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Exact solution to the velocity field description for Couette–Poiseulle flows of binary liquids

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Abstract

Exact solution of the Oberbeck–Boussinesq equations for describing steady flows of binary Poiseuille-type fluids is proposed and studied. The fluid motion is considered in the infinite horizontal layer. Shear flows are described by overdetermined system of equations. Nontrivial exact solution for the Oberbeck–Boussinesq system exists in the class of velocities with two vector components and depends only on the transverse coordinate. This structure of the velocity vector coordinates ensures naturally the fulfillment of the continuity equation as an “extra” equation. The pressure field, the temperature field, and the concentration field of the dissolved substance are described by linear functions of horizontal (longitudinal) coordinates with coefficients that functionally depend on the third coordinate. Fluid layer, as it is shown, can have two points where the velocity becomes zero. In this case, the spiral flow is realized (the hodograph of the velocity vector has a turning point).

Mathematical Modeling, Numerical Methods and Software Complexes
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1. Introduction. Theoretical study of convection started with consideration of the motion of liquids of homogeneous composition [1–9]. It is known, the assumption of liquid homogeneity does not always satisfy accuracy. Sea water and sugar syrup are the classic examples of non-homogeneous liquids where the distribution of salt and sugar induces more intense mixing of the medium than the motion caused by non-uniform temperature distribution [10–13]. Thus, the influence of dissolved substances in the solution on the structure of convection in a hydrodynamic thermal diffusion flow must be taken into account while studying the convective motion of a real liquid [10–12, 14].

The cross dissipative effects of Soret and Dufour should be regarded to describe thermal diffusion in solutions [15–24]. The Soret effect (the influence of temperature change on the impurity distribution) is traditionally included in solution while studying the flows of binary liquids considering the Dufour effect (inverse effect) to be negligibly small [19–22].

The study of binary fluid flows is carried out similarly to the study of convective flows in the Boussinesq approximation. The density dependence on temperature and concentration of the dissolved substance according to the linear law is considered in the term for the density of the Archimedes force and is neglected for the inertial forces [1–3, 18, 22, 25–33]. Consequently, the structure of exact solutions of the equations of binary fluid convection coincides with the expressions of motions generated by heat sources [30].

The exact solution for unidirectional steady convective flows can be presented as the Ostroumov–Birikh–Shliomis solution type $V_x = U(z)$, $P = P_0(z) + xP_1(z)$, $T = T_0(z) + xT_1(z)$ [25, 26, 34–38]. This exact solution describes the superposition of gravitational convection and fluid motion caused by horizontal temperature gradients (Marangoni convection) [26, 39–45]. The Ostroumov–Birikh–Shliomis solution type was used to solve various one-dimensional convective boundary value problems with subsequent study of ansatzes on hydrodynamic stability for various classes of disturbances [2, 3, 27, 29]. The application of this type of exact solutions to unsteady flows was undertaken in several scientific researches [2, 3, 27].

The generalization of the Ostroumov–Birikh–Shliomis solution type was implemented in the papers [18, 19, 21, 22, 32, 46–51] to describe steady-state shear flows in velocity field $V_x = U(z)$, $V_y = V(z)$ with linear two coordinate forms for the temperature and pressure fields: $T = T_0(z) + xT_1(z) + yT_2(z)$ and $P = P_0(z) + xP_1(z) + yT_2(z)$. This exact solution for description of binary fluids was announced in [18]. The linear forms of the force fields of pressure, concentration, and temperature were used to construct classes of exact solutions describing inhomogeneous shear flows [19, 21, 32].

After publication of paper [46], the announced exact solution was used to study shear thermal convective flows. The study of shear flows of binary fluids in infinite horizontal layer was started with the description of Couette-type flows in paper [18]. In this paper, the influence of horizontal pressure gradients (Poiseuille flow [52–55]) on the structure of hydrodynamic fields of moving solutions with one dissolved substance is studied.

