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ON NECESSARY AND SUFFICIENT CONDITIONS OF SIMPLY REDUCIBILITY OF WREATH PRODUCT OF FINITE GROUPS

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A finite group is considered to be real if all the values of its complex irreducible characters lie in the field of real numbers. We note that the above reality condition is equivalent to the fact that each element of the group is conjugate to its inverse. A finite group is called simply reducible or a SR-group if it is real and all the coefficients of the decomposition of the tensor product of any two of its irreducible characters are zero or one.

The notion of a SR-group arose in the paper of R. Wiener in connection with the solution of eigenvalue problems in quantum theory. At present, there is a sufficient amount of literature on the theory of SR-groups and their applications in physics. The simplest examples of SR-groups are elementary Abelian 2-groups, dihedral groups, and generalized quaternion groups.

From the point of view of a group theory questions of interest are connected first of all with the structure of simply reducible groups. For example A. I. Kostrikin formulated the following question: how to express the belonging of a finite group to the class of SR-groups in terms of the structural properties of the group itself. Also, for a long time it was not known whether a simply reducible group is solvable (S. P. Stunkov's question). A positive answer to the last question was obtained in the works of L. S. Kazarin, V. V. Yanishevskiy, and E. I. Chankov. Questions concerning the portability of the properties of a group to subgroups, factor groups, and also their preservation in the transition to direct (Cartesian) and semidirect products or wreath products are always of interest.

The paper proves that the reality of H is the necessary condition of simply reducibility of the wreath product of the finite group H with the finite group K and the group K must be an elementary Abelian 2-group. We also indicate sufficient conditions for simply reducibility of a wreath product of a simply reducible group with a cyclic group of order 2.

Keywords: simply reducible group, real group, wreath product.

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О НЕОБХОДИМЫХ И ДОСТАТОЧНЫХ УСЛОВИЯХ ПРОСТО ПРИВОДИМОСТИ СПЛЕТЕНИЯ КОНЕЧНЫХ ГРУПП

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Конечную группу назовем вещественной, если все значения её комплексных неприводимых характеров лежат в поле вещественных чисел. Отметим, что сформулированное выше условие вещественности равносильно тому, что каждый элемент группы сопряжен со своим обратным. Конечная группа называется просто приводимой, или SR-группой, если она вещественна и все коэффициенты разложения тензорного произведения любых двух её неприводимых характеров равны нулю или единице.

Понятие SR-группы возникло в работе Р. Винера в связи с решением задач на собственные значения в квантовой теории. В настоящее время имеется достаточно литературы по теории SR-групп и их приложениям в физике. Простейшие примеры SR-групп дают элементарные абелевы 2-группы, диэдральные группы и обобщенные группы кватернионов.

С точки зрения теории групп прежде всего представляют интерес вопросы, связанные со строением просто приводимых групп. Например, А. И. Кострикин отмечает следующий вопрос: как выразить принадлежность конечной группы к классу SR-групп в терминах структурных свойств самой группы. Также продолжительное время не было известно, является ли просто приводимая группа разрешимой (вопрос С. П. Стрункова). Положительный ответ на последний вопрос был получен в работах Л. С. Казарина, В. В. Янишевского и Е. И. Чанкова. Вопросы, касающиеся переносимости свойств группы на подгруппы, фактор-группы, а также сохранения их при переходе к прямым (декартовым) и полупрямым произведениям или сплетениям, всегда вызывают интерес.

Доказано, что необходимым условием просто приводимости сплетения конечной группы H с конечной группой K является вещественность H, а группа K должна быть элементарной абелевой 2-группой. Также указаны достаточные условия просто приводимости сплетения просто приводимой группы с циклической группой порядка 2.

Ключевые слова: просто приводимая группа, вещественная группа, сплетение.

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Introduction. Simply reducible groups introduced by Wigner in [1] in connection with the questions of the quantum theory have found applications in other branches of physics as well (for example, [2–6]). The questions formulated above were noted in [7–9]. A positive solution of the problem of a finite simply reducible group solvability was published in a series of papers [10–12]. Questions of the portability of various properties of groups to wreath products were considered in [13–15].

