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Steady thermo-diffusive shear Couette flow of incompressible fluid. Velocity field analysis

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Abstract

An exact solution that describes steady flow of viscous incompressible fluid with coupled convective and diffusion effects (coupled dissipative Soret and Dufour effects) has been found. To analyze shear fluid flow an over-determined boundary value problem has been solved. The over-determination of the boundary value problem is caused by the advantage of number of equations in non-linear Oberbeck–Boussinesq system against number of unknown functions (two components of velocity vector, pressure, temperature and concentration of dissolved substance). Non-trivial exact solution of system consisting of Oberbeck–Boussinesq equations, incompressibility equation, heat conductivity equation and concentration equation has been built as Birich–Ostroumov class exact solution. Since the exact solution a priori satisfies the incompressibility equation the over-determined system is solvable. Existence of stagnation points is shown both in general flow and in secondary fluid motion without vorticity. Conditions of countercurrent appearance are found.

Keywords: Navier–Stokes equations, exact solution, stratified fluid, mass force field, overdetermined reduced system.

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Introduction. The most frequent reason to induce and support fluid motion is known to be convection [1–5]. Convective mixture principles depend on the nonhomogeneous distribution of temperature and impurities of the dissolved substance or solid inclusions, on the presence of magnetic and electric fields, mechanical impacts (vibrations, mixing, rotation) and other reasons [1–5]. Studying convective flow, two principles of force field (especially temperature field) stratification is distinguished. The vertical temperature field stratification corresponds to Rayleigh convection [1, 3]. The Marangoni effect that excites horizontal convection was ignored in the classical Benard experiments under heating of the spermaceti layer (sperm whale brain fat). The selection of convection from the two trends is enough conditionally because due to the Onsager principle they complement each other [1, 6].

Horizontal convection analysis began later than the study of Rayleigh principle of impulse moment transfer in fluids. However, the non-homogeneity of the exciting force field is frequent in nature. Water and air mass flow, astrophysical interstellar medium motion, crystal growing, biological fluid flow, and other processes are caused by horizontal (longitudinal) density gradients. These gradients may be a sequence of density dependence upon temperature, upon the concentration of dissolved substances, upon pressure, upon magnetic and electrical fluid properties [3–6].

Experimental study of convection is difficult as it is seen in the theoretical description. The Navier–Stokes equations and the continuity equation together with transfer correlation are written in Boussinesq approximation [1–6]. The Oberbeck–Boussinesq equations are constructed due to the principle of density linear dependence upon the scalar field in normal gravitation conditions, neglecting the density variance in mixing forces and low compressibility [1].

To understand the convection mechanism, it is important to have an extensive library of exact Oberbeck–Boussinesq equations. Theoretical study in this scientific direction begins with the pioneer publications of Ostroumov and Birich [7, 8] where the unidirectional convective flows take its origin. By now, several classes of exact solutions of three-dimensional Oberbeck–Boussinesq equation system have been built to describe viscous incompressible fluid flow [3–6, 9–16]. The main idea in the construction of classes of exact solutions of Navier–Stokes equations is based on velocity field modification with linear dependence on spatial acceleration. The generalization of Ostroumov–Birich exact solution for layered and shear flows is realized in [3, 4, 9, 17–25].

The essential gap in the horizontal convection study is found in binary liquid investigations. The coupled dissipative Soret and Dufour effects prevail in this case [1, 6, 18, 21, 25]. Dufour effect is neglected in the majority of analyses [1, 18, 21, 25]. This research work deals with the large-scale flow where one geometric variable is negligible in comparison to the other ones. Hence, a plain horizontal layer can be taken as a hydrodynamic model where the fluid motion represents Couette flow class. The exact solutions for different boundary value problems taking into account the convection effect were built earlier in works [3, 4, 8, 10–16, 19–21, 23]. The exact solution class published in [18] was taken as the foundation of articles [3, 4, 8, 10–16, 19–21, 23] and remains the most wide-expanded solution among the famous polynomial exact solutions of hydrodynamic equations in our day. The literature review of this equation class construction is presented in [18] and its references.

