left(82 + 82Z + 82Z^2 - 58Z^3 + 5Z^4 + 5Z^5\right),$$

$$f_5(Z) = (1 - Z)\left(1 + Z + Z^2\right).$$

The functions f_3, f_4, f_5 are also strictly monotonic on the interval $[0, 1]$. There are only four coefficients in front of the polynomials f_i $(i = \overline{1, 6})$ in the solution (8). All these coefficients can be considered independent of each other due to the arbitrary choice of the values of the shear stress ξ_1, the temperature gradient A, the temperature ϑ, and the physical constants determining the viscous fluid under study. The analysis of the properties of the polynomial (8) has shown that, in view of these circumstances, the maximum number of zero points of the background temperature (8) does not exceed three (Fig. 2).

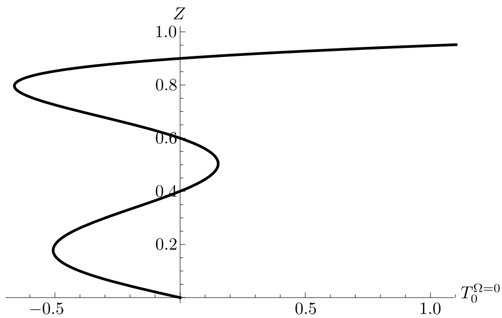

Figure 2. Profile of the temperature $T_0^{\Omega=0}$ defined by the expression (8)

Consequently, the thermal field $T_0^{\Omega=0}$ can both heat and cool the fluid layer; the type of the thermal effect can change no more than three times with the distancing from the lower boundary $Z = 0$ in the direction of the upper boundary $Z = 1$.

Now, let the vertical twist Ω be nonzero. In this case, according to (7), the terms reflecting the presence of nonzero vorticity in the fluid layer are added to the above-mentioned individual thermal fields of various nature.

It is easy to verify that the number of points at which the background temperature T_0 (7) takes a zero value inside the fluid layer $[0, 1]$ does not exceed eleven since the exact solution (7) is an 11th degree polynomial. Moreover, the number of polynomials in the exact solution (7) increases sharply compared with the same number for the thermal field $T_0^{\Omega=0}$. Their number increases to fifteen, all of them are strictly monotonic at $Z \in [0, 1]$. However, the number of independent coefficients in front of these polynomials increases to a lesser extent, i.e., only four coefficients are added, which are determined by two new independent parameters, namely the stress ξ_2 and the actual vertical vorticity component Ω. The study of the localization of the zeros of the polynomial (7) has shown that their number in the layer $[0, 1]$ does not exceed eight (Fig. 3).

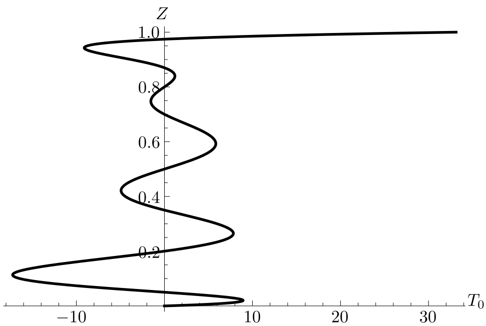

Figure 3. Profile of the temperature T_0 determined by the exact solution (7)

Thus, taking into account the constant spatial acceleration $u(z) = \Omega = \text{const}$ leads to a significant increase in the number of zero points of the thermal field T_0.

4. Superposition of thermal fields

The resulting temperature field is determined by the interaction of three thermal fields: $T_1 x$, $T_2 y$, and T_0. As a result of their superposition, the number of zero points of the temperature T can change as the values of the longitudinal coordinates x and y change. As an illustrative example, we consider the case that the background temperature has four zero points (Fig. 4). This case corresponds to the value $c = -2.04182$ of the dimensionless parameter characterizing the ratio of the longitudinal temperature gradients A and C.

Since the ratio A/C proves to be negative, according to the above analysis, the thermal field $T_1 x$ admits one stratification point (Fig. 5). Herewith, the parameter $a = (C + 2A)/(C - A) = -1.01375$ determining the presence of zero points of the longitudinal temperature gradient T_2 does not fall in the interval $(-1, 0)$, and this means the absence of zero points of the gradient T_2 (Fig. 5). For definiteness, when constructing the profiles of the temperature field components, the following values of the parameters were taken: $C = 1$, $\Omega h^2/(6\chi) = 1$.

The resulting temperature field isolines T are shown in Figs. 6 and 7.

The change in the location of the isolines is considered as an example of the displacement of the zero isotherm in the characteristic isolines of the sections $y = 0$ (Fig. 8) and $x = 0$ (Fig. 9).

