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ON NONPARAMETRIC CONTROL OF A DYNAMIC SYSTEM

E. D. Agafonov[1], A. V. Shishkina[2*]

[1]Reshetnev Siberian State University of Science and Technology
31, Krasnoyarsky Rabochy Av., Krasnoyarsk, 660037, Russian Federation
[2]Siberian Federal University
26b, Academica Kirenskogo Str., Krasnoyarsk, 660074, Russian Federation
[*]E-mail: nastya.shishkina9666@mail.ru

The paper considers the problem of dual control of an inertia-free object whose input is affected by a control variable and an observable but uncontrollable variable. The idea of dual control belongs to A. Feldbaum and was developed on the basis of Bayesian approach. In this case, the probability densities of interference, as well as input output variables are known. In particular, the case of Gaussian probability densities was investigated. As a result, algorithms of dual control with the simplest objects of the inertia-free class were obtained. In the case of combined control systems, these studies were carried out by Feldbaum's followers.

Further development of dual control theory was described by Y. Tsypkin. The probability density of the interference was unknown, but the stage of selecting the structure of the control device and the equation of the describing object were necessary. In Tsypkin's works, the corresponding parametric algorithms for dual control are given. In this case, a simultaneous estimation of the coefficient of the model and the regulator was made on the basis of the method of stochastic approximations.

Later on nonparametric dual control algorithms were proposed. It is this way of control design that is discussed in the current paper. The parametric model of the object and the parametric structure of the controller were unknown in the problem statement. Nevertheless, it was known from a priori information that the characteristic of the object was one-to-one in control. Below we consider nonparametric control algorithms that combine the processes of simultaneous control and the study of the object with an accuracy of the structure unknown to the parameters, but a priori with a given number of delayed elements of the output variable of the object. In other words, the depth of the memory of the managed object was determined. In this case, a nonparametric algorithm for dual control can function in conditions of passive and active accumulation of information. The technique for representing a one-dimensional inertial-free dynamic system to a multidimensional static one is presented in the paper. Some results of numerical investigation of nonparametric algorithms of dual control are also presented.

Keywords: object with memory, dual control, combined system, nonparametric algorithms, bandwidth parameter, parameter setting.

О НЕПАРАМЕТРИЧЕСКОМ УПРАВЛЕНИИ ДИНАМИЧЕСКОЙ СИСТЕМОЙ

Е. Д. Агафонов[1], А. В. Шишкина[2*]

Сибирский государственный университет науки и технологий имени академика М. Ф. Решетнева
Российская Федерация, 660037, г. Красноярск, просп. им. газ. «Красноярский рабочий», 31
[2]Сибирский федеральный университет
Российская Федерация, 660074, г. Красноярск, ул. Академика Киренского, 26б
[*]E-mail: nastya.shishkina9666@mail.ru

Рассматривается задача дуального управления безынерционным объектом, на вход которого действует управляющая переменная и неуправляемая, но контролируемая переменная. Идея дуального управления принадлежит А. А. Фельдбауму и была развита на основе байесовского подхода, в этом случае плотности вероятности помех, а также входных-выходных переменных известны, в частности, исследовался случай гауссовых плотностей вероятностей. В результате были получены алгоритмы дуального управления с самыми простыми объектами класса безынерционных. На случай комбинированных систем управления эти исследования проводились его последователями.

Дальнейшее развитие теории дуального управления описывается в книге Я. З. Цыпкина, где плотность вероятности помех уже неизвестна, но также необходим был этап выбора структуры управляющего устройства и уравнения описывающего объекта. В работах Цыпкина приведены соответствующие параметрические алгоритмы дуального управления. В этом случае осуществлялась одновременная оценка коэффициента модели и регулятора на основе метода стохастических аппроксимаций.

Несколько позже были предложены непараметрические алгоритмы дуального управления. Именно этот путь и рассматривается в настоящей статье. Здесь уже не известна ни параметрическая модель объекта, ни параметрическая структура регулятора. Тем не менее, из априорных сведений известно, что характеристика объекта является взаимно однозначной по управлению. Рассматриваются непараметрические алгоритмы управления, совмещающие процессы одновременного управления объектом и его изучения с точностью неизвестной до параметров структуры, но априори с заданным числом запаздывающих элементов выходной переменной объекта. Иными словами, глубина памяти управляемого объекта определена. В этом случае непараметрический алгоритм дуального управления может функционировать в условиях пассивного и активного накопления информации. Приведена методика представления одномерной безынерционной динамической системы к многомерной статической. Представлены также некоторые результаты численного исследования рассматриваемых непараметрических алгоритмов дуального управления.