The analyses within the exact equation class [18] to study binary liquid flows regarding Soret effect and ignoring the cross Dufour effect were published in articles [1, 2, 4, 6, 21, 25]. The diffusion processes were added in the development of the mentioned exact equation class [18] to enable a sufficient and more accurate description of fluid flow and evaluation of its influence on the formation of whirling fluid countercurrents.

Motion equations and their exact solution. The steady shear flow of a binary viscous incompressible fluid is studied between two parallel planes where the lower one forms the coordinate plane xOy and the Oz axis is normal to the upper one (Fig. 1).

The lower plane is considered absolutely solid and unmovable and the upper one is free with no deformation. The deformation neglecting of the free surface of the fluid layer can avoid from consideration the fluid flow with scale matching to layer depth. Due to this approach, we cannot consider, for example, surface waves of different origin (gravitation waves, thermos-capillary waves, etc.) [1–4]. Fluid layer depth (distance between planes) is equal to h. Hence, the lower boundary of the infinite horizontal fluid layer is related to $z = 0$, and the upper boundary equation is $z = h$.

Equation system of viscous incompressible fluid in Boussinesq approximation for thermos-diffusive shear flow is written as [1, 18]:

$$
\begin{gathered}
V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}=-\frac{\partial P}{\partial x}+\nu\Big(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_x}{\partial z^2}\Big),\\
V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}=-\frac{\partial P}{\partial y}+\nu\Big(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_y}{\partial z^2}\Big),\\
\frac{\partial P}{\partial z}=g(\beta_1 T+\beta_2 C),\\
V_x\frac{\partial T}{\partial x}+V_y\frac{\partial T}{\partial y}=(\chi+\alpha^2 dn)\Big(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\Big)+\\
+\alpha dn\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right),\\
V_x\frac{\partial C}{\partial x}+V_y\frac{\partial C}{\partial y}=\alpha\Big(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\Big)+\alpha d\Big(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\Big),\\
\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0.
\end{gathered}
\tag{1}
$$

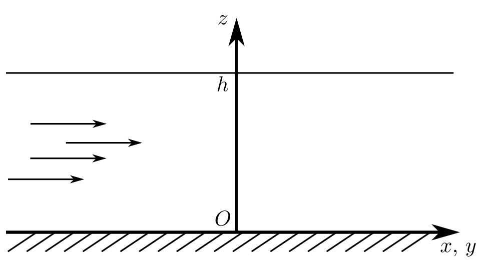

Figure 1. Fluid flow (the Ox and Oy axes are coincided on the figure, although we mean the three-dimensional space)

Here V_x, V_y are the velocity vector components, P is the pressure related to the constant average fluid density ρ; ν is the kinematic (molecular) mixture viscosity; C, T are the concentration of light component and fluid temperature, respectively, deviated from equilibrium value; g is the gravity acceleration; χ, d, α are the temperature conductivity, diffusion, thermo-diffusion coefficients, respectively; $n = \left[\frac{T}{c_p}\left(\frac{\partial\mu}{\partial C}\right)_{T,P}\right]_0$ is the thermo-dynamical parameter.

The system of equations (1) is overdetermined. It consists of six equations to determine two velocity components, related pressure, temperature, and concentration. We will use a further approach based on the method of differential correlations [18, 25, 26]. It is necessary to find correlation between hydrodynamic fields that enables to eliminate "extra" equation in the system (1) [18, 25, 26].

The analysis of the solvability of thermodiffusion equations (1) will be made in the exact solution class presented in articles [18, 25, 26]:

$$
\begin{aligned}
&V_x = U(z), \quad V_y = V(z),\\
&P = P_0(z) + xP_1(z) + yP_2(z),\\
&T = T_0(z) + xT_1(z) + yT_2(z),\\
&C = C_0(z) + xC_1(z) + yC_2(z).
\end{aligned} \tag{2}
$$

The velocity field (2) depend only upon the vertical (transversal) coordinate z. Other hydrodynamic fields depend on three coordinates and are expressed linearly in the coordinates x and y. Thus, the formulae (2) generalize the class of Ostroumov–Birich exact solutions proposed for the first time to solve Marangoni convection problems [7, 8]. The hydrodynamic field presentation (2) describes horizontal convection induced by special gradients of pressure, temperature, and concentration [18].