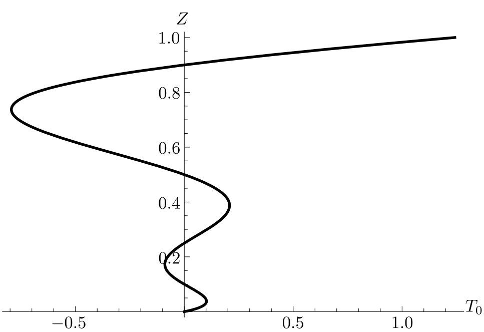

Figure 4. Profile of the temperature T_0 with four zero points

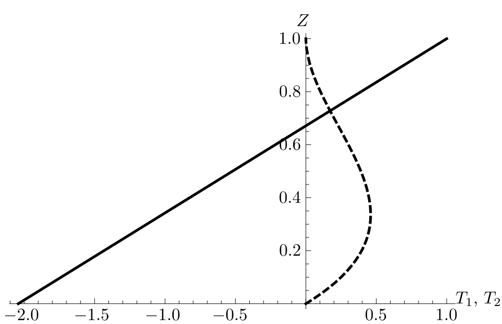

Figure 5. Profile of temperature gradients T_1 (solid line) and T_2 (dashed line)

Figures 8, and 9 clearly illustrate the change (decrease) in the number of stratification points of the final temperature field T, even for small values of the longitudinal coordinates x and y, compared to the number of zero points of the background temperature T_0 marked by a bold line in Figs. 8 and 9.

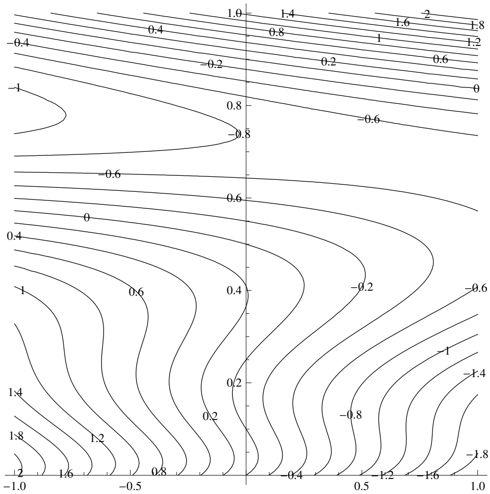

Figure 6. Isolines of the temperature T in the section $y = 0$

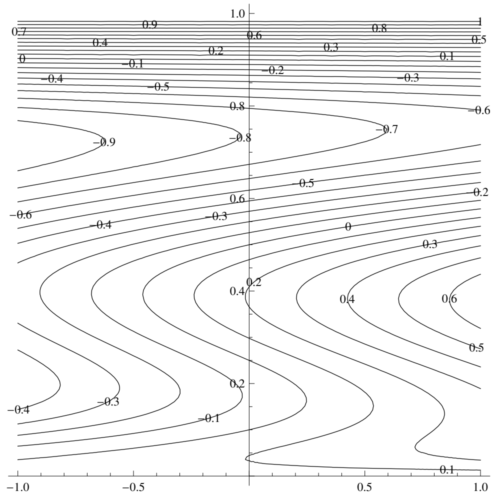

Figure 7. Isolines of the temperature T in the section $x = 0$

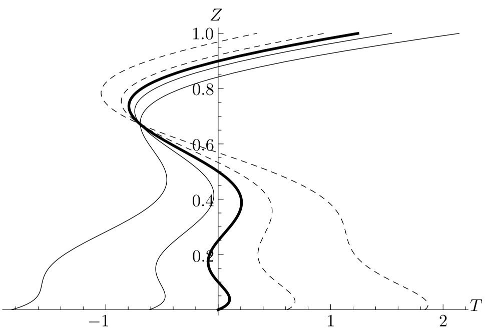

Figure 8. Zero isotherm in the section $y = 0$ for $x = 0.9$, $x = 0.3$, $x = 0$, $x = -0.3$ and $x = -0.9$ (from left to right)

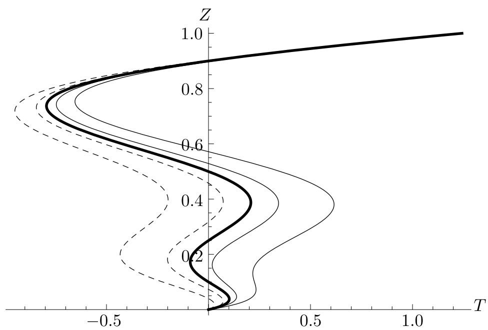

Figure 9. Zero isotherm in the section $x = 0$ for $y = -0.9$, $y = -0.3$, $y = 0$, $y = 0.3$ and $y = 0.9$ (from left to right)

Competing interests. We declare that we have no conflicts of interest in the authorship and publication of this article.

Authors' contributions and responsibilities. We are fully responsible for submitting the final manuscript in print. Each of us has approved the final version of manuscript.

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Конвективные слоистые течения вертикально завихренной вязкой несжимаемой жидкости. Исследование температурного поля

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Аннотация

Приведен класс точных решений уравнений Обербека—Буссинеска, подходящих для описания трехмерных нелинейных слоистых течений вертикально завихренной вязкой несжимаемой жидкости. Неоднородное распределение поля скорости (имеет место зависимость компонент поля от горизонтальных координат) генерирует вертикальную закрутку в жидкости без внешнего вращения (без учета Кориолисова ускорения). Задание на границах области течения линейно распределенных теплового поля и поля касательных напряжений является одной из причин, индуцирующих конвекцию в вязкой несжимаемой жидкости. Основное внимание уделено исследованию свойств температурного поля. Изучено влияние вертикальной закрутки на распределение изолиний этого поля. Показано, что однородная составляющая температурного поля может стратифицироваться на несколько зон относительно отсчетного значения, причем число таких зон не превосходит девяти. Учет неоднородных составляющих поля температуры может приводить только к уменьшению этого числа. Также показано, что представленный в статье класс позволяет обобщить ранее полученные результаты по моделированию конвективных течений вязких несжимаемых жидкостей.

Научная статья

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