DOI: https://doi.org/10.17816/2074-0530-501750

Original study article

Analysis of collisions of precipitating solid particles with a wall in a viscous liquid

Alexander S. Kondratiev[1], Pavel P. Shvydko[2]

[1] Moscow Polytechnic University, Moscow, Russian Federation;
[2] Sistemotekhnika, Moscow, Russian Federation

ABSTRACT

BACKGROUND: When calculating velocities of free precipitation of solid particle in a viscous Newtonian liquid, it is assumed that influence of incoming vessel head can be neglected. Necessity of considering this factor grows with a vessel head getting closer and with decrease of particles' geometrical size.

AIMS: Development of the method of calculating solid particles' precipitation rate in a Newtonian fluid considering the incoming vessel head and definition the limits of its application.

METHODS: The known analytical model of motion of precipitating disperses solid particles when approaching a vessel head is analyzed. It is shown that the known model of calculation of solid particles' precipitation rate near the vessel head needs to be refined.

RESULTS: The proposed modification of the analytical model of definition the solid particles' precipitation rate considering the existence of a vessel head helps to ensure more credible results from physical standpoint. Recommendations regarding the influence of the Brownian motion of fluid's molecules on motion of precipitating solid particles are given.

CONCLUSION: Practical value of the study lies in ability to calculate solid particles' precipitation rate near to a vessel head.

Keywords: solid particles; precipitation rate; influence of a vessel head.

To cite this article:
Kondratiev AS, Shvydko PP. Analysis of collisions of precipitating solid particles with a wall in a viscous liquid. *Izvestiya MGTU «MAMI»*. 2023;17(3):233–239. DOI: https://doi.org/10.17816/2074-0530-501750

Received: 17.05.2023 **Accepted:** 15.08.2023 **Published online:** 15.10.2023

DOI: https://doi.org/10.17816/2074-0530-501750

Оригинальное исследование

Анализ столкновения осаждающихся твёрдых частиц со стенкой в вязкой жидкости

А.С. Кондратьев[1], П.П. Швыдько[2]

[1] Московский политехнический университет, Москва, Российская Федерация;
[2] Системотехника, Москва, Российская Федерация

АННОТАЦИЯ

Обоснование. При расчётах скоростей свободного осаждения твёрдых частиц в вязкой ньютоновской жидкости предполагается, что влиянием приближающегося днища сосуда можно пренебречь. Необходимость учёта этого фактора возрастает с приближением днища сосуда и с уменьшением геометрического размера частиц.

Цель — разработка методики расчёта скорости осаждения твёрдых частиц в ньютоновской жидкости с учётом приближающегося днища сосуда и определение границы его применимости.

Материалы и методы. Анализируется известная расчётная модель движения осаждающихся дисперсных твёрдых частиц при приближении к днищу сосуда. Показано, что известная модель расчёта скорости осаждения твёрдых частиц при вблизи днища сосуда нуждается в уточнении.

Результаты. Предложенная модификация расчётной модели определения скорости осаждения твёрдых частиц, учитывающая наличие днища сосуда, позволяет обеспечить физически более достоверные результаты. Даны разъяснения о влиянии броуновского движения молекул жидкости на движение осаждающихся твёрдых частиц.

Заключение. Практическая ценность исследования заключается в возможности расчёта скорости осаждения твёрдых частиц при приближении к днищу сосуда.

Ключевые слова: твёрдые частицы; скорость осаждения; влияние днища сосуда.

Как цитировать:
Кондратьев А.С., Швыдько П.П. Анализ столкновения осаждающихся твёрдых частиц со стенкой в вязкой жидкости // Известия МГТУ «МАМИ». 2023. Т.17, № 3. С. 233–239. DOI: https://doi.org/10.17816/2074-0530-501750

Рукопись получена: 17.05.2023 **Рукопись одобрена:** 15.08.2023 **Опубликована онлайн:** 15.10.2023

INTRODUCTION

The deposition of solid particles in a liquid medium is studied by assuming that the medium comprises particles of a dispersed solid phase and a viscous Newtonian fluid. In separate works, the precipitation process of small solid phase particles in the liquid phase in the lower bottom part of a two-phase mixture is analyzed.