Substitution of the exact solution class (2) into the incompressibility equation

$$
\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0
$$

turns it into identity. In this case, we have a system of eleven ordinary differential equations to determine eleven unknown functions:

$$
\begin{gathered}
(\chi + \alpha^2 dn)T_1'' + \alpha dn C_1'' = 0,\\
(\chi + \alpha^2 dn)T_2'' + \alpha dn C_2'' = 0,\\
C_1'' + dT_1'' = 0, \quad \alpha C_2'' + \alpha dT_2'' = 0,\\
P_1' = g\beta_1 T_1 + g\beta_2 C_1,\\
P_2' = g\beta_1 T_2 + g\beta_2 C_2,\\
\nu U'' = P_1, \quad \nu V'' = P_2,\\
UT_1 + VT_2 = (\chi + \alpha^2 dn)T_0'' + \alpha dn C_0'',\\
UC_1 + VC_2 = \alpha C_0'' + \alpha dT_0'',\\
P_0' = g\beta_1 T_0 + g\beta_2 C_0.
\end{gathered} \tag{3}
$$

Hence, the choice of exact solution (2) enables to avoid overdetermination of the initial Oberbeck–Boussinesq system (1).

Derivation stroke means z variable derivation in the system (3). To calculate the exact solution of the ordinary differential equations system (3) we consider a subsystem:

$$\begin{gathered}(\chi+\alpha^2 dn)T_1''+\alpha dnC_1''=0,\quad (\chi+\alpha^2 dn)T_2''+\alpha dnC_2''=0,\\ C_1''+dT_1''=0,\quad \alpha C_2''+\alpha dT_2''=0.\end{gathered}\tag{4}$$

The subsystem (4) is studied separately because it consists of differential equations concerning horizontal gradients of temperature T_1, T_2 and concentration C_1, C_2 with different dissipation coefficients. In this case, the situation is possible when the differential equations system (4) can have no exact equation with explicit physical interpretation.

We rewrite the system (4) in vector-matrix view for convenience:

$$\begin{pmatrix}\chi+\alpha^2 dn & 0 & \alpha dn & 0\\ 0 & \chi+\alpha^2 dn & 0 & \alpha dn\\ d & 0 & 1 & 0\\ 0 & \alpha d & 0 & \alpha\end{pmatrix}\begin{pmatrix}T_1''\\ T_2''\\ C_1''\\ C_2''\end{pmatrix}=0.\tag{5}$$

We analyze the equations (5) as the system of linear algebraic equations concerning second derivatives. The matrix determinant is not equal to zero. Consequently, due to the Kronecker–Capelli theorem, the solution of the system of linear algebraic equations (5) is trivial (null solution):

$$T_1''=0,\quad C_1''=0,\quad T_2''=0,\quad C_2''=0.$$

In this case, the horizontal temperature and concentration gradients are presented as linear polynomial functions:

$$\begin{aligned}T_1&=c_1z+c_2, &\quad T_2&=c_3z+c_4,\\ C_1&=c_5z+c_6, &\quad C_2&=c_7z+c_8.\end{aligned}\tag{6}$$