In this paper we determine necessary and sufficient conditions for the simply reducibility of the wreath product of finite groups. In the first section we find necessary and sufficient conditions for the reality of the wreath product of two finite groups. In the second section we give necessary and sufficient conditions for the simply reducibility of the wreath product of a simply reducible group and a cyclic group of order 2, and we present an infinite series of simply reducible groups built using the construction of a wreath product.

The standard group-theoretical notation is used in this paper (for example, [16]). We also use the following equivalent definition of a simply reducible group that is more convenient for computations. A finite real group is called simply reducible if

$$\sum_{g\in G}\theta_G^3(g)=\sum_{g\in G}|C_G(g)|^2,$$

where $\theta_G(g)=|\{h\in G\,|\,h^2=g\}|$ and $C_G(g)$ is a centralizer of the element g.

Necessary conditions for simply reducibility of the wreath product of finite groups

Theorem 1. *Let H and K be finite groups, and their wreath product HsK is a simply reducible group. Then H is a real group, K is an elementary Abelian 2-group.*

Proof. According to its definition, a simply reducible group is real. We show that the reality *G* = *HsK* implies that *H* is real, and the group *K* must be an elementary Abelian 2-group.

So, let the group *G* = *HsK* be real, $k_1=1,k_2,\dots,k_n$ are all the different elements of the group *K*. Let us take an arbitrary element $h\in H$ and consider the element $x=h^{k_1}$ of the group G. According to the definition of the reality, there will be found such an element

$$y=k_i\cdot h_1^{k_1}\dots h_n^{k_n},\quad h_1,\dots,h_n\in H,$$

of the group G, that $x^y=x^{-1}$. We have

$$(h^{-1})^{k_1}=x^{-1}=x^y=(h_n^{-1})^{k_n}\dots(h_1^{-1})^{k_1}\cdot k_i^{-1}h^{k_1}k_i\cdot h_1^{k_1}\dots h_n^{k_n}=$$
$$=(h_1^{-1}h_1)^{k_1}\dots(h_i^{-1}hh_i)^{k_i}\dots(h_n^{-1}h_n)^{k_n}=(h_i^{-1}hh_i)^{k_i},$$

whence it follows that $k_i=1$ and $h^{-1}=h_i^{-1}hh_i$. Thus, every element of the group H is conjugate in it with its inverse and, hence, H is a real group.

Let us prove that K is an elementary Abelian 2-group. We assume the contrary, let $|k_j|=s>2$ for some j. Without restricting the generality, we may assume that $k_2=k_j,k_3=k_j^2,\dots,k_s=k_j^{s-1}$. Let us consider the element $g=k_2\cdot h^{k_1}$ of the group G, where $1\neq h\in H$. Again according to the definition of the reality there will be found such an element

$$y=k_i\cdot h_1^{k_1}\dots h_n^{k_n},\quad h_1,\dots,h_n\in H,$$

of the group G, that $g^y=g^{-1}$. We have

$$k_2^{-1}(h^{-1})^{k_1k_2^{-1}}=g^{-1}=g^y=$$
$$=(h_n^{-1})^{k_n}\dots(h_1^{-1})^{k_1}k_i^{-1}k_2\,h^{k_1}k_i\,h_1^{k_1}\dots h_n^{k_n}=$$
$$=k_2^{k_i}(h_n^{-1})^{k_nk_2^{k_i}}\dots(h_1^{-1})^{k_1k_2^{k_i}}\,h^{k_1k_i}\,h_1^{k_1}\dots h_n^{k_n},$$

whence it follows that $k_2^{k_i}=k_2^{-1}$ (in particular, k_i does not lie in a cyclic subgroup $\langle k_2\rangle$) and, therefore,

$$(h^{-1})^{k_1k_2^{-1}}=(h_n^{-1})^{k_nk_2^{-1}}\dots(h_1^{-1})^{k_1k_2^{-1}}\,h^{k_1k_i}\,h_1^{k_1}\dots h_n^{k_n}.\qquad(1)$$