Thus, in [1], the possibility of a collision between precipitating small solids in a viscous liquid and the lower horizontal wall (bottom of the vessel) was analyzed. The author of work [1] indicates in a note that this section of the article on the collision of a precipitating solid particle of the body with the vessel bottom was coauthored with N.E. Zababakhin.

Article [2] almost completely reproduces the essence of the work in [1] concerning the original theoretical and computational material. In the graphic part of [2], as in [1], a solid particle is presented as a cylinder with an upper horizontal bottom of a given radius and a lower spherical bottom of an unspecified radius. The height of the vertical cylindrical part of the particle is unstated, and its size is not indicated.

Article [2] additionally presents the results of the first experimental studies, which qualitatively confirm the theoretical conclusion about the impossibility, under certain conditions, of the collision of a convex body with the underlying flat wall of the vessel. However, no results of a comparison of the behavior of spherical particles with calculated estimates are presented.

RESEARCH METHOD

As shown in Figure 1, the particle shape adopted in [1, 2] is cylindrical with a radius a, an upper flat bottom, and a convex lower bottom of radius R. *In addition*, we indicate the height of the cylindrical part H. The particle approaches the vessel's flat bottom at a speed of υ.

The gap width between the bottom surface of the particle and the flat wall $h = Y + r^2/(2R)$ decreases with time. Thus, the particle is assumed to be cylindrical with a flat top surface and a spherical bottom surface. Moreover, the flow regime in the annular section between the downward-moving particle and the flat wall is assumed to be laminar. The distribution of velocity υ over the gap thickness is assumed to be parabolic, and for a given r, the average velocity in height is $u_{cp} = (2/3)u_{\max}$, and the velocity gradient at the surface is equal to $4u_{\max}/h$ or $6u_{cp}/h$.

Next, we consider the element of the liquid disk and the pressure P and friction forces F, acting on it, the sum of which must be equal to zero. Consequently, the pressure distribution in the surrounding fluid is determined. Next, considering the fluid pressure on the ends of the selected cylindrical element of the fluid, the total retarding force is

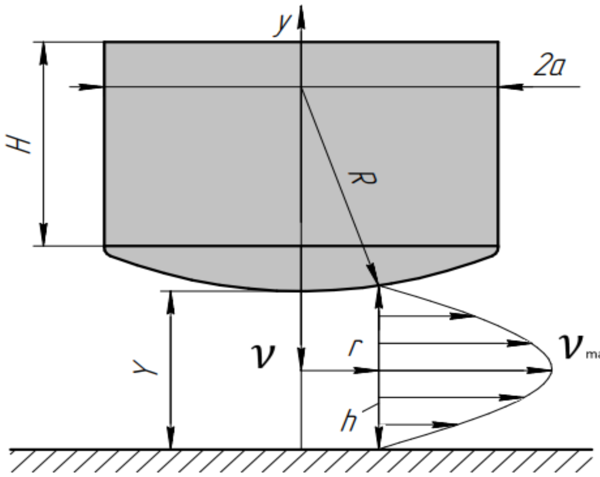

Fig. 1. Configuration of collocation of a precipitating solid particle and a flat surface of a vessel head.

Рис. 1. Конфигурация взаиморасположения осаждающейся твёрдой частицы и плоской поверхности днища сосуда.

determined, and the equation of motion of the precipitating body is presented as follows [1, 2]:

$$M\frac{d\upsilon}{dt} = -\frac{3}{2}\pi\eta\upsilon\frac{a^4}{Y\cdot\left(Y + a^2/2R\right)^2}. \quad (1)$$

In Eq. (1), M denotes the effective mass of the body, which exceeds its true mass by the amount of added liquid mass. This mass is added to the mass of a body moving unevenly in a liquid medium to account for the effect of the medium on this body [3]. The amount of added mass depends on the body shape, movement direction, and medium density.

Further, to simplify the final calculation expressions, the distance from the lower, near-bottom contour of the particle to the stationary bottom of the vessel is assumed to be small, which makes it possible to considerably simplify the developed calculation equations.