Ключевые слова: объект с памятью, дуальное управление, комбинированная система, непараметрические алгоритмы, параметр размытости, настройка параметров.

Introduction. The problem to be solved in the paper is the design and implementation of nonparametric control algorithms. An object under control is assumed to be inertia-free and it is described by an equation with an unknown structure with respect to its parameters. Together with the control effect the object is influenced by uncontrollable but observable input. The nonparametric dual control algorithms under consideration were investigated for various tactics of determining the bandwidth parameters at each clock cycle. A concept of dual control was created by A. Feldbaum [1] and developed by Y. Tsypkin [2]. It was originally intended for Bayes control problem statement when the object under control was inertia-free. The main idea of the concept was simultaneous control and learning of the object. That involves procedures of parametric identification [3] together with implementation of control methods in the parametric formulation [4–7]. It should be noted that a dual control system is an example of a control device with memory.

Problem statement. The following notations are introduced: let $x = (x_1 \ldots x_n) \in R^n$ be output of the object, $u = (u_1 \ldots u_k) \in R^k$ be controlled input effect, $\mu = (\mu_1 \ldots \mu_m) \in R^m$ be uncontrollable but observable input, and $x^* = (x_1^* \ldots x_n^*) \in R^n$ be reference of the object under control (fig. 1) [8].

In fig. 1 the control device is denoted by 'yy', the object under control is denoted by 'O', and random stationary noise effects influencing both the object itself and the object measurement channels are denoted by ξ_t, h_t. We assume these noise effects to be unbiased and to have limited variance. The unknown model curve of the object $x = f(u, \mu)$ is assumed to be one-to-one with respect to control effect $u \in \Omega(u) \in R^k$ for the fixed vector $\mu \in \Omega(\mu) \in R^m$ in the feasible domain of $u \in \Omega(u)$.
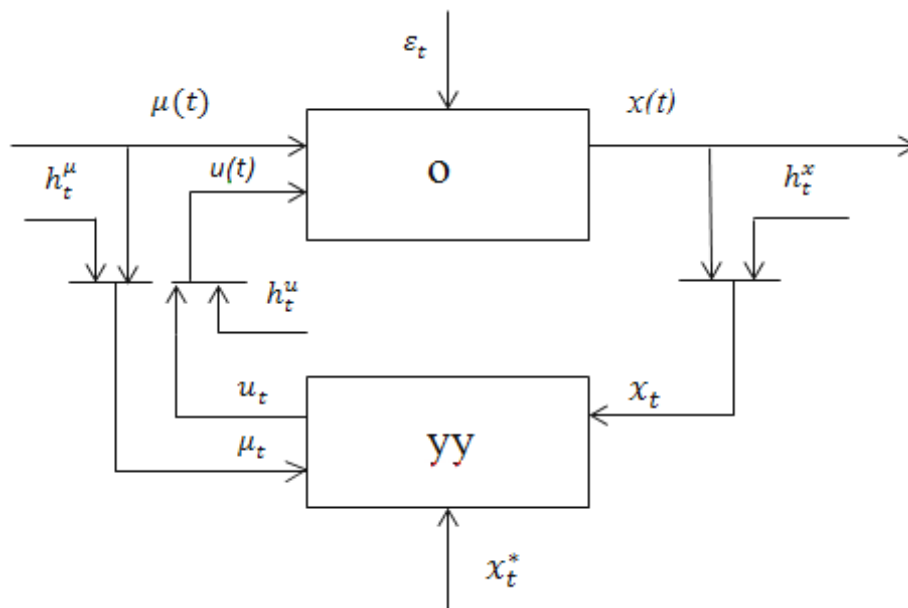

Fig. 1. The scheme of a nonparametric control system, where (*t*) denotes continuous time,
and the subscript *t* indicates discreet time moments of measurements

Рис. 1. Схема непараметрической системы управления:
t – непрерывное время; *t* в качестве индекса – дискретное время контроля измерения