2. Motion equations and exact solution. We consider steady-state shear flow of binary viscous incompressible fluid in extended horizontal layer with boundaries formed by a pair of non-deformable parallel planes spaced apart by a distance h. We will also assume that the lower plane is absolutely rigid and motionless, and the upper plane is free. The assumption of negligible deformation of the upper boundary does not allow us to consider fluid motions comparable in scale to the thickness of the studied layer, for example, gravitational, thermocapillary and other types of surface waves [1–3, 6]. We introduce Cartesian coordinate system $Oxyz$ where the Oxy plane coincides with the lower boundary of the layer, and the Oz axis is directed perpendicular to this boundary toward the upper plane (boundary), spaced by a distance h (Fig. 1).

We use the system of thermal diffusion equations regarding the Boussinesq hypothesis of the density dependence to describe the steady-state liquid [1, 18, 30]:

$$
\begin{gathered}
V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}=-\frac{\partial P}{\partial x}+\nu\Big(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\Big),\\
V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}=-\frac{\partial P}{\partial y}+\nu\Big(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2}\Big),\\
\frac{\partial P}{\partial z}=g(\beta_1 T+\beta_2 C),\\
V_x\frac{\partial T}{\partial x}+V_y\frac{\partial T}{\partial y}=(\chi+\alpha^2 dn)\Big(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\Big)+\alpha dn\Big(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\Big),\\
V_x\frac{\partial C}{\partial x}+V_y\frac{\partial C}{\partial y}=\alpha\Big(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\Big)+\alpha d\Big(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\Big),\\
\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0.
\end{gathered}
\tag{1}
$$

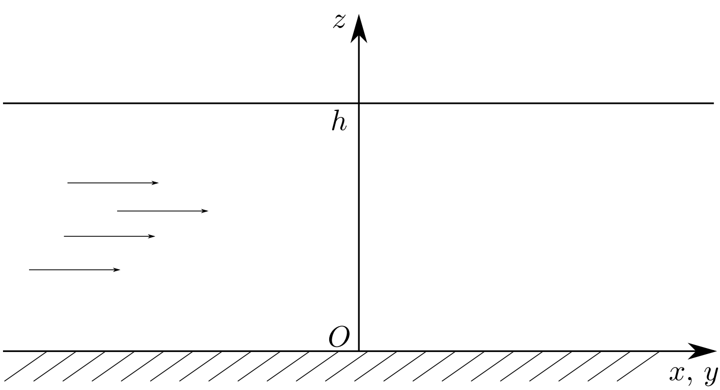

Figure 1. Liquid flow diagram (the Ox and Oy axes are "glued" together in figure; in reality the space is considered as three-dimensional)

We use the following designations in system (1): V_x, V_y are the components of the velocity vector; P is the pressure normalized to the constant average density of the liquid ρ; ν is the kinematic (molecular) viscosity of the mixture; C, T are the concentration of the light component and the temperature of the liquid, respectively, measured from the equilibrium value; g is the gravity acceleration; χ, d, α are the coefficients of thermal diffusivity, diffusion, thermal diffusion, respectively; β_1 and β_2 are the coefficients of temperature and concentration volume expansion of the liquid, respectively; $n = \left[\frac{T}{c_p}\left(\frac{\partial\mu}{\partial C}\right)_{T,P}\right]_0$ is the thermodynamic parameter.

The main feature of system (1) (in addition to its nonlinearity) is its overdetermination: there are six equations to determine the five unknown functions that describe the behavior of the velocity, pressure, temperature, and concentration fields. We assume the relationship between the hydrodynamic fields that allows us to restore the balance between the number of unknown functions and the number of their relations solving the system (1) [30, 49–51]. We consider such relationship as following:

$$
\begin{gathered}
V_x = U(z), \quad V_y = V(z), \\
T = T_0(z) + xT_1(z) + yT_2(z), \quad P = P_0(z) + xP_1(z) + yT_2(z), \\
C = C_0(z) + xC_1(z) + yC_2(z).
\end{gathered}
\tag{2}
$$

The velocity field of the form (2) describes many classical flows, for example, the Couette flow [56], the Poiseuille flow [52–55], the Birikh–Ostroumov flow [25, 26] and many others. The use of the class of exact solutions (2) generalizes the well-known exact Ostroumov–Birikh solution for describing unidirectional convective flows. In [46], it was shown how the exact solution was modified for two-dimensional flows in velocities. Such flows cannot be reduced to unidirectional ones [46]. In the articles [18, 48, 49] similar solutions of shear flows were constructed. The exact solution (2) allows to linearize the equation for the transfer of angular momentum, but this is not an artificial postulation of linear approximation of the Oberbeck–Boussinesq equations (1). It is an opportunity to study the transverse structure of the flow, in which direct calculations of the flow characteristics are impossible. This solution exactly allows to study the structure of horizontal and vertical convection of binary fluid for the boundary value problem announced below for infinite horizontal layer. The exact solution (2) is the simplest solution for the overdetermined system (1) and is necessary for studying flows with non-uniform velocities.