Remembering, that $k_1k_2^{-1}=k_s$, $k_2k_2^{-1}=k_1$, $k_3k_2^{-1}=k_2,\dots,k_sk_2^{-1}=k_{s-1}$, we obtain (after comparing the right-hand and left-hand sides of relation (1)) the system of equations

$$1=h_1^{-1}h_s,1=h_2^{-1}h_1,1=h_3^{-1}h_2,\dots,1=h_{s-1}^{-1}h_{s-2},h=h_s^{-1}h_{s-1},$$

from which

$$h=(h_s^{-1}h_{s-1})(h_{s-1}^{-1}h_{s-2})\dots(h_3^{-1}h_2)(h_2^{-1}h_1)(h_1^{-1}h_s)=1,$$

contradiction with the choice of h. Hence, the group K is an elementary Abelian 2-group. The theorem is proved.

We show that the conditions formulated in Theorem 1 are sufficient for the realness of the group *G* = *HsK*. Let *G* = *HsK*, where the group *H* is real, and *K* is an elementary Abelian 2-group. We choose an arbitrary element $g\in G$ and establish its realness.

Situation 1. Let $g=h_1^{k_1}\dots h_n^{k_n}$, where $h_1,\dots,h_n\in H$. From realness H it follows that there exist such elements $r_1,\dots,r_n\in H$, that $h_i^{r_i}=h_i^{-1}$, $i=1,\dots,n$. The element $y=r_1^{k_1}\dots r_n^{k_n}$, obviously, inverts g.

Situation 2. Let $g=k\cdot h_1^{k_1}\dots h_n^{k_n}$, where $k\in K$ и $h_1,\dots,h_n\in H$. Without restricting the generality, we may assume, that the elements of the group K are arranged in such a way that $k_ik=k_{i+m}$, $i=1,\dots,m=n/2$. According to this the element g (and any other element from K) can be expressed by the following product

$$g = k \cdot \prod_{i=1}^{m} h_i^{k_i} h_{i+m}^{k_i k}.$$

We sort out for each product $h_i h_{i+m}$, $i = 1, \ldots, m$, such an element $f_i \in H$, that $(h_i h_{i+m})^{f_i} = (h_i h_{i+m})^{-1}$ and assume

$$y = k \cdot \prod_{i=1}^{m} (f_i h_i)^{k_i} (h_i^{-1} f_i)^{k_i k}.$$

Then

$$g^y = \prod_{i=1}^{m} (h_i^{-1} f_i^{-1})^{k_i} (f_i^{-1} h_i)^{k_i k} \cdot k\, k \times$$

$$\times \prod_{i=1}^{m} h_i^{k_i} h_{i+m}^{k_i k}\, k \cdot \prod_{i=1}^{m} (f_i h_i)^{k_i} (h_i^{-1} f_i)^{k_i k} =$$

$$= k \cdot \prod_{i=1}^{m} (h_i^{-1} f_i^{-1})^{k_i k} (f_i^{-1} h_i)^{k_i} h_i^{k_i k} h_{i+m}^{k_i} (f_i h_i)^{k_i} (h_i^{-1} f_i)^{k_i k} =$$

$$= k \cdot \prod_{i=1}^{m} (f_i^{-1} h_i h_{i+m} f_i h_i)^{k_i} (h_i^{-1} f_i^{-1} h_i h_i^{-1} f_i)^{k_i k} =$$

$$= k \cdot \prod_{i=1}^{m} (h_{i+m}^{-1})^{k_i} (h_i^{-1})^{k_i k} = g^{-1}.$$

A sufficient condition of simply reducibility of the wreath product of a simply reducible group and a cyclic group of order 2

Theorem 2. *In order that the wreath product G=HsZ$_2$ of a simply reducible group H and a cyclic group of order 2 be a simply reducible group it is necessary and sufficient that the equality*

$$\sum_{(u,v)\in H\times H,\, u\in v^H} (3\theta_H^4(u)\,|\,C_H(u)\,| + 3\theta_H^2(u)\,|\,C_H(u)\,|^2 + |\,C_H(u)\,|^3) =$$

$$= 4|H| \sum_{h\in H} |C_H(h)|^2 + 3 \sum_{(h,f)\in H\times H,\, h\in f^H} |\,C_H(h)\,|^4. \qquad (2)$$

be satisfied.

