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Asymptotics of the eigenvalues of a boundary value problem for the operator Schrödinger equation with boundary conditions nonlinearly dependent on the spectral parameter

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Abstract

On the space $H_1 = L_2(H, [0,1])$, where H is a separable Hilbert space, we study the asymptotic behavior of the eigenvalues of a boundary value problem for the operator Schrödinger equation for the case when one, and the same, spectral parameter participates linearly in the equation and quadratically in the boundary condition. Asymptotic formulae are obtained for the eigenvalues of the considered boundary value problem.

Keywords: operator differential equations, spectrum, eigenvalue, asymptotic formula, Hilbert space.

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Introduction

In this paper, on the space $H_1 = L_2(H, [0,1])$, where H is a separable Hilbert space, we study the asymptotics of the eigenvalues of the following boundary value problem for the operator Schrödinger equation:

$$-u''(x) + \frac{\nu^2 - \frac{1}{4}}{x^2} u(x) + Au(x) = \lambda u(x), \quad x \in (0,1), \quad \nu \geqslant \frac{1}{2}, \tag{1}$$

$$u(0) = 0, \quad u'(1) - \lambda^2 u(1) = 0, \tag{2}$$

where λ is a spectral parameter, A is a self-adjoint, positive-definite operator in H and the inverse operator A^{-1} is completely continuous in H.

In [1], the discreetness and some other properties of the spectrum are investigated for the Schrödinger operator on the space $H_1 = L_2(H, [0,\infty))$.

In the papers [2] and [3], the asymptotic behavior of the eigenvalues of the following boundary value problem for a second-order elliptic differential operator equation

$$-u''(x) + Au(x) = \lambda u(x), \quad x \in (0,1), \tag{3}$$

with boundary conditions

$$u'(0) + \lambda u(0) = 0, \quad u(1) = 0 \tag{4}$$

was studied on the space $H_1 = L_2(H, [0,1])$.

In [4], Aliev investigated the eigenvalues of two boundary value problems: (3), (4) and (3) with boundary conditions

$$u'(0) + \lambda u(0) = 0, \quad u'(1) - \lambda u(1) = 0$$

and proved that the set of eigenvalues of these problems is discrete and has two series of eigenvalues: $\lambda_k \sim \sqrt{\mu_k}$ and $\lambda_{n,k} \sim n^2\pi^2 + \mu_k$, $k, n \in \mathbb{N}$.

In [5], similar problem for equation (4) with boundary conditions of the form

$$u'(0) + d\lambda^2 u(0) = 0, \quad u(1) = 0$$

was studied.

In the paper [6], the author considered the asymptotic behavior of eigenvalues for the equation (3) with the boundary conditions

$$u'(0) + \lambda^2 u(0) = 0, \quad u'(1) - \lambda^2 u(1) = 0. \tag{5}$$

It was shown that the problem (3), (5) has three series of eigenvalues, one of which converges to zero, and the other two asymptotically behave as $(2n)^2\pi^2 + \mu_k$ and $(2n-1)^2\pi^2 + \mu_k$, where $\mu_k = \mu_k(A)$ are the eigenvalues of the operator A.

In [7] and [8], a boundary value problem for a second-order ordinary differential equation is considered in the case when one and the same spectral parameter λ linearly participates in the equation and quadratically in one of the boundary conditions. The asymptotic behavior of the eigenvalues of the considered boundary value problem was studied.

In [9], the authors considered a boundary value problem for a second-order ordinary differential equation, when one and the same, spectral parameter λ

quadratically participates in the equation, while in the boundary conditions it appears as a quadratic trinomial (with respect to λ) and it is studied the asymptotic behavior of eigenvalues of the considered boundary value problem.