The structure of correlation (2) for the velocity field allows identical satisfaction of the incompressibility equation in system (1). And substitution of correlation (2) into the remaining equations of system (1) leads (due to the independence of the coordinates of the selected Cartesian system) to the system of eleven ordinary differential equations to define eleven unknown functions:

$$
\begin{gathered}
(\chi + \alpha^2 dn)T_1'' + \alpha dnC_1'' = 0, \quad (\chi + \alpha^2 dn)T_2'' + \alpha dnC_2'' = 0; \\
C_1'' + dT_1'' = 0, \quad \alpha C_2'' + \alpha dT_2'' = 0; \\
P_1' = g\beta_1 T_1 + g\beta_2 C_1, \quad P_2' = g\beta_1 T_2 + g\beta_2 C_2, \\
\nu U'' = P_1, \quad \nu V'' = P_2;
\end{gathered}
\tag{3}
$$

$$UT_1 + VT_2 = (\chi + \alpha^2 dn)T_0'' + \alpha dnC_0'', \quad UC_1 + VC_2 = \alpha C_0'' + \alpha dT_0'';$$

$$P_0' = g\beta_1 T_0 + g\beta_2 C_0.$$

Here the prime denotes the derivative with respect to the vertical coordinate z. The coincidence of the number of equations and the number of unknown functions in the system (3) indicates that the selected class of solutions (2) removes the overdetermination of the original Oberbeck–Boussinesq system (1).

The second derivatives of the gradients of the temperature and concentration fields are shown in [18] to be only zero values:

$$T_1'' = 0, \quad T_2'' = 0, \quad C_1'' = 0, \quad C_2'' = 0. \tag{4}$$

The constant coefficients in the linear forms (4) are determined from the boundary conditions. Based on expressions (4), one can obtain exact solution for the horizontal pressure gradients and expression for the components of the velocity vector:

$$\begin{aligned}
P_1 &= g\beta_1\Big(c_1\frac{z^2}{2} + c_2 z\Big) + g\beta_2\Big(c_5\frac{z^2}{2} + c_6 z\Big) + c_9,\\
P_2 &= g\beta_1\Big(c_3\frac{z^2}{2} + c_4 z\Big) + g\beta_2\Big(c_7\frac{z^2}{2} + c_8 z\Big) + c_{10},\\
U &= \frac{g\beta_1}{\nu}\Big(c_1\frac{z^4}{24} + c_2\frac{z^3}{6}\Big) + \frac{g\beta_2}{\nu}\Big(c_5\frac{z^4}{24} + c_6\frac{z^3}{6}\Big) + c_9\frac{z^2}{2} + c_{11}z + c_{12},\\
V &= \frac{g\beta_1}{\nu}\Big(c_3\frac{z^4}{24} + c_4\frac{z^3}{6}\Big) + \frac{g\beta_2}{\nu}\Big(c_7\frac{z^4}{24} + c_8\frac{z^3}{6}\Big) + c_{10}\frac{z^2}{2} + c_{13}z + c_{14}.
\end{aligned} \tag{5}$$

The final integration of the equation system (3) needs in definition of exact solution for uniform (background) components of pressure, temperature and concentration fields. The revealed view of these bulky relations are not shown but can be easily obtained on the base of above written relations. The background temperature T_0 and background concentration C_0 are presented as seventh power polynomials upon the z variable. The background pressure P_0 is eighth power polynomials upon the z variable. The exact solution for velocity field is studied in details in this paper.

3. Boundary value problem. The coefficients appeared from the integration of some equations of system (3), as mentioned above, must be determined from the boundary conditions. We formulate them.