In the following three lemmas, the structure of the centralizers of the group *G* elements is determined.

Lemma 1. If $g = (h, f)$, $f, h \in H$, and $f^v = h$ for some $v \in H$, then

$$C_G(g) = \{(t,u), \sigma(uv, tv^{-1}) \mid t \in C_H(h), u \in C_H(f)\},$$

in particular,

$$|C_G(g)| = 2 \cdot |C_H(h)|^2.$$

Proof. Let $z \in C_G(g)$. Let us cosider two situations.

Situation 1. Let $z = (x, y)$, $x, y \in H$. Then

$$z^{-1} g z = (x^{-1}, y^{-1})(h, f)(x, y) = (h^x, f^y) = (h, f),$$

from which $x \in C_H(h)$ and $y \in C_H(f)$.

Situation 2. Let $z = \sigma(x, y)$, $x, y \in H$. Then $z^{-1} = \sigma(y^{-1}, x^{-1})$,

$$z^{-1} g z = \sigma(y^{-1}, x^{-1})(h, f)\sigma(x, y) =$$
$$= (x^{-1}, y^{-1})(f, h)(x, y) = (f^x, h^y) = (h, f),$$

from which $f^x = h$ and $h^y = f$. From the equalities $f^x = h$ and $f^v = h$ it follows, that $x = uv$, $u \in C_H(f)$. Similarly, from the equalities $h^y = f$ and $h^{v^{-1}} = f$ it follows, that $y = tv^{-1}$, where $t \in C_H(h)$. Thus, $z = \sigma(uv, tv^{-1})$, where $t \in C_H(h)$ and $u \in C_H(f)$.

The second assertion of the lemma follows from the isomorphism $C_H(h) \cong C_H(f)$. The lemma is proved.

Lemma 2. *If* $g = (h, f)$ *and the elements* h *and* f *are not conjugate in* H, *then*

$$C_G(g) = \{(x, y) \mid x \in C_H(h), y \in C_H(f)\},$$

in particular,

$$|C_G(g)| = |C_H(h)| \cdot |C_H(f)|.$$

Proof. Let $z \in C_G(g)$. If $z = (x, y)$, then

$$g^z = (x^{-1}, y^{-1})(h, f)(x, y) = (h^x, f^y) = (h, f).$$

From which $x \in C_H(h)$ and $y \in C_H(f)$. If we assume that $z = \sigma(x, y)$, then

$$g^z = \sigma(y^{-1}, x^{-1})(h, f)\sigma(x, y) = (f^x, h^y) = (h, f).$$

which contradicts the disconjugacy of h and f. The lemma is proved.

Lemma 3. *If* $g = \sigma(h, f)$, *then*

$$C_G(g) =$$
$$= \{(y^h, y) \mid y \in C_H(h \cdot f)\} \bigcup \{\sigma(f^{-1}t, th^{-1}) \mid t \in C_H(f \cdot h)\},$$

in particular,

$$|C_G(g)| = 2 \cdot |C_H(h \cdot f)|.$$

Proof. Let $z \in C_G(g)$. Again we consider two situations.

Situation 1. Let $z = (x, y)$, $x, y \in H$. We show that $z \in C_G(g)$ when and only when $y \in C_H(hf)$ and $x = y^h$. We have

$$\sigma(h, f) = z^{-1} g z = (x^{-1}, y^{-1})\sigma(h, f)(x, y) =$$
$$= \sigma(y^{-1}, x^{-1})(h, f)(x, y) = \sigma(y^{-1} h x, x^{-1} f y),$$

therefore

$$y^{-1} h x = h, \qquad x^{-1} f y = f. \qquad (3)$$

Multiplying these equalities, we obtain $y^{-1} h f y = hf$, that is, $y \in C_H(hf)$. The first equality (3) in this case gives $x = y^h$.