Since $\frac{d\upsilon}{dt} = \upsilon\cdot\frac{d\upsilon}{dY}$ and assuming that $Y \ll (a^2/2R)$, Eq. (1) is simplified as follows:

$$\frac{d\upsilon}{dY} = -6\pi\eta\cdot\left(\frac{R^2}{MY}\right)^2. \quad (2)$$

Integrating Eq. (2) with the boundary condition that $Y = Y_0$, $\upsilon = \upsilon_0$, we obtain

$$\upsilon - \upsilon_0 = -6\pi\eta\cdot\frac{R^2}{M}\cdot\ln\left(\frac{Y}{Y_0}\right). \quad (3)$$

Assuming that the particle has stopped at $\upsilon = 0$, we transform Eq. (3) into

DOI: https://doi.org/10.17816/2074-0530-501750

$$Y_k = Y_0 \exp\left(\frac{M\upsilon_0}{6\pi\eta R^2}\right). \quad (4)$$

The works [1, 2] comprise the same example of the calculation, where M =1 mg, R =0,05 cm, Y_0 =1 mm, and η =0,01 g/cm·s at υ_0 =– 1 cm/s. In both articles, the calculation using Eq. (4) gives a value of Y_k =0,012 cm, i.e., the particle does not touch the bottom of the vessel.

In addition, work [1] provides an example of a calculation with different initial data: M =1 g, R =0,5 cm, Y_0 =1 cm, and η =0,01 g/cm·s at υ_0 = –1 cm/s. Calculation using Eq. (4) gives a value of Y_k =10^{-8} cm, which is much less than the expected surface roughness, i.e., the particle practically touches the bottom of the vessel [1].

In these two examples, despite the substantial thousand-fold difference in particle masses, the initial velocity υ_0 = –1 cm/s is assumed to be identical, which traditionally requires special justification, which is absent.

CLARIFICATION OF THE METHODOLOGY FOR CALCULATING THE BOTTOM PRECIPITATION OF PARTICLES

At the same time, the use of the assumption $Y \ll (a^2/2R)$ is not an obstacle to integrating Eq. (1). Having integrated differential Eq. (1) in the range from Y_0 to Y_k, we obtain the following equation:

$$\upsilon - \upsilon_0 = 6\pi\eta \cdot \frac{R^2}{M} \cdot \left[\begin{array}{l} \ln\left(1+\frac{a^2}{2RY}\right) + \frac{1}{1+a^2/(2RY)} - \\ -\ln\left(1+\frac{a^2}{2RY_o}\right) - \frac{1}{1+a^2/(2RY_0)} \end{array} \right]. \quad (5)$$

Assuming that the particle stopped at $\upsilon = 0$ or rather close to zero, we transform Eq. (5) into the following equation:

$$\upsilon_0 = 6\pi\eta \cdot \frac{R^2}{M} \cdot \left[\begin{array}{l} \ln\left(1+\frac{a^2}{2RY_0}\right) + \frac{1}{1+a^2/(2RY_0)} - \\ -\ln\left(1+\frac{a^2}{2RY_k}\right) - \frac{1}{1+a^2/(2RY_k)} \end{array} \right]. \quad (6)$$

In comparison with the previous version considered in [1, 2] (Eq. (4)), a dependence on the particle shape arose. In [2], the particle was considered a cylinder of radius a with a convex bottom of radius R facing the vessel bottom.

Table 1 presents the results of the calculations given in [1, 2] and obtained in this work. In all calculations, the dynamic viscosity of water is η = 0.01 g/cm·s.

Compared to the results of calculations performed in [1, 2], the results obtained from the relations obtained in this work show considerable refinement in the value of the final distance Y_k from the top of the body to the fixed wall. In addition, the local value of particle velocity v_k in cm/s could be determined at these points.

Notably, the calculated values of Y_k are at the micron level or less, which corresponds to the surface roughness achieved by its grinding [4] and seems physically unattainable in the real conditions of a liquid-filled vessel.

Using the results obtained, we evaluate the acceptability of the assumptions used in [1, 2], namely, the condition

$$Y \ll (a^2/2R). \quad (7)$$

The works [1, 2] do not contain information about the particle radius a. Therefore, when performing calculations within the model used, the radii of the cylindrical part of the particle and the radius of its convex bottom part were assumed to be equal, i.e., $a = R$. In this case, Eq. (7) is transformed into

$$Y \ll R/2. \quad (8)$$

The calculated data in Table 1 show that Eq. (8) is satisfied in all cases when the particles are located near the lower stationary surface. Eq. (8) is not satisfied in the initial state using the relations given in [1, 2].