We assume that the no-slip condition is satisfied at the lower boundary (bottom):

$$V_x(0) = V_y(0) = 0.$$

At the upper boundary $z = h$ we assume given uniform distribution of velocities, which corresponds to the motion of the upper boundary as a solid surface:

$$V_x(h) = W\cos\phi, \quad V_y(h) = W\sin\phi.$$

Here W is the value of the velocity at the upper boundary, ψ is the angle the velocity vector forms with the abscissa axis Ox. In other words, the movement of the solid boundary (plate) kinematically causes steady convective flow of the binary liquid.

The boundary condition for pressure is presented as

$$P(h) = S_0 + S_1 x + S_2 y,$$

where S_0 is the atmospheric pressure on the free surface. The condition of impermeability and ideal heat exchange is established for the concentration and temperature at the boundary $z = 0$, respectively:

$$\frac{\partial C}{\partial z} = 0, \quad \frac{\partial T}{\partial z} = 0.$$

In addition, we assume that the temperature and concentration are determined by the following linear forms at the upper boundary of the layer:

$$T(h) = ax + by, \quad C(h) = mx + ny.$$

According to representation (2) for hydrodynamic fields, the formulated boundary conditions are reduced to the following conditions:

$$U = V = 0, \quad \frac{dT_0}{dz} = 0, \quad \frac{dT_1}{dz} = \frac{dT_2}{dz} = 0, \quad \frac{dC_0}{dz} = 0, \quad \frac{dC_1}{dz} = \frac{dC_2}{dz} = 0 \tag{6}$$

at the lower boundary;

$$\begin{gathered} U = W\cos\phi, \quad V = W\sin\phi, \quad T_0 = 0, \quad T_1 = a, \quad T_2 = b, \\ P_0 = S_0, \quad P_1 = S_1, \quad P_2 = S_2, \quad C_0 = 0, \quad C_1 = m, \quad C_2 = n \end{gathered} \tag{7}$$

at the upper boundary of the studied layer.

4. Solution of the boundary value problem. The fulfilment of the boundary conditions (6) and (7) in expressions (4) and (5) leads to the following exact solution:

$$\begin{aligned} V_x = U(z) &= \frac{Z}{6\nu}\left[6W\nu\cos\varphi + h^2(-1+Z)\big(3\nu(S_1 - ghE) + ghE(1+Z)\big)\right], \\ V_y = V(z) &= \frac{Z}{6\nu}\left[6W\nu\sin\varphi + h^2(-1+Z)\big(3\nu(S_2 - ghF) + ghF(1+Z)\big)\right]. \end{aligned} \tag{8}$$

$Z = z/h \in [0, 1]$ is the normalized vertical coordinate in expressions (8), and the following notations are introduced for the coefficients:

$$E = (a\beta_1 + m\beta_2), \quad F = (b\beta_1 + n\beta_2).$$

As was already mentioned above (constructing the general solution (5)), the main focus of the article is concentrated on the study of the velocity field, therefore the exact solutions for the temperature, concentration and pressure fields are bulky and are not given here. We will only note that they have the following structure:

$$T = T_0(z) + ax + by, \quad C = C_0(z) + mx + ny,$$

$$P = P_0(z) + P_1(z)x + P(z)y.$$

In other words, the gradients of the temperature field and the concentration field are constant values determined by the boundary conditions (7). Contrarily the pressure gradients in the studied problem can vary across the layer:

$$P_1 = S_1 + Egh(-1+Z), \quad P_2 = S_2 + Fgh(-1+Z).$$

Moreover, these components of the pressure field can change sign for a certain correlation of values specified at the boundary of the flow region, i.e. the pressure can either increase or decrease across the layer.

The expressions (8) for both components of the velocity field have the same structure and can be obtained from each other by simultaneous replacement $\cos\phi \to \sin\phi$, $S_1 \to S_2$ and $E \to F$. This is the reason that we will study in detail the properties of one component (e.g., the velocity U) and extend the obtained conclusions to the behavior of the second component (the velocity V). For a uniform form of notation, we do not assume that $\sin\phi = 0$, which, generally speaking, we have the right to do without loss of generality, since the orientation of the axes Ox, Oy has not been specified anywhere above.