In [10], Aslanova studied the asymptotics of the eigenvalues of boundary problem for the operator Schrödinger equation where spectral parameter participates linearly in the equation and in the boundary condition. She proved that the Schrödinger equation (1) with the boundary conditions

$$u(0)=0, \quad u'(1)-hu(1)=\lambda u(1) \tag{6}$$

has a discrete spectrum and two series of eigenvalues: $\lambda_k \sim -h\sqrt{\mu_k}$ and $\lambda_{m,k} \sim \left(\pi m+\frac{1}{2}\nu\pi-\frac{1}{4}\pi\right)^2+\mu_k$.

Problems such as (1), (2) and (1), (6) arise upon separation of variables in heat or wave equations, where one of the boundary conditions contains a partial derivative with respect to time. This is why it is so important to study such problems. Especially, problems with nonlinear boundary conditions are important.

The purpose of this study is to determine the asymptotics of the eigenvalues of boundary value problem for the operator Schrödinger equation in the case when one and the same spectral parameter participates linearly in the equation and quadratically in the boundary condition. We prove that the eigenvalues of the problem (1), (2) are real and simple. Furthermore, it will be shown that the eigenvalues asymptotically behave as $\left(\frac{1}{2}\nu\pi+\frac{3}{4}\pi+\pi n\right)^2+\mu_k$ or $(j_{n+1})^2+\mu_k$, where the numbers $j_1<j_2<j_3<\cdots<j_n<\cdots$ are the roots of the equation $J_\nu(z)=0$, and $\mu_k=\mu_k(A)$ are the eigenvalues of the operator A.

Some properties of eigenvalues

LEMMA 1. *The eigenvalues of the boundary value problem (1), (2) are real numbers.*

Proof. Let $\mu_1\leqslant\mu_2\leqslant\cdots\leqslant\mu_n\leqslant\cdots$ be the eigenvalues and $\{e_k\}_{k=1}^{\infty}$ be the corresponding eigenvectors of the operator A. The set $\{e_k\}_{k=1}^{\infty}$ forms a complete orthonormal basis in the space H, and each vector-function $u(x)$ can be expanded as $u(x)=\sum_{k=1}^{\infty}\big(u(x),e_k\big)_H\cdot e_k$. For the Fourier coefficients $u_k(x)=\big(u(x),e_k\big)_H$, we get the following spectral problem

$$-u_k''(x)+\frac{\nu^2-\frac{1}{4}}{x^2}u_k(x)+\mu_k u_k(x)=\lambda u_k(x), \quad x\in(0,1), \quad \nu\geqslant\frac{1}{2}, \tag{7}$$

$$u_k(0)=0, \quad u_k'(1)-\lambda^2u_k(1)=0. \tag{8}$$

The study of the eigenvalue problem (1), (2) is reduced to the study of eigenvalues of the problem (7), (8) for different natural k. The spectrum of the problem (1), (2) consists of the union of eigenvalues of the problem (7), (8) for all natural k.

Let λ be an eigenvalue of the problem (7), (8) and $u_k(x)$ be the corresponding eigenfunction. Multiplying both sides of equality (7) by the function $\overline{u_k(x,\lambda)}$ and integrating the identity with respect to x from 0 to 1; we have

$$-\int_0^1 u_k''(x,\lambda)\overline{u_k(x,\lambda)}dx + \mu_k\int_0^1 \left|u_k(x,\lambda)\right|^2 dx + \int_0^1 \frac{\nu^2-\frac{1}{4}}{x^2}\left|u_k(x,\lambda)\right|^2 dx = \\ = \lambda\int_0^1 \left|u_k(x,\lambda)\right|^2 dx \quad (9)$$

Calculating the first integral and using the boundary conditions (8), we get

$$-\int_0^1 u_k''(x,\lambda)\overline{u_k(x,\lambda)}dx = u_k'(x,\lambda)\overline{u_k(x,\lambda)}\Big|_0^1 - \int_0^1 u_k'(x,\lambda)\overline{u_k'(x,\lambda)}dx = \\ = u_k'(1,\lambda)\overline{u_k(1,\lambda)} - u_k'(0,\lambda)\overline{u_k(0,\lambda)} - \int_0^1 \left|u_k'(x,\lambda)\right|^2 dx = \\ = \lambda^2\left|u_k(1,\lambda)\right|^2 - \int_0^1 \left|u_k'(x,\lambda)\right|^2 dx.$$