Previous studies [5] showed that in the case of solid particles of arbitrary shape, three characteristic particle diameters should be used, namely, $d_v^3 = 6a^2(H + 2a/3)$ as the cube of the sphere diameter, equivalent to the particle volume; $d_s^2 = a(3a + 2H)$ as the square of

Table 1. Comparison of calculated properties of solid particles precipitation near a horizontal vessel head

Таблица 1. Сравнение расчётных характеристик осаждения твёрдых частиц вблизи горизонтального днища

Symbols, dimensions	Reference			
	[1]	(6)	[1], [2]	(6)
M, g	1	1	0,001	0,001
R, cm	0,5	0,5	0,05	0,05
υ_0, cm/s	-1	-1	-1	-1
Y_0, cm	1	1	0,1	0,1
Y_k, cm	10-8	$5,4\times10^{-11}$	0,01	$1,177\times10^{-3}$
, cm/s		$5,54\times10^{-5}$		$2,777\times10^{-4}$

DOI: https://doi.org/10.17816/2074-0530-501750

the sphere diameter, equivalent in area to the lateral surface of the particle; and $d_m = 2a$ as the sphere diameter, equivalent in area to the middle section of the particle. The equation for the Reynolds number uses the equivalent particle diameter $d_e = (2d_s + d_m)/3$. Using such concepts, hydraulic resistance forces were calculated for solid particles comprising two spherical particles, a disk, and needle-like ellipsoids. In all cases, for two orientations of the particles relative to the vertical direction of particle deposition, satisfactory agreement with the results of exact numerical calculations was obtained [5]. This approach has been extended to the precipitation of multimodal two-phase mixtures. In this case, the collision between particles of different fractions of the solid phase was also considered. Subsequently, this approach was used to calculate the movement of polymodal mixtures of solid particles in horizontal, vertical, and inclined pipelines [6]. In this case, the process of diffusion of solid particles in the vertical direction was also considered.

In all cases, complete information about the actual geometric configuration of solid particles is required to study precipitating solid particles and their movement in fluid flows. In previous studies [1, 2], only one direct geometric parameter appears, namely, the radius R of the lower convex bottom of the solid particle. The value M determines the particle mass but without indicating the substance density of the particle, which does not enable the determination of its volume and further determine, or even estimate, the values a and H. Therefore, unfortunately, the initial value of velocity υ_0 cannot be correctly calculated.

Using the hydraulic resistance coefficient according to the Stokes formula for spherical particles, [5] represented the precipitation rate of a solid particle of arbitrary shape as follows:

$$\upsilon_0 = (\rho_s - \rho) \cdot \frac{g d_v^3 d_e}{18 \eta d_m^2}. \qquad (9)$$

Let us assume, in the same way as was accepted in [1, 2], that $a = H = R = 0{,}5$ cm, when large particles are deposited, and $a_* = H_* = R_* = 0{,}5$ mm in the case of small particles.

Using Eq. (9), we determine the ratio of the sedimentation rates of large particles to those of small particles.

$$\frac{\upsilon_{01}}{\upsilon_{02}} = \left(\frac{d_{v1}}{d_{v2}}\right)^3 \cdot \left(\frac{d_{e1}}{d_{e2}}\right) \cdot \left(\frac{d_{m2}}{d_{m1}}\right)^2, \qquad (10)$$

where ${d_{v1}}^3 = 6a^2(H + 2a/3) = 1{,}25\ \text{cm}^3$;
$d_{v2}^3 = 6{a_*}^2(H_* + 2a_*/3) = 1{,}25\ \text{mm}^3$;
$d_{m1} = 2a = 1$ cm; $d_{s1} = [a(3a + 2H)]^{1/2} = 1{,}118$ cm;
$d_{e1} = (2d_{s1} + d_{m1})/3 = 1{,}0787$ cm;
$d_{s2} = [a_*(3a_* + 2H_*)]^{1/2} = 1{,}118$ mm;
$d_{m2} = 2a_* = 2 \times 0{,}5 = 1$ mm;
$d_{e2} = (2d_{s2} + d_{m2})/3 = 1{,}0787$ mm.