Nonparametric combined control algorithms. A nonparametric control algorithm is based upon Nadaraya and Watson [9] nonparametric estimates of regression. A nonparametric dual control algorithm in the form [10] for a single-input case is represented by the expression:

$$u_{t+1} =$$
$$= \frac{\sum\limits_{i=1}^{t} u_i \Phi\left(\frac{x_{t+1}^* - x_i}{c_t^x}\right) \Phi\left(\frac{\mu_{t+1} - \mu_i}{c_t^\mu}\right)}{\sum\limits_{i=1}^{t} \Phi\left(\frac{x_{t+1}^* - x_i}{c_t^x}\right) \Phi\left(\frac{\mu_{t+1} - \mu_i}{c_t^\mu}\right)} + \Delta u_{t+1}, i = \overline{1,t}. \quad (1)$$

For the multiple-input object a nonparametric dual control algorithm can be expressed by

$$u_{t+1}^\eta =$$
$$= \frac{\sum\limits_{i=1}^{t} u_i^\eta \prod\limits_{j=1}^{n} \Phi\left(\frac{x^{*j} - x_i^j}{c_t^x}\right) \prod\limits_{j=1}^{m} \Phi\left(\frac{\mu^j - \mu_u^j}{c_t^\mu}\right)}{\sum\limits_{i=1}^{t} \prod\limits_{j=1}^{n} \Phi\left(\frac{x^{*j} - x_i^j}{c_t^x}\right) \prod\limits_{j=1}^{m} \Phi\left(\frac{\mu^j - \mu_u^j}{c_t^\mu}\right)} + \Delta u_{t+1}, \quad (2)$$
$$\eta = \overline{1,k},$$

where u_i are learning sample elements, $\Phi(\cdot)$ is a kernel function with the following features:

$$0 \le \Phi(\cdot) < \infty; \frac{1}{c_t}\int \Phi\left(\frac{x}{c_t}\right) dx = 1; \lim_{t\to\infty} \frac{1}{c_t} \Phi\left(\frac{x}{c_t}\right) = \delta(x);$$

v is an arbitrary argument; c_t^x, c_t^μ are bandwidth parameters that satisfy the following convergence conditions [8; 10]:

$$c_t > 0; \lim_{t\to\infty} tc_t \to \infty; \lim_{t\to\infty} c_t \to 0.$$

Estimate (1) is control algorithm for a combined system described by the following equation:

$$x_t = f(u_t, \mu_t, \xi_t), \quad (3)$$

where the shape of the function $f(\cdot)$ is unknown, but we know that $f(\cdot)$ is a continuous one-to-one stationary function up to its arguments [11].

A single-dimension nonparametric dual algorithm functioning can be explained as follows. A learning process begins with the first ternary of observed variables: x_1, u_1, μ_1. The initial stage of the control process is mostly concentrated on active accumulation of information in order to bring the object to the target state.

Here two key problems should be solved: how to fit bandwidth parameters c_t^x and c_t^μ, and which of them should be found first.

The bandwidth parameter c_t^x is found by means of the weighted residual calculation [12–14]:

$$c_t^x = l_1 \left| x_t^* - x_t^0 \right|, \quad (4)$$

where $x_t^0 = \min\limits_{i} \left| x^* - x_i \right|, i = \overline{1,t},$ and coefficient $l_1 > 1$. After repeating the control process ρ times one receives the data set $\{x_i, u_i, \mu_i\}, i = \overline{1,\rho}$.

Further we reduce the set $\{x_i, u_i, \mu_i\}, i = \overline{1,\rho}$ taking into consideration only measurements that satisfy the following condition:

$$c_t^\mu = l_2 \left| \mu_{t+1} - \mu_\rho^0 \right|, \quad (5)$$

where l_2 is a coefficient to be found experimentally: $l_2 > 1$. μ_ρ^0 is found from the optimization process $\mu_\rho^0 = \min\limits_{i} \left| \mu - \mu_i \right|, i = \overline{1,\rho}$. Finally we get a new reduced data set $\{x_i, u_i, \mu_i\}, i = \overline{1,s},$ whose size s satisfies the inequality $s < \rho$.

Judging from our experience, the algorithm (1) is insensitive to the sequence of c_t^x and c_t^μ bandwidth parameters fitting.