Inversely, let $y \in C_H(hf)$ and $x = y^h$. Then $y^{-1}hx = h$ and, consequently, $x^{-1}fy = h^{-1}y^{-1}hfy = h^{-1}hf = f$, that is, equalities (3) are satisfied, hence, $z^{-1}gz = g$.

Situation 2. Let $z = \sigma(x,y),\ x,y \in H$. Then

$$g = z^{-1}gz = \sigma(y^{-1},x^{-1})\sigma(h,f)\sigma(x,y) =$$

$$= \sigma(y^{-1},x^{-1})(f,h)(x,y) = \sigma(y^{-1}fx, x^{-1}hy)$$

and the equality $z^{-1}gz = g$ is equivalent to the following two equalities:

$$y^{-1}fx = h, \qquad x^{-1}hy = f. \qquad (4)$$

Multiplying the first equality by the second from the right, we obtain

$$y^{-1}fhy = hf = (fh)^{h^{-1}},$$

from which $y = h^{-1}t$ for some $t \in C_H(fh)$ and

$$x = f^{-1}yh = f^{-1}th^{-1}h = f^{-1}t.$$

Inversely, let $x = f^{-1}t$ and $y = th^{-1}$ for some $t \in C_H(fh)$. Then

$$y^{-1}fx = ht^{-1}ff^{-1}t = h$$

and

$$x^{-1}hy = t^{-1}fhth^{-1} = fhh^{-1} = f,$$

that is, equalities (4) are satisfied. The lemma is proved.

Using lemmas 1–3, we find the sum of the squares of the centralizers of the elements of the group G. We have

$$\sum_{g\in G}|C_G(g)|^2 = \sum_{h,f\in H}|C_G(\sigma(h,f))|^2 + \sum_{h,f\in H}|C_G((h,f))|^2 =$$

$$= \sum_{h,f\in H}|2C_H(hf)|^2 + \sum_{(h,f)\in H\times H, h\in f^H}(2|C_H(h)|^2)^2 +$$

$$+ \sum_{(h,f)\in H\times H, h\notin f^H}(|C_H(h)||C_H(f)|)^2 = 4|H|\sum_{h\in H}|C_H(h)|^2 +$$

$$+ \sum_{h,f\in H}|C_H(h)|^2|C_H(f)|^2 + 3\sum_{(h,f)\in H\times H, h\in f^H}|C_H(h)|^4. \quad (5)$$

Lemma 4. *Let* $g \in G$. *Then*

1) $\theta_G(g) = 0$, if $g = \sigma(u,v)$;

2) $\theta_G(g) = \theta_H(u)\theta_H(v)$, if $g = (u,v)$ and, u and v are not conjugate in H;

3) $\theta_G(g) = \theta_H(u)\theta_H(v) + |C_H(u)|$, if u and v are conjugate in H.

Proof. The assertion 1) is obvious, since the square of any element from G lies in $H \times H$.

Let $g = (u,v)$ and $z \in G$. If $z = (x,y)$, then the equality $z^2 = g$ is satisfied when and only when $u = x^2$ and $v = y^2$. If $z = \sigma(x,y)$, then the equality $z^2 = g$ is satisfied when and only when

$$u = yx, \qquad v = xy. \qquad (6)$$

We show that the conditions (6) are equivalent to the fact that both u and v are conjugate in some element $h \in H$ and

$$x = h^{-1}t, \quad y = ut^{-1}h, \quad t \in C_H(u). \qquad (7)$$

Indeed, $(yx)^y = xy$, therefore, u and v are conjugate in H. Let $v = u^h$. Then from (6) it follows that $y = ux^{-1}$ and

$$u^h = v = xy = xux^{-1} = u^{x^{-1}}.$$

From which $x = h^{-1}t,\ y = ut^{-1}h$ for some $t \in C_H(h)$. The lemma is proved.