The linear form coefficients in the correlations (6) are the constants of integration. Regarding the integration of the system (3) due to the formulae (6) we obtain the exact solution for horizontal pressure and velocity gradients:

$$\begin{aligned}P_1&=g\beta_1\Big(c_1\frac{z^2}{2}+c_2z\Big)+g\beta_2\Big(c_5\frac{z^2}{2}+c_6z\Big)+c_9,\\ P_2&=g\beta_1\Big(c_3\frac{z^2}{2}+c_4z\Big)+g\beta_2\Big(c_7\frac{z^2}{2}+c_8z\Big)+c_{10},\\ U&=\frac{g\beta_1}{\nu}\Big(c_1\frac{z^4}{24}+c_2\frac{z^3}{6}\Big)+\frac{g\beta_2}{\nu}\Big(c_5\frac{z^4}{24}+c_6\frac{z^3}{6}\Big)+c_9\frac{z^2}{2}+c_{11}z+c_{12},\\ V&=\frac{g\beta_1}{\nu}\Big(c_3\frac{z^4}{24}+c_4\frac{z^3}{6}\Big)+\frac{g\beta_2}{\nu}\Big(c_7\frac{z^4}{24}+c_8\frac{z^3}{6}\Big)+c_{10}\frac{z^2}{2}+c_{13}z+c_{14}.\end{aligned}\tag{7}$$

The last three system equations give us the exact expressions for T_0 and C_0 as seventh power polynomial functions of z and P_0 is presented as eighth power polynomial function of z. As the expressions for the background field components of T_0, C_0 and P_0 are bulky, they are not presented. Furthermore, we will consider only the field of velocity.

Boundary value problem. The boundary conditions are needed to be written to calculate the constants of integration in formulae (6) and (7). The adhesion condition is realized on the lower boundary (bottom) $z = 0$:

$$V_x(0) = V_y(0) = 0.$$

The homogeneous velocity distribution is given on the upper boundary (it moves as a solid surface):

$$V_x(h) = W\cos\psi, \quad V_y(h) = W\sin\psi.$$

Here W is the velocity value on the upper boundary, ψ is the angle formed by the velocity vector and abscises axis. Boundary condition for the pressure is written as

$$P(h) = S,$$

where S is the atmosphere pressure on the free surface. The conditions of impenetrability and ideal heat exchange are given for concentration and temperature on the border $z = 0$, respectively:

$$\left.\frac{\partial C}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial T}{\partial z}\right|_{z=0} = 0.$$

The temperature on the upper boundary is defined as

$$T(h) = ax + by,$$

and concentration is determined as

$$C(h) = mx + ny.$$

Using the formulae (2) and the linearity of the boundary conditions, we obtain the following relations at $z = 0$:

$$U = V = 0, \quad \frac{dT_0}{dz} = 0, \quad \frac{dT_1}{dz} = \frac{dT_2}{dz} = 0, \quad \frac{dC_0}{dz} = 0, \quad \frac{dC_1}{dz} = \frac{dC_2}{dz} = 0. \tag{8}$$

The next equalities are defined at $z = h$:

$$\begin{gathered} U = W\cos\psi, \quad V = W\sin\psi, \quad T_0 = 0, \quad T_1 = a, \quad T_2 = b, \\ P_0 = S, \quad P_1 = 0, \quad P_2 = 0, \quad C_0 = 0, \quad C_1 = m, \quad C_2 = n. \end{gathered} \tag{9}$$

The written boundary conditions (8) and (9) describe Couette type fluid flow (horizontal pressure gradients are not regarded).

Boundary value problem solution. The exact solution for the boundary value problem is obtained using boundary conditions (8) and (9) for the formulae (6) and (7). The velocity field is described by the following correlations:

$$\begin{aligned} V_x = U(z) &= \frac{h^2 gF}{3\nu} z + \frac{W\cos\psi}{h} z - \frac{ghF}{2\nu} z^2 + \frac{gF}{6\nu} z^3, \\ V_y = V(z) &= \frac{h^2 gE}{3\nu} z + \frac{W\sin\psi}{h} z - \frac{ghE}{2\nu} z^2 + \frac{gE}{6\nu} z^3. \end{aligned} \tag{10}$$