Consequently, from (9), it follows that

$$\lambda^2\left|u_k(1,\lambda)\right|^2 + \lambda\int_0^1 \left|u_k(x,\lambda)\right|^2 dx - \int_0^1 \left|u_k'(x,\lambda)\right|^2 dx - \\ - \int_0^1 \frac{\nu^2-\frac{1}{4}}{x^2}\left|u_k(x,\lambda)\right|^2 dx - \mu_k\int_0^1 \left|u_k(x,\lambda)\right|^2 dx = 0. \quad (10)$$

Denote

$$a_k(\lambda) = |u_k(1,\lambda)|^2, \quad b_k(\lambda) = \int_0^1 \left|u_k(x,\lambda)\right|^2 dx,$$

$$c_k(\lambda) = -\int_0^1 \left|u_k'(x,\lambda)\right|^2 dx - \int_0^1 \frac{\nu^2-\frac{1}{4}}{x^2}\left|u_k(x,\lambda)\right|^2 dx - \mu_k\int_0^1 \left|u_k(x,\lambda)\right|^2 dx.$$

From (10), it follows that the eigenvalue λ is a root of the equation

$$a_k(\lambda)z^2 + b_k(\lambda)z + c_k(\lambda) = 0,$$

for every k.

Since $a_k(\lambda) \geqslant 0$, $b_k(\lambda) > 0$, $c_k(\lambda) < 0$ for any $k \in \mathbb{N}$, then

$$b_k^2(\lambda) - 4a_k(\lambda)c_k(\lambda) > 0.$$

Consequently, for every k, the problem (7), (8) has only real roots. Lemma 1 is proved. □

LEMMA 2. *The number $\lambda = 0$ is not an eigenvalue of the boundary value problem* (1), (2).

Proof. It is sufficient to prove that the boundary value problem (7), (8) for $\lambda = 0$, i.e., the problem

$$-u_k''(x) + \frac{\nu^2-\frac{1}{4}}{x^2}u_k(x) + \mu_k u_k(x) = 0, \quad x \in (0,1), \quad \nu \geqslant \frac{1}{2}, \quad (11)$$

$$u_k(0) = 0, \quad u_k'(1) = 0 \quad (12)$$

has only a trivial solution for every natural k.

Denote by L_0 the differential operator acting in the space $L_2(0,1)$ generated by the differential expression

$$l_0[u] = -u''(x) + \frac{\nu^2 - \frac{1}{4}}{x^2}u(x) + \mu_k u(x)$$

and boundary conditions

$$u(0) = 0, \quad u'(1) = 0.$$

Consider the scalar product $(L_0 u, u)$. We have:

$$(L_0 u, u) = -\int_0^1 u''(x)\overline{u(x)}dx + \int_0^1 \frac{\nu^2 - \frac{1}{4}}{x^2}u(x)\overline{u(x)}dx + \mu_k \int_0^1 u(x)\overline{u(x)}dx =$$
$$= -u'(x)\overline{u(x)}\Big|_0^1 + \int_0^1 u'(x)\overline{u'(x)}dx + \int_0^1 \frac{\nu^2 - \frac{1}{4}}{x^2}u(x)\overline{u(x)}dx + \mu_k \int_0^1 u(x)\overline{u(x)}dx.$$

Therefore,

$$(L_0 u, u) = \int_0^1 \left|u'(x)\right|^2 dx + \int_0^1 \frac{\nu^2 - \frac{1}{4}}{x^2}\left|u(x)\right|^2 dx + \mu_k \int_0^1 \left|u(x)\right|^2 dx. \tag{13}$$

From (13), it follows that $(L_0 u, u) > 0$, when $u(x) \neq 0$; and $L_0 u = 0$, when $u(x) = 0$. Hence, for every natural k, the problem (11),(12) has only a trivial solution. This proves the Lemma 2. □

Lemma 3. *The eigenvalues of the boundary value problem* (1), (2) *are simple.*

Proof. The problem (1), (2) is reduced to the study of problem (7), (8), for every k.