Using Lemma 4, we find the sum of the cubes of the values of the function θ. We have

$$\sum_{g\in G}\theta_G^3(g) = \sum_{u,v\in H}\theta_G^3(\sigma(u,v)) + \sum_{u,v\in H}\theta_G^3((u,v)) =$$

$$= \sum_{u,v\in H}\theta_G^3((u,v)) = \sum_{(u,v)\in H\times H, u\notin v^H}\theta_H^3(u)\theta_H^3(v) +$$

$$+ \sum_{(u,v)\in H\times H, u\in v^H}(\theta_H(u)\theta_H(v) + |C_H(u)|)^3 = \sum_{u,v\in H}\theta_H^3(u)\theta_H^3(v) +$$

$$+ \sum_{(u,v)\in H\times H, u\in v^H}\begin{pmatrix}3\theta_H^4(u)|C_H(u)| + \\ +3\theta_H^2(u)|C_H(u)|^2 + |C_H(u)|^3\end{pmatrix}. \qquad (8)$$

Comparing (5) with (8) and taking into account that, by virtue of the simply reducibility of the group H, the following equality is true

$$\sum_{u,v\in H}\theta_H^3(u)\theta_H^3(v) = \sum_{h,f\in H}|C_H(h)|^2|C_H(f)|^2,$$

we obtain that condition (2) is necessary and sufficient for simply reducibility of the wreath product of a simply reducible group with a cyclic group of order 2. The theorem is proved.

The proof of Theorem 2 allows us to construct the following infinite series of simply reducible 2-groups.

Corollary 1. *Let G = HsZ₂, where H is an elementary Abelian 2-group of order* 2^n. *Then* G *is simply reducible.*

Proof. Let $Z_2 = \{1,-1\}$, $H = Z_2 \times \ldots \times Z_2$ and $h = (h_1,\ldots,h_k)$. We find $\theta(h)$. Obviously, if $h = (1,\ldots,1)$, then $\theta(h) = |H| = 2^n$. If $h_i = -1$ for at least one i, then $\theta(h) = 0$, as the equation $x_i^2 = -1$ is not solvable in group Z_2. As H is Abelian, then $|C_{E_n}(h)| = |H|$ for any $h \in H$. From which,

$$\sum_{(u,v)\in H\times H, u\in v^H}\begin{pmatrix}3\theta_H^4(u)|C_H(u)| + \\ +3\theta_H^2(u)|C_H(u)|^2 + |C_H(u)|^3\end{pmatrix} =$$

$$= 2^{4n}(4 + 3\cdot 2^n) =$$

$$= 4|H|\sum_{h\in H}|C_H(h)|^2 + 3\sum_{(h,f)\in H\times H, h\in f^H}|C_H(h)|^4.$$

The corollary is proved.

Conclusion. It is proved that the reality of *H* is the necessary condition of simply reducibility of the wreath product of the finite group *H* with the finite group *K*, and the group *K* must be an elementary Abelian 2-group. We also indicate sufficient conditions for simply reducibility of a wreath product of a simply reducible group with a cyclic group of order 2.

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References

1. Wigner E. P. On representations of certain finite groups. *American Journal of Mathematics.* 1941, Vol. 63, No. 1, P. 57–63.

2. Mackey G. W. Multiplicity free representations of finite groups. *Pacific Journal of Mathematics.* 1958, Vol. 8, P. 503–510.

3. Lomont J. S. Application of finite groups. New York, Academic Press, 1959, 346 p.

4. Wigner E. P. Group Theory and its Application to the Quantum Mechanics of Atomic Spectra. Academic Press, 1959, 372 p.

5. Hamermesh M. *Prilozheniya teorii grupp k fizicheskim problemam* [Group theory and its application to physical problems]. Moscow, Mir Publ, 1966, 588 p.