So, the velocity profile U is determined, in particular, by the number of its zero points. And we begin with analysis of their number and position.

First of all, we note that if the condition $E = S_1 = 0$ is fulfilled (i.e. the influence of thermal diffusion factors and pressure is ignored), then the solution (8) degenerates into a linear dependence

$$U\Big|_{S_1=E=0} = W\cos\varphi\nu Z.$$

In other words, the velocity profile is described by exact Couette-type solution [56], which means that the counter-flows in the direction of the Ox axis are not possible. Therefore the counter-flows are induced by the superposition of the temperature and concentration fields in combination with the non-uniformity of the pressure distribution.

Obviously, some point $Z_* \in (0,1)$ is zero if it is zero solution of the function

$$\begin{aligned} f(z) &= 6W\nu\cos\varphi + h^2(-1+Z)\big(3\nu(S_1 - ghE) + ghE(1+Z)\big) = \\ &= gh^3E(z^2-1) + 3\nu(S_1 - ghE)h^2(-1+Z) + 6W\nu\cos\varphi = \\ &= gh^3Ez^2 + 3\nu(S_1 - ghE)h^2 Z + \big(6W\nu\cos\varphi - gh^3E - 3\nu(S_1 - ghE)h^2\big). \end{aligned}$$

There are no more than two such zeros, since the function f defines a quadratic dependence on the vertical coordinate. There is only one zero point in the layer (near which a zone with reverse flow appears), if the function f takes values of different signs at the ends of the interval $(0,1)$:

$$f(0)f(1) = 6W\nu\cos\varphi\big(6W\nu\cos\varphi - gh^3E - 3\nu(S_1 - ghE)h^2\big) < 0.$$

The values of the function f at the boundaries of the interval $(0,1)$ must coincide in sign for the existence of two points of velocity stratification U:

$$f(0)f(1) = 6W\nu\cos\varphi\big(6W\nu\cos\varphi - gh^3E - 3\nu(S_1 - ghE)h^2\big) > 0.$$

This condition is necessary, but not sufficient. Additionally, it is necessary to require that the values of the function f at its extremum point and at any end of the interval $(0,1)$ should be of different signs (the values at the ends coincide in sign due to the necessary condition, so, for simplicity, we will take the point $Z=1$):

$$f(1)f(Z_{extr})<0.$$

Regarding the structure of the function f, this condition takes an equivalent form:

$$W\cos\varphi\big(-h\big(-3S_1\nu+Egh(-2+3\nu)\big)^2+24WEg\nu\cos\varphi\big)E^{-1}<0.$$

The coefficient E is naturally assumed to be non-zero when we consider the presence of two zero points for the velocity U, since this parameter is a multiplier in the coefficient at the highest (second) power of the polynomial U.

Figure 2 shows the velocity U profiles with and without countercurrents. The velocity profile U is not monotonic in all cases shown in Fig. 2. Nonlinear dependence of expressions (8) on the normalized vertical coordinate Z explains this effect. Constructing the profiles (Fig. 2), the values of the parameters specified at the boundaries of the fluid layer were varied. The values of the angle, layer thickness h, and kinematic viscosity ν were taken constant ($\nu=1.006\cdot10^{-6}$ m^2/s, $h=1$ m, $\phi=\pi/3$). The mentioned conclusions about the number of counter-current zones are also applicable to the velocity V due to the structure of expressions (8).

Figure 3 shows the hodograph of the velocity vector in the case where the velocity U has two zero points in the layer, and the velocity V has one zero point.