To prove the lemma, it suffices to establish that the equation

$$u_k'(1,\lambda) - \lambda^2 u_k(1,\lambda) = 0 \tag{14}$$

has only simple roots. We will prove this assertion by the method proposed in [9]. Let $u_k(x,\lambda)$ be a solution to the equation (11) with the initial condition $u_k(0,\lambda) = 0$. Then, if $\lambda = \lambda^*$ is a multiple root of equation (14), then the following equalities hold:

$$u_k'(1,\lambda) - \lambda^2 u_k(1,\lambda) = 0, \tag{15}$$
$$\frac{\partial u_k'(1,\lambda)}{\partial \lambda} - 2\lambda u_k(1,\lambda) - \lambda^2 \frac{\partial u_k(1,\lambda)}{\partial \lambda} = 0. \tag{16}$$

Since $u_k(x,\lambda)$ is a solution of the equation (11), we have

$$\frac{d}{dx}\left[u_k(x,\lambda)u_k'(x,\mu) - u_k(x,\mu)u_k'(x,\lambda)\right] = (\lambda^2 - \mu^2)u_k(x,\lambda)u_k(x,\mu).$$

Integrating this equality in the interval $[0,1]$, and taking $u_k(0,\lambda) = 0$, we obtain:

$$\frac{u_k(1,\lambda)u_k'(1,\mu) - u_k(1,\mu)u_k'(1,\lambda)}{\lambda - \mu} = (\lambda + \mu)\int_0^1 u_k(x,\lambda)u_k(x,\mu)dx,$$

where $\lambda \neq \mu$. By taking the limit when $\mu \to \lambda$ in the last equality and putting $\lambda = \lambda^*$, we have:

$$u_k'(1,\lambda^*)\frac{\partial u_k(1,\lambda^*)}{\partial \lambda} - u_k(1,\lambda^*)\frac{\partial u_k'(1,\lambda^*)}{\partial \lambda} = 2\lambda^* \int_0^1 \left|u_k(x,\lambda^*)\right|^2 dx. \qquad (17)$$

Equalities (15) and (16), respectively, imply the equalities

$$u_k'(1,\lambda^*) = (\lambda^*)^2 u_k(1,\lambda^*),$$

$$\frac{\partial u_k'(1,\lambda^*)}{\partial \lambda} = 2\lambda^* u_k(1,\lambda^*) + (\lambda^*)^2 \frac{\partial u_k(1,\lambda^*)}{\partial \lambda}.$$

Using these equalities in (17), we obtain

$$(\lambda^*)^2 u_k(1,\lambda^*)\frac{\partial u_k(1,\lambda^*)}{\partial \lambda} - u_k(1,\lambda^*)\Big(2\lambda^* u_k(1,\lambda^*) + (\lambda^*)^2 \frac{\partial u_k(1,\lambda^*)}{\partial \lambda}\Big) = $$
$$= 2\lambda^* \int_0^1 \left|u_k(x,\lambda^*\ )\right|^2 dx,$$

or

$$2\lambda^*\left(u_k^2(1,\lambda^*) + 2\lambda^* \int_0^1 \left|u_k(x,\lambda^*)\right|^2 dx\right) = 0. \qquad (18)$$

Lemma 2 implies that $\lambda^* \neq 0$. On the other hand,

$$u_k^2(1,\lambda^*) + 2\lambda^* \int_0^1 \left|u_k(x,\lambda^*)\right|^2 dx > 0.$$