Let us pay attention to learning process of the nonparametric dual control algorithm. It was discussed in [10], and is represented by the iterative scheme:

$$u_{t+1} = u_t^* + \Delta u_{t+1}. \quad (6)$$

Information about the object under control is contained in u_t^*, and learning capabilities are fulfilled by the search additive Δu_{t+1}:

$$\Delta u_{t+1} = \alpha\left(x_{t+1}^* - x_{t+1}\right), \quad (7)$$

where α is a coefficient defining search amplitude that should be fitted. We require that $\Delta u_{t+1} \to 0$ with the growth of t.

It is a well-known fact that the output of a discrete dynamic object can be represented as follows [3]:

$$x_t = f(x_{t-1} \dots x_{t-k}, u_t). \quad (8)$$

In this case $x_{t-1} \dots x_{t-k}$ can be interpreted as supplementary incontrollable inputs in terms of previously introduced static object modeling routine. Fig. 2 describes the approach.

In fig. 2 the following notations are given: x_t^* is a reference output variable of the object; t with round brackets is a continuous time variable; subscript t denotes discrete time indices; h_t^u, h_t^x are random noise in measurement channels corresponding to the variables of the object; $\xi(t)$ is an unobservable random effect.

Thus, for the uncontrollable but observable variable x_{t-i}, $i = \overline{1,k}$ as the input effect of the object the algorithm (1) can be rewritten in the following form [15]:

$$u_{t+1} =$$
$$= \frac{\sum\limits_{i=1}^{t} u_i \Phi\left(\frac{x_{t+1}^* - x_i}{c_t^x}\right) \prod\limits_{j=1}^{k} \Phi\left(\frac{x_{t-j} - x_{i-j}}{c_t^x}\right)}{\sum\limits_{i=1}^{t} \Phi\left(\frac{x_{t+1}^* - x_i}{c_t^x}\right) \prod\limits_{j=1}^{k} \Phi\left(\frac{x_{t-j} - x_{i-j}}{c_t^x}\right)} + \quad (9)$$
$$+ \Delta u_{t+1}, i = \overline{1,t}.$$

A general control theory of similar objects control is explicated in [2]. Implementation of the corresponding dual control algorithms can be found in [16; 17]. Below we will focus on numerical experiments with a nonparametric dual control algorithm (9). During the experiments the object under control will be substituted by either an inertia-free (memory-free) operator or a dynamic operator.

It should be noted that the control algorithm does not possess information on the equation (operator) of the object under control excepting the type of the operator. The use of dual control algorithms is presented in [16; 17].

Numerical experiments. To perform the first batch of numerical experiments the object was substituted by the expression

$$x_{t+1} = u_{t+1} + \mu_{t+1}, \qquad (10)$$

where $x(t)$ is an output variable; $u(t)$ is a controllable input; $\mu(t)$ is observable but uncontrollable effect, taken as the process

$$\mu_t = 0.5 + 0.3\sin(0.2t). \qquad (11)$$

The control procedure starts with the first point (x_1, u_1, μ_1). Further data accumulation results in active learning of the control algorithm. As a consequence the object can be more effectively driven to a reference state or follow a reference trajectory.

Functioning of a nonparametric control algorithm is illustrated by fig. 3 and 4. A particular experiment was conducted to demonstrate the ability to follow the step-wise reference trajectory $x*(t)$.

Fig. 4, *a* shows an enlarged scale of reference $x*(t)$ and control $x(t)$ processes depicted in fig. 4, *b*. A non-parametric dual control algorithm can effectively solve the control problem even for a noticeable level of noise. The case when random noise with amplitude up to 3 % of the output value is presented in fig 5.

Let the reference to the control process be given by the expression $x_t^* = 2 + \sin(0.1t)$. For the case the corresponding control process is depicted in fig. 6.

Except stepwise functions and continuous reference functions one can construct other references using even random functions. To illustrate capabilities of the control algorithm (1) to follow random reference the following experiment was carried out (fig. 7). x_t^* is defined here as a sequence of sine function and purely random effect evenly distributed in the interval [0.5; 2.5].
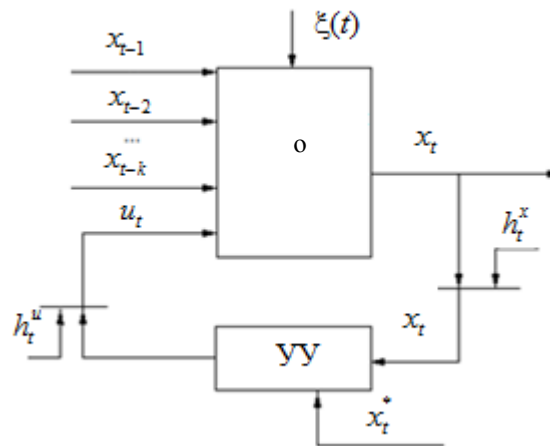