Two designations are introduced for the coefficients:

$$E = \beta_1 b + \beta_2 n, \quad F = \beta_1 a + \beta_2 m.$$

The temperature field stratification along the vertical coordinates and horizontal variables is expressed in a linear form:

$$T = T_0(z) + ax + by.$$

Background temperature

$$T_0(z) = T_0^0 + T_0^3 z^3 + T_0^4 z^4 + T_0^5 z^5 \tag{11}$$

is formed by several components:

$$T_0^0 = -\frac{AgFh^5}{45\nu K} - \frac{AWh^2\cos\psi}{6K} - \frac{MgEh^5}{45K\nu} - \frac{WMh^2\sin\psi}{6K},$$
$$T_0^3 = \frac{AgFh^2}{18\nu K} + \frac{AW\cos\psi}{6Kh} + \frac{MgEh^2}{18K\nu} + \frac{WM\sin\psi}{6Kh},$$
$$T_0^4 = \frac{-AgFh}{24\nu K} - \frac{MgEh}{24K\nu}, \qquad T_0^5 = \frac{AgF}{120\nu K} + \frac{MgE}{120K\nu},$$

where $K = -d^2 n\alpha + \chi + \alpha^2 dn$, $A = a - mnd$, $M = b - dn^2$.

The concentration distribution has the similar correlation

$$C = C_0(z) + mx + ny, \tag{12}$$

where $C_0(z) = C_0^0 + C_0^3 z^3 + C_0^4 z^4 + C_0^5 z^5$ with components

$$C_0^0 = -\frac{BgFh^5}{45\nu K} - \frac{BWh^2\cos\psi}{6K} - \frac{IgEh^5}{45K\nu} - \frac{WIh^2\sin\psi}{6K},$$
$$C_0^3 = \frac{BgFh^2}{18\nu K} + \frac{BW\cos\psi}{6Kh} + \frac{IgEh^2}{18K\nu} + \frac{WI\sin\psi}{6Kh},$$
$$C_0^4 = -\frac{BgFh}{24\nu K} - \frac{IgEh}{24K\nu}, \qquad C_0^5 = \frac{BgF}{120\nu K} + \frac{IgE}{120K\nu}.$$

Here we add the designations $B = m(\chi\alpha^{-1} + \alpha dn) - da$, $I = -(n\chi\alpha^{-1} + \alpha dn^2 - bd)$.

The pressure distribution has the sixth power polynomial function of the coordinate z:

$$P = P_0(z) + xP_1(z) + yP_2(z), \tag{13}$$

where $P_0(z) = P_0^0 + P_0^1 z + P_0^4 z^4 + P_0^5 z^5 + P_0^6 z^6$ with components

$$P_0^0 = S - \Big(-\frac{11DgFh^6}{720\nu K} - \frac{DWh^3\cos\psi}{8K} - \frac{11LgEh^6}{720K\nu} - \frac{WLh^3\sin\psi}{8K}\Big),$$
$$P_0^1 = -\frac{DgFh^5}{45\nu K} - \frac{DWh^2\cos\psi}{6K} - \frac{LgEh^5}{45K\nu} - \frac{WLh^2\sin\psi}{6K},$$
$$P_0^4 = \frac{DgFh^2}{72\nu K} + \frac{DW\cos\psi}{24Kh} + \frac{LgEh^2}{72K\nu} + \frac{WL\sin\psi}{24Kh},$$

$$P_0^5 = -\frac{DgFh}{120\nu K} - \frac{LgEh}{120K\nu}, \qquad P_0^6 = \frac{DgF}{720\nu K} + \frac{LgE}{720K\nu},$$
$$P_1(z) = gFz - gFh, \qquad P_2(z) = gEz - gEh.$$

We add the designations

$$D = \beta_1 A + \beta_2 B, \quad L = \beta_1 M + \beta_2 I$$

to get the final recording of the exact solution (10)–(13) of the boundary value problem (3), (8) and (9) and it is written in polynomial function class.