The solution of the boundary value problem is locally spiral flow with the illustrations in Fig. 3. This effect (the formation of a loop on the hodograph) is characteristic of the classical Ekman flow, which describes the rotation of liquid. But there is no explicit rotation, only the vorticity is present:

$$\Omega=\operatorname{rot}V=\begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ V_x & V_y & 0\end{vmatrix}=-V'i+U'j.$$

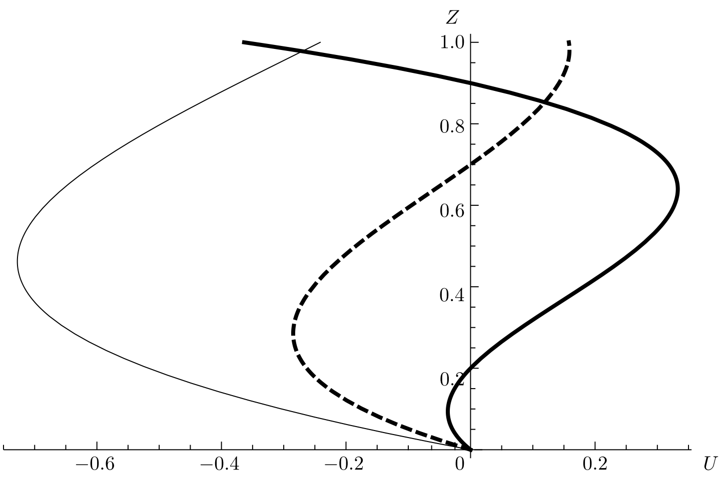

Figure 2. Velocity U profiles with countercurrents (solid heavy line and dash heavy line) and without countercurrents (solid thin line)

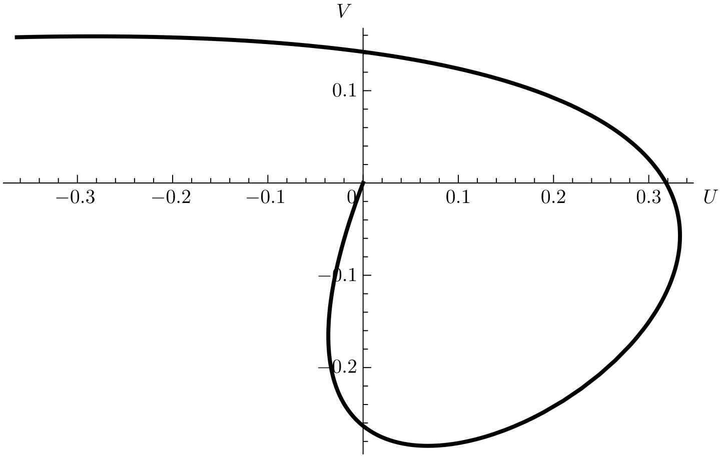

Figure 3. Hodograph of the velocity vector

The considered velocity vector field can be locally potential if the first derivatives of both velocities U, V are simultaneously zero, for some value of the normalized vertical coordinate Z within the interval. In other words, we indicate only on the extremum points of the functions U, V, and these points must coincide.

5. Conclusion. The paper presents a new exact solution describing steady-state thermos-diffusion flows of the Couette type. The solution is obtained in the class of linear functions of the coordinates with nonlinear dependence of the coefficients on remaining coordinates. The presented solution structure, with its formal external simplicity, allows to mark nonlinear effects observed in liquid during its flow and to study the methods of control of these effects. The main attention in the article is focused on the analysis of the number of zero points of the velocity field components and constructing zones with reverse flow. It is shown that the velocity field under certain conditions can be stratified into three zones, in each of which the flow has its own direction. Characteristic flow profiles are given. The local helicity of the flow is illustrated.

Competing interests. The authors declare that they have no competing interests.

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Точное решение для описания поля скоростей течений Куэтта–Пуазейля бинарных жидкостей

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Аннотация

Предложено и изучено точное решение уравнений Обербека–Буссинеска для описания установившихся течений бинарных жидкостей типа Пуазейля. Движение жидкости рассматривается в бесконечном горизонтальном слое. Сдвиговые течения описываются переопределенной системой уравнений. Нетривиальное точное решение для системы Обербека–Буссинеска существует в классе скоростей (две компоненты вектора), зависящих только от поперечной координаты. Данная структура координат вектора скорости обеспечивает автоматическое выполнение уравнения неразрывности («лишнего» уравнения). Поле давления, поле температуры и поле концентрации растворенного вещества описываются линейными формами от горизонтальных (продольных) координат с коэффициентами, функционально зависящими от третьей координаты. Показано, что в слое жидкости могут существовать две точки, где скорость обращается в нуль. В этом случае реализуется спиралевидное течение (годограф вектора скорости имеет точку поворота).

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