Fig. 2. A control scheme for an object with memory (dynamic object)
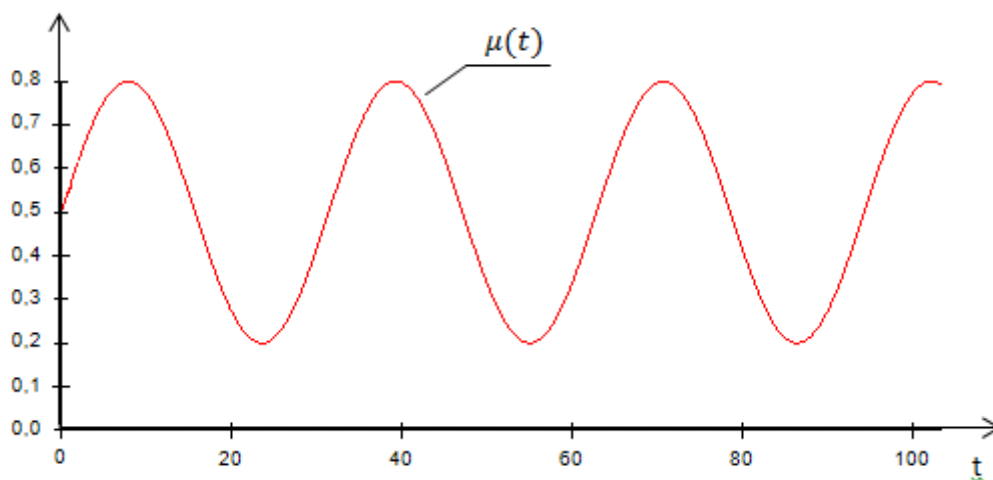
Рис. 2. Управление объектом с памятью

Fig. 3. Uncontrollable effect $\mu(t)$

Рис. 3. Неуправляемое входное воздействие
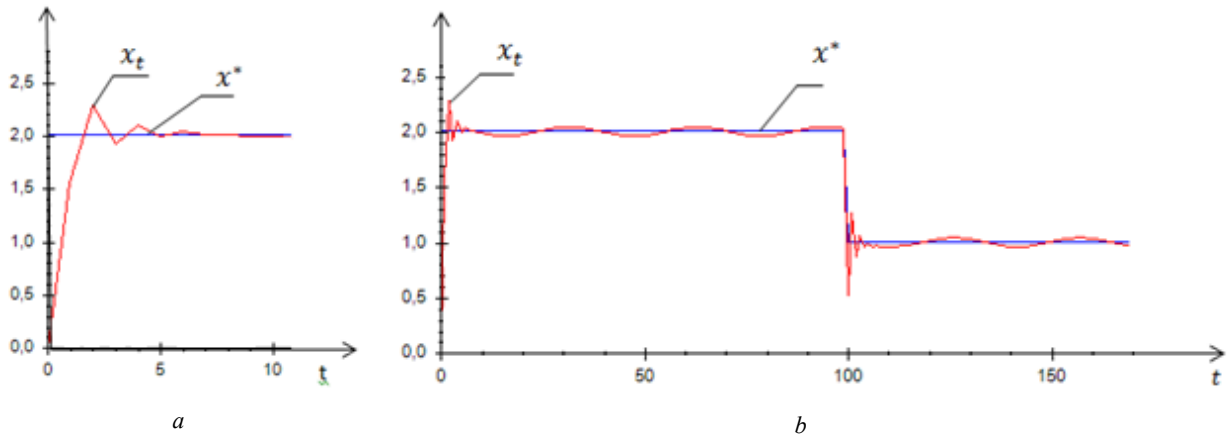
a   b