Velocity field analysis. To analyze velocity field, we introduce dimensionless values $u = U/W$, $v = V/W$, and $Z = z/h$ ($0 \leqslant Z \leqslant 1$). The third power polynomial functions are written to analyze the cinematic characteristics of hydrodynamic flow:

$$u = Z\Big[\cos\psi + \frac{h^3 gF}{6\nu W}(Z-1)(Z-2)\Big],$$
$$v = Z\Big[\sin\psi + \frac{h^3 gE}{6\nu W}(Z-1)(Z-2)\Big]. \tag{14}$$

One can note that the dimension of two-dimensional velocity field (10) reduces due to the turning transformation

$$\tan\psi = \frac{E}{F} = \frac{\beta_1 b + \beta_2 n}{\beta_1 a + \beta_2 m}. \tag{15}$$

In this case, the fluid motion transforms from shear motion into a layered (one direction) one.

Every component of the velocity vector can have maximally one null point where the sign change of velocity and it corresponds to the direction change of fluid flow. The fulfilment of the following correlations for each velocity component is needed to find this null point:

$$[u(0)] \cdot [u(1)] < 0, \quad [v(0)] \cdot [v(1)] < 0,$$

where $[u(\,\cdot\,)]$, $[v(\,\cdot\,)]$ describe the expressions in square brackets for each velocity. Using the correlations for the exact solution (14) we get

$$\cos\psi\Big(\cos\psi + \frac{h^3 gF}{3\nu W}\Big) < 0, \quad \sin\psi\Big(\sin\psi + \frac{h^3 gE}{3\nu W}\Big) < 0.$$

We can note that if $F = 0$ ($\beta_1 a = -\beta_2 m$) or $E = 0$ ($\beta_1 b = -\beta_2 n$), the countercurrents do not happen in fluid flow as the velocity profile within the given values of the parameters F and E is described by the classical exact Couette solution [26, 28]. Hence, the fluid countercurrents can be found due to the superposition of the temperature and concentration effects on the structure of hydrodynamic flow.

The velocity $u(z)$ profiles for the cases of countercurrent absence and presence are shown on the Fig. 2. The velocity profile (14) is not monotonous. The velocity function can have the extremum and depending on the coefficient values, the extremum can be maximum or minimum. Analyzing the formulae (14) it can be easily shown that the countercurrent formation is connected with the mutual influence of the boundary conditions of velocity, temperature, and concentration on physical fluid constants characterizing the flow of dissipation processes in fluid.

The velocity hodographs are presented on the Fig. 3 with the conditions $E = F$ and $\psi = 0$ (a), $\psi = \pi/4$ (b), $\psi = 3\pi/4$ (c), $\psi = \pi$ (d). The velocity hodograph shows that the flows are said to be locally spiral (Fig. 3, a, c, d). The hodograph loop formation in stable motion is typical for two-dimensional velocity field. If the fluid flow has one direction due to the fulfilment of correlation (15) then the velocity hodograph becomes a segment (Fig. 3, b).

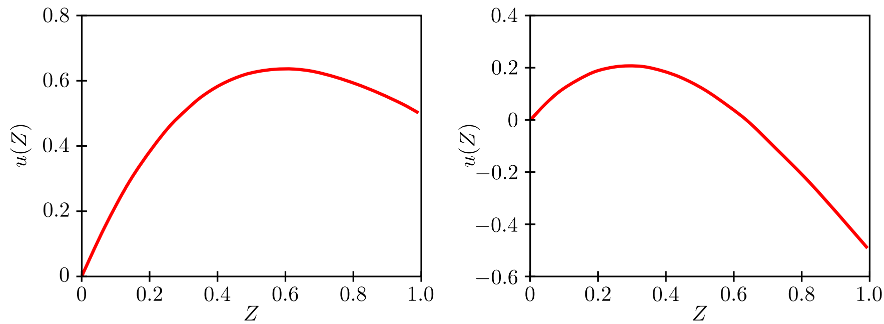

Figure 2. Velocity profiles without countercurrent (left figure) and with countercurrent (right figure)