Therefore, equality (18) cannot hold. Thus, Lemma 3 is proved. □

Asymptotic formulae for eigenvalues

THEOREM 1. *Let A be a self-adjoint, positive-definite operator in a separable Hilbert space H and A^{-1} be completely continuous in H. Then the following asymptotic expression holds for the eigenvalues of the boundary value problem* (1), (2):

$$\lambda_{k,n} \sim \mu_k + \left(\frac{1}{2}\nu\pi + \frac{3}{4}\pi + \pi n\right)^2 = \mu_k + (j_{n+1})^2,$$

where the numbers $j_1 < j_2 < j_3 < \cdots < j_n < \cdots$ are the roots of the equation $J_\nu(z) = 0$, and $\mu_k = \mu_k(A) \to \infty$ (as $k \to \infty$) are the eigenvalues of the operator A.

Proof. The general solution of the differential equations (7) is of the form

$$u_k(x,\lambda) = k_1\sqrt{2x}\cdot J_\nu\big(\sqrt{\lambda-\mu_k}\,x\big) + k_2\sqrt{2x}\cdot Y_\nu\big(\sqrt{\lambda-\mu_k}\,x\big),$$

where k_1 and k_2 are arbitrary constants; $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions of the first and second kinds, respectively. From the first boundary condition $u_k(0,\lambda) = 0$ it follows that $k_2 = 0$. Therefore,

$$u_k(x,\lambda) = k_1\sqrt{2x}\cdot J_\nu\big(\sqrt{\lambda-\mu_k}\,x\big).$$

From the second condition of (8) we get

$$u'_k(1,\lambda) - \lambda^2 u_k(1,\lambda) = \Big(\frac{1}{\sqrt{2}} - \sqrt{2}\nu\Big) J_\nu\big(\sqrt{\lambda-\mu_k}\big) + \\ + \sqrt{2(\lambda-\mu_k)} J_{\nu-1}\big(\sqrt{\lambda-\mu_k}\big) - \sqrt{2}\lambda^2 J_\nu\big(\sqrt{\lambda-\mu_k}\big) = 0,$$

or

$$(1-2\nu-2\lambda^2)\cdot J_\nu\big(\sqrt{\lambda-\mu_k}\big) + 2\sqrt{\lambda-\mu_k}\cdot J_{\nu-1}\big(\sqrt{\lambda-\mu_k}\big) = 0. \tag{19}$$

Denote $\sqrt{\lambda-\mu_k} = y$. Hence, $\lambda = \mu_k + y^2$, $\lambda^2 = (\mu_k+y^2)^2$ and $y\to\infty$ as $\lambda\to\infty$. Then equation (19) takes the form

$$\big(1-2\nu-2(\mu_k+y^2)^2\big)\cdot J_\nu(y) + 2y\cdot J_{\nu-1}(y) = 0.$$

Using the asymptotic expression for $J_\nu(y)$ at $y\to\infty$ we obtain

$$\big(1-2\nu-2(\mu_k+y^2)^2\big)\cdot\Big[\cos\Big(y-\frac{1}{2}\nu\pi-\frac{1}{4}\pi\Big)\Big(1-\frac{c_1}{y^2}+\frac{c_2}{y^4}+O\Big(\frac{1}{y^6}\Big)\Big) - \\ - \sin\Big(y-\frac{1}{2}\nu\pi-\frac{1}{4}\pi\Big)\Big(\frac{4\nu^2-1}{8y}-\frac{c_3}{y^3}+O\Big(\frac{1}{y^5}\Big)\Big)\Big] - \\ - 2y\Big[\sin\Big(y-\frac{1}{2}\nu\pi-\frac{1}{4}\pi\Big)\Big(1+O\Big(\frac{1}{y^2}\Big)\Big) + \\ + \cos\Big(y-\frac{1}{2}\nu\pi-\frac{1}{4}\pi\Big)\Big(\frac{4\nu^2-1}{8y}+O\Big(\frac{1}{y^3}\Big)\Big)\Big] = 0,$$

where c_1, c_2 and c_3 are some positive real constants.