Substituting the given numerical values into Eq. (10), we obtain the ratio $\upsilon_{01}/\upsilon_{02} = 100$, i.e., particle precipitation rates differ considerably and are not equal, as assumed in previous studies [1, 2].

With regard to spherical precipitating particles, the same conclusion follows from the well-known Stokes formula for precipitating spherical particles, $\upsilon_0 = (\rho_s - \rho)\dfrac{gd^2}{18\eta}$. In this case, when the particle diameter differs by a factor of 10, the precipitation rates differ by a factor of 100.

When determining the free precipitation rate of solid particles, the influence of Brownian motion on the free sedimentation rate of particles is neglected. As noted in [7], Brownian motion has an important influence on two-phase flows involving tiny solid particles. If we proceed from the condition that the kinetic energy of precipitating particles is much greater than the energy of the thermal motion of molecules, then the condition that Brownian motion can be neglected is presented as follows [5]:

$$\frac{1}{2} \cdot \frac{\pi d^3}{6} \cdot \rho \upsilon^2 >> \frac{3}{2} \cdot kT, \qquad (11)$$

where $k = 1{,}38 \times 10^{-23}$ J/K is Boltzmann's constant, and T is the temperature in Kelvin.

The characteristic value of the particle diameter, at which the influence of Brownian motion on the precipitation rate of solid particles must be considered, is determined by the equation [5]

$$d \gg \left[\frac{5832 kT \nu^2}{\pi\left(\dfrac{\rho_s}{\rho} - 1\right)^2 g^2 \rho_s}\right]^{\frac{1}{7}}. \qquad (12)$$

The values of the right side of Eq. (10) calculated for different values of the material density of solid particles in water at T = 293 K is presented in Table 2.

According to these calculated data, the influence of the Brownian motion of the carrier liquid molecules on the movement of precipitating solid particles can be neglected only for high-density particles.

CONCLUSION

The analysis showed that the use of the assumption $Y \ll (a^2/2R)$ in works [1, 2] is physically incorrect

Table 2. Values of critical diameter of solid particles

Таблица 2. Значения критического диаметра частиц твёрдой фазы

ρ_s, kg/m^3	1250	1500	2000	2500	3000	3500	4000	4500
d, mm	0,116	0,095	0,078	0,069	0,064	0,060	0,057	0,054

when analyzing the deposition of solid particles near a flat bottom. It is shown that an analytical solution can be obtained without using this assumption, which enables a more accurate determination of the coordinates of a particle near a flat bottom.

ADDITIONAL INFORMATION

Authors' contribution. A.S. Kondratiev — statement of the research problem, search for publications on the topic of the article, writing the text of the manuscript, development of a generalized analytical model, analysis of calculation results, editing the text of the manuscript; P.P. Shvydko — bibliographic search, numerical research, preparation of the text of the manuscript, creating images. All authors made a substantial contribution to the conception of the work, acquisition, analysis, interpretation of data for the work, drafting and revising the work, final approval of the version to be published and agree to be accountable for all aspects of the work.

Competing interests. The authors declare that they have no competing interests.

Funding source. This study was not supported by any external sources of funding.

ДОПОЛНИТЕЛЬНО

Вклад авторов. А.С. Кондратьев — постановка задачи исследования, анализ публикаций по теме статьи, разработка обобщённой расчётной модели, анализ результатов расчётов, редактирование текста рукописи; П.П. Швыдько — библиографический поиск, проведение численных исследований, подготовка текста рукописи, создание изображений. Авторы подтверждают соответствие своего авторства международным критериям *ICMJE* (все авторы внесли существенный вклад в разработку концепции, проведение исследования и подготовку статьи, прочли и одобрили финальную версию перед публикацией).

Конфликт интересов. Авторы декларируют отсутствие явных и потенциальных конфликтов интересов, связанных с публикацией настоящей статьи.

Источник финансирования. Авторы заявляют об отсутствии внешнего финансирования при проведении исследования.