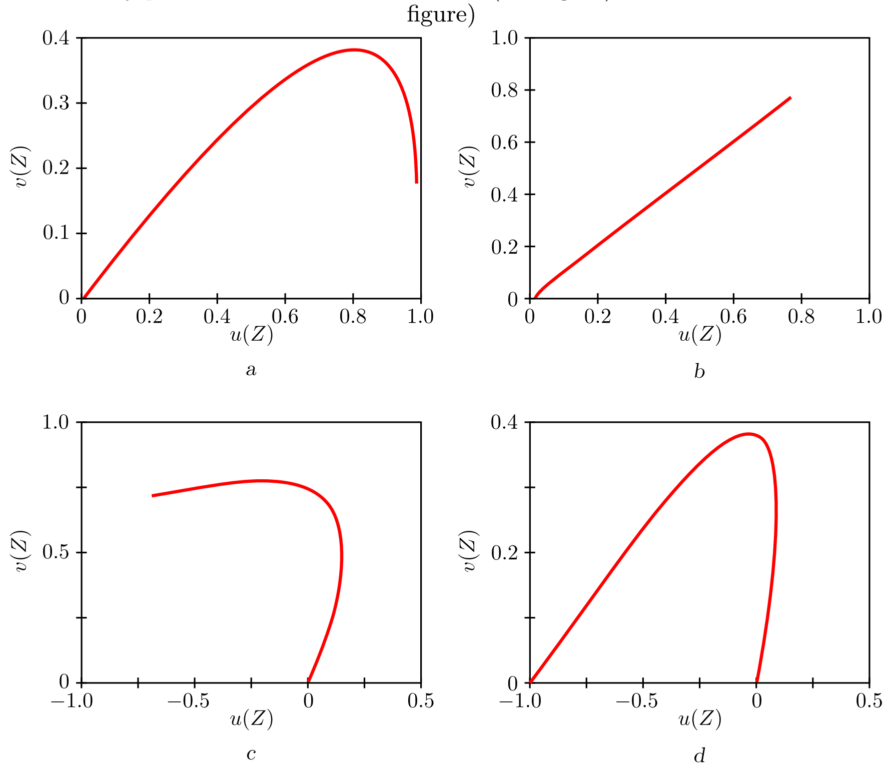

Figure 3. Velocity hodographs with the conditions $E = F$ and $\psi = 0$ (a), $\psi = \pi/4$ (b), $\psi = 3\pi/4$ (c), $\psi = \pi$ (d)

Such effect was noticed in the classical Couette solution for rotating liquid and its generalizations [3, 4, 27].

Conclusion. The exact solution to describe large-scale stationary Couette flow is presented. The solution is calculated in the class of velocities distributed due to the certain dependence on the transversal coordinate and linearly on one horizontal value. The distribution of zeroes of the regarded polynomial functions is studied. The connection of zeroes with the fluid countercurrent formation is shown.

Competing interests. The authors declare no conflicts of interests.

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Установившееся термодиффузионное сдвиговое течение Куэтта несжимаемой жидкости. Исследование поля скоростей

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Аннотация

Найдено точное решение, описывающее установившееся течение вязкой несжимаемой жидкости с учетом перекрестного влияния конвективного и диффузионного эффектов (перекрестное влияние диссипативных эффектов Соре и Дюфора). Для исследования сдвигового потока жидкости была решена переопределенная краевая задача. Переопределенность краевой задачи обусловлена тем, что количество уравнений в нелинейной системе уравнений Обербека—Буссинеска больше, чем количество неизвестных функций (две компоненты вектора скорости, давление, температура и концентрация растворенного вещества). Нетривиальное точное решение системы, состоящей из уравнений Обербека—Буссинеска, уравнения непрерывности, уравнения теплопроводности и уравнения концентрации, было построено в классе точных решений Бириха—Остроумова. Разрешимость переопределенной системы уравнений обусловлена тем, что точное решение автоматически удовлетворяет уравнению непрерывности. Показано существование застойных точек как в общем течении, так и во вторичном движении жидкости без завихренности. Найдены условия, при которых возможны противотечения.

Ключевые слова: уравнения Навье–Стокса, точное решение, многослойная жидкость, поле массовых сил, переопределенная приведенная система.

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