Denote $y-\frac{1}{2}\nu\pi-\frac{1}{4}\pi = z$. After some processing we get

$$\cos(z)\Big[\big(1-2\nu-2(\mu_k+y^2)^2\big)\Big(1-\frac{c_1}{y^2}+\frac{c_2}{y^4}+O\Big(\frac{1}{y^6}\Big)\Big) + \frac{4\nu^2-1}{4} + O\Big(\frac{1}{y^2}\Big)\Big] - \\ - \sin(z)\Big[\big(1-2\nu-2(\mu_k+y^2)^2\big)\Big(\frac{4\nu^2-1}{8y}-\frac{c_3}{y^3}+O\Big(\frac{1}{y^5}\Big)\Big) + 2y + O\Big(\frac{1}{y}\Big)\Big] = 0. \tag{20}$$

Equation (20) takes the form

$$\tan(z) = \frac{\big(1-2\nu-2(\mu_k+y^2)^2\big)\cdot\big(1-\frac{c_1}{y^2}+\frac{c_2}{y^4}+O\big(\frac{1}{y^6}\big)\big) + \frac{4\nu^2-1}{4} + O\big(\frac{1}{y^2}\big)}{\big(1-2\nu-2(\mu_k+y^2)^2\big)\cdot\big(\frac{4\nu^2-1}{8y}-\frac{c_3}{y^3}+O\big(\frac{1}{y^5}\big)\big) + 2y + O\big(\frac{1}{y}\big)}.$$

When $y\to\infty$,

$$\tan(z) = \frac{\big(1-2\nu-2(\mu_k+y^2)^2\big)\big(1+O\big(\frac{1}{y^2}\big)\big)}{\big(1-2\nu-2(\mu_k+y^2)^2\big)\big(\frac{4\nu^2-1}{8y}+O\big(\frac{1}{y^3}\big)\big)} = \frac{8y}{4\nu^2-1} + O\Big(\frac{1}{y}\Big).$$

Equation $\tan(z) = \frac{8y}{4\nu^2-1} + O\big(\frac{1}{y}\big)$ has roots $z_k = y_k - \frac{1}{2}\nu\pi - \frac{1}{4}\pi \sim \frac{\pi}{2} + \pi k$, $k\in\mathbb{Z}^+$ when $y\to\infty$.

From here: $y_k = \sqrt{\lambda - \mu_k} = \frac{1}{2}\nu\pi + \frac{3}{4}\pi + \pi k$ or

$$\lambda_{k,n} \sim \mu_k + \left(\frac{1}{2}\nu\pi + \frac{3}{4}\pi + \pi n\right)^2. \tag{21}$$

Let numbers $j_1 < j_2 < j_3 < \cdots < j_n < \cdots$ satisfy the equality $J_\nu(j_n) = 0$ for $n = 1, 2, 3, \ldots$.

Taking into account the asymptotics of the numbers j_n, i.e. $j_n \sim \frac{1}{2}\nu\pi - \frac{1}{4}\pi + \pi n$, for $n \to \infty$, from (21) we obtain the following evaluation:

$$\lambda_{k,n} \sim \mu_k + (j_{n+1})^2.$$

Theorem 1 is proved. □

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Асимптотика собственных значений краевой задачи для операторного уравнения Шредингера с граничными условиями нелинейно зависящими от спектрального параметра

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Аннотация

В пространстве $H_1 = L_2(H, [0,1])$, где H — сепарабельное гильбертово пространство, изучается асимптотическое поведение собственных значений краевой задачи для операторного уравнения Шредингера для случая, когда один и тот же спектральный параметр участвует в уравнении линейно, а в граничном условии — квадратично. Получены асимптотические формулы для собственных значений рассматриваемой краевой задачи.

Ключевые слова: операторно-дифференциальные уравнения, спектр, собственное значение, асимптотическая формула, гильбертово пространство.

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