Fig. 4. Control process for a stepwise reference value

Рис. 4. Управление при задающем воздействии в виде ступенчатой функции
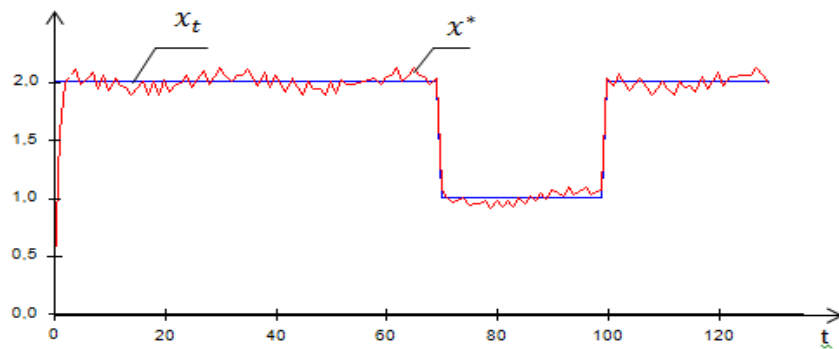

Fig. 5. Control process in case of 3 % random noise applied to the object output

Рис. 5. Управление при задающем воздействии с помехой
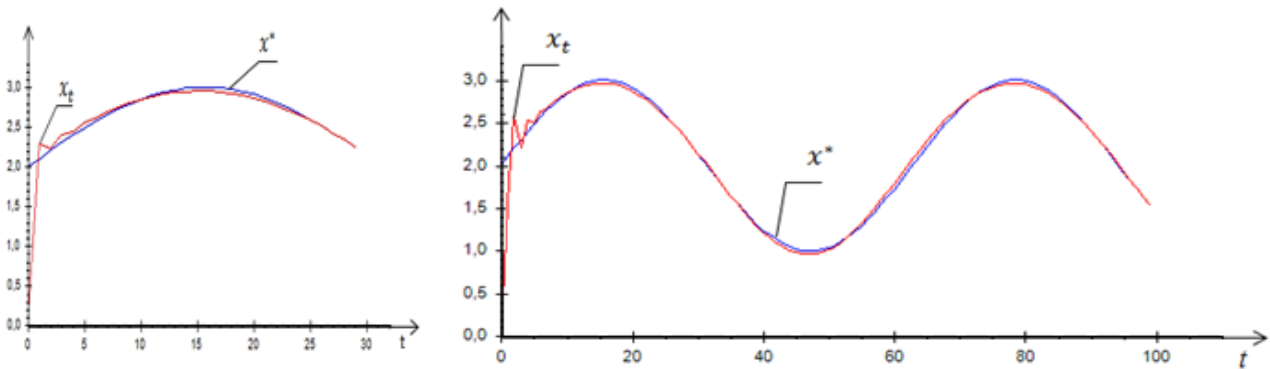

Fig. 6. Control process for continuous reference

Рис. 6. Управление при задающем воздействии в виде траектории

Experiments with the control algorithm operation make it evident that it is able to deal even with random references. On the contrary, standard P, PI, PID controllers cannot reach the level of control quality, because they are not based on data accumulation and analysis. Moreover, settling time is expected to be much worse for the controllers.

The control process in fig. 7 demonstrates satisfactory quality. That is an exceptional functionality of the control algorithm (1) can be noticed. It should be noted that none of already existing controllers can reach the same level of control accuracy and velocity.

Let us take into consideration another case when the object is represented by the dynamic operator (8). The equation is accepted in the form of the first-order discrete operator:

$$x_t = f\left(x_{t-1}, u_t\right). \tag{12}$$

Particularly, the linear first-order object is described by

$$x_t = \beta_1 u_t + \beta_2 x_{t-1}, \tag{13}$$

where β_1 and β_2 are finite constants.

715

For the case we describe peculiarities of the control procedure. The learning process begins with a pair of measurements, namely (x_0, u_0) and (x_1, u_1). The initial phase of control is devoted to data accumulation needed to bring the object to the target state. Further, time to reach the target diminishes to a great extent.

Let us demonstrate functioning of the algorithm (9). Let control reference be a stepwise function. Control process for the reference is depicted in fig. 8.