REFRENCES

1. Zababakhin EI. Some cases of viscous fluid motion. In: *Questions of modern experimental and theoretical physics.* Leningrad: Nauka; 1984:58–68. (in Russ).

2. Zdeshchits VM, Sidorenko VD. Collision of solids in a viscous fluid. *Visnik KTU.* 2010;26:59–61. (in Russ).

3. Sedov LI. Plane problems of hydrodynamics and aerodynamics. Moscow: Nauka; 1980. (in Russ).

4. GOST 9378-93. Obraztsy sherokhovatosti poverkhnosti (sravneniya). Mezhgosudarstvennyy standart. Minsk: Mezhgosudarstvennyy sovet po standartizatsii, metrologii i sertifi-katsii; 2002. (in Russ). [дата обращения: 17.05.2023] Режим доступа: https://meganorm.ru/Data2/1/4294847/4294847416.pdf

5. Kondratyev AS. *Settlement of polymodal solids in Newtonian fluids.* Moscow: Sputnik+; 2014. (in Russ).

6. Kondratyev AS, Nya TL, Shvydko PP. *Methodology and methods for calculating the movement of dispersed solid particles in pipelines.* Moscow: Mos. Politekh; 2020. (in Russ).

7. Batchelor J. The influence of Brownian motion on the average stress in a suspension of spherical particles. In: *Hydrodynamic interaction of particles in suspensions. Mechanics. New in foreign science.* Moscow: Mir;22:124–153. (in Russ).

СПИСОК ЛИТЕРАТУРЫ

1. Забабахин Е.И. Некоторые случаи движения вязкой жидкости // Вопросы современной экспериментальной и теоретической физики. Л.: Наука, 1984. С. 58–68.

2. Здещиц В.М., Сидоренко В.Д. Столкновение твердых тел в вязкой жидкости // Вісник КТУ. 2010. № 26. С. 59–61.

3. Седов Л.И. Плоские задачи гидродинамики и аэродинамики. М.: Наука, 1980.

4. ГОСТ 9378-93. Образцы шероховатости поверхности (сравнения). Межгосударственный стандарт. Минск: Межгосударственный совет по стандартизации, метрологии и сертификации, 2002. [дата обращения: 17.05.2023] Режим доступа: https://meganorm.ru/Data2/1/4294847/4294847416.pdf

5. Кондратьев А.С. Осаждение полимодальных твердых частиц в ньютоновских жидкостях. М.: Спутник+, 2014.

6. Кондратьев А.С., Нья Т.Л., Швыдько П.П. Методология и методы расчета движения дисперсных твердых частиц в трубопроводах. М.: Мос. Политех. 2020.

7. Бэтчелор Дж. Влияние броуновского движения на среднее напряжение в суспензии сферических частиц // Гидродинамическое взаимодействие частиц в суспензиях. Механика. Новое в зарубежной науке. М.: Мир, 1980. Вып. 22. С. 124–153.

AUTHORS' INFO

*** Alexander S. Kondratiev,**
Dr. Sci. (Tech.),
Professor of the Industrial Heat Power Engineering Department;
address: 38 Bolshaya Semenovskaya street, 107023 Moscow, Russian Federation;
ORCID: 0009-0001-2030-5165;
eLibrary SPIN: 6739-0496;
e-mail: ask41@mail.ru

Pavel P. Shvydko,
Project Manager;
ORCID: 0009-0007-8030-6403;
eLibrary SPIN: 8347-3886;
e-mail: Shvydko.P@gmail.com

* Corresponding author / Автор, ответственный за переписку

ОБ АВТОРАХ

*** Кондратьев Александр Сергеевич,**
д-р техн. наук,
профессор кафедры «Промышленная теплоэнергетика»;
адрес: Российская Федерация, 107023, Москва, ул. Большая Семеновская, д. 38;
ORCID: 0009-0001-2030-5165;
eLibrary SPIN: 6739-0496;
e-mail: ask41@mail.ru

Швыдько Павел Петрович,
руководитель проекта;
ORCID: 0009-0007-8030-6403;
eLibrary SPIN: 8347-3886;
e-mail: Shvydko.P@gmail.com