Fig. 9 demonstrates functioning of the algorithm when reference is a combination of a sine function and a random function. Again, the control process can be qualified as highly effective.
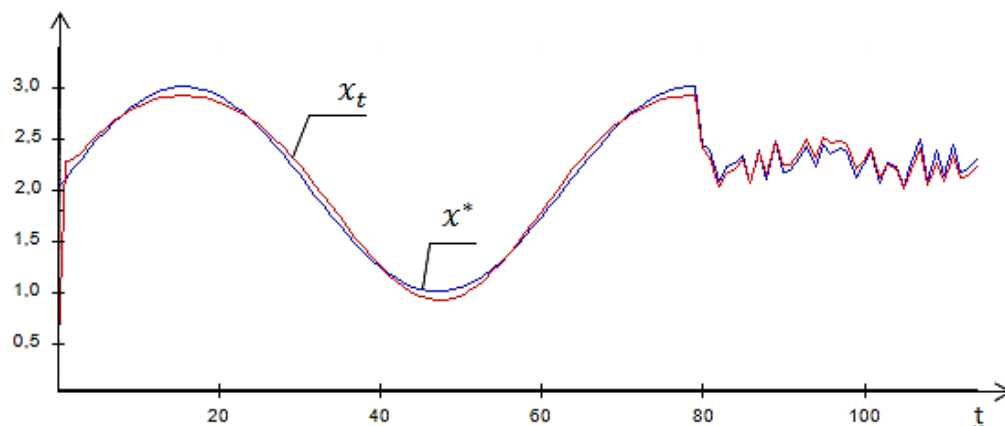
Fig. 7. Control procedure for combined reference containing random noise

Рис. 7. Управление при задающем воздействии в виде траектории и случайного задания
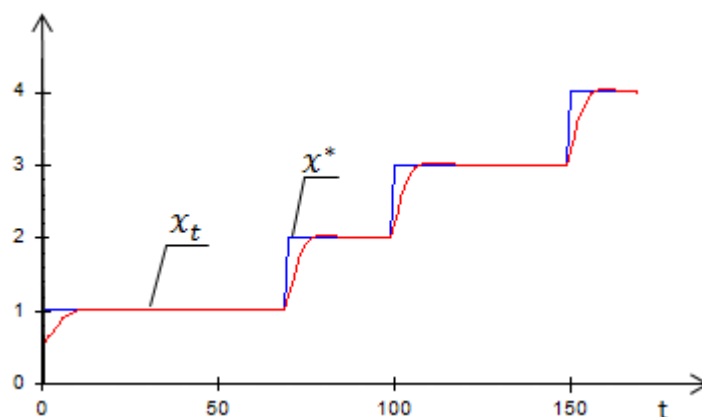
Fig. 8. Control process in case of stepwise reference

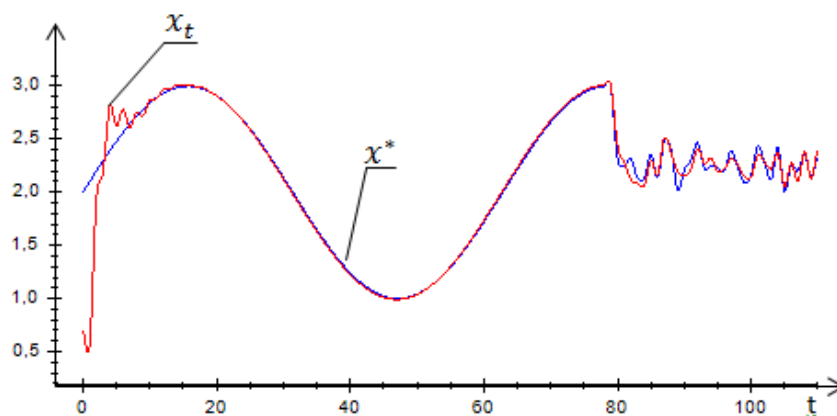Рис. 8. Управление при задающем воздействии в виде ступенчатой функции

Fig. 9. A random reference test for dynamic system dual control

Рис. 9. Результаты управления при случайном задании

Thus, the algorithm (9) is able to control a dynamic object with memory, providing good quality due to data accumulation and proper model-based control synthesis.

Conclusion. The problem of dynamic system control in case of nonparametric uncertainty conditions is discussed in the paper. After re-designation of object variables this problem can be reformulated in terms of multidimensional inertia-free object control. Bandwidth determination techniques for both controllable and uncontrollable input effects are proposed. Two variants of nonparametric control algorithm learning are discussed. Illustrations of some numerical experiments with the algorithm prove that it can be used in various computer-added systems of adaptive control. The key point of the algorithm is the capability to control continuous production processes with discrete-time measurement equipment.

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