6. Van Zanten A. J., De Vries E. Number of roots of certain equations in finite simply reducible groups. *Physica*. 1970, Vol. 49, P. 536–548.

7. Strunkov S. P. On character location of simply reducible groups. *Matematicheskie zametki*. 1982, Vol. 31, No. 3, P. 357–362 (In Russ.).

8. Kostrikin A. I. *Vvedeniye v algebru. Chast' III. Osnovnyye struktury algebry* [Introduction to algebra. Part III. Basic structures od algebra]. Moscow, Phizmatlit, 2000, 272 p.

9. Mazurov V. D., Merzlyakov Yu. I., Churkin V. A. et al. *Kourovskaya tetrad'. Nereshennye problemy teorii grupp* [The Kourovka notebook. Unsolved problems in group theory]. Ed. 16, Novosibirsk, Institute of Math., 2006.

10. Kazarin L. S., YAnishevskij V. V. [SR-groups of order $p2^n$]. *Matematika v Yaroslavskom universitete*. Yaroslavl', 2006, P. 257–262 (In Russ.).

11. Kazarin L. S., Yanishevskij V. V. [On finite simply reducible groups]. *Algebra i analiz*. 2007, Vol. 19, No. 6, P. 86–116 (In Russ.).

12. Kazarin L. S., CHankov E. I. [Finite simply reducible groups are soluble]. *Matematicheskiy sbornik*. 2010, Vol. 201, No. 5, P. 27–40 (In Russ.).

13. Kolesnikov S. G. [On rationality and 2-reflexivity of wreath product finite groups]. *Matematicheskie zametki*. 2006, Vol. 80, No. 3, P. 395–402 (In Russ.).

14. Baumslag G. Wreath products and *p*-groups. *Proceeding Cambridge Philosophical Society*. 1959, Vol. 55, P. 224–231.

15. Remeslennikov V. N. [Finite approximability of groups relative to conjugacy]. *Sibirskiy matematicheskiy zhurnal*. 1971, Vol. 12, No. 5, P. 1085–1099 (In Russ.).

16. Kargapolov M. I., Merzlyakov Yu. I. *Osnovy teorii grupp* [Fundamentals of group theory. Second edition], Moscow, Nauka Publ., 1977, 240 p.

Библиографические ссылки

1. Wigner E. P. On representations of certain finite groups // American Journal of Mathematics. 1941. Vol. 63, No. 1. P. 57–63.

2. Mackey G. W. Multiplicity free representations of finite groups // Pacific Journal of Mathematics. 1958. Vol. 8. P. 503–510.

3. Lomont J. S. Application of finite groups. New York : Academic Press, 1959. 346 p.

4. Wigner E. P. Group Theory and its Application to the Quantum Mechanics of Atomic Spectra. Academic Press, 1959. 372 p.

5. Хамермеш М. Приложения теории групп к физическим проблемам. М. : Мир, 1966. 588 с.

6. Van Zanten A. J., De Vries E. Number of roots of certain equations in finite simply reducible groups // Physica. 1970. Vol. 49. P. 536–548.

7. Струнков С. П. On character location of simply reducible groups // Математические заметки. 1982. Т. 31, № 3. Р. 357–362.

8. Кострикин А. И. Введение в алгебру. Ч. III. Основные структуры алгебры. М. : Физматлит, 2000. 272 с.

9. Коуровская тетрадь. Нерешенные проблемы теории групп / В. Д. Мазуров [и др.]. 16-е изд. Новосибирск : Ин-т математики, 2006.

10. Казарин Л. С., Яничевский В. В. SR-группы порядка $p2^n$ // Математика в Ярославском университете. Ярославль, 2006. С. 257–262.

11. Казарин Л. С., Яничевский В. В. Конечные просто приводимые группы // Алгебра и анализ. 2007. Т. 19, No. 6. Р. 86–116.

12. Конечные просто приводимые группы разрешимы / Л. С. Казарин [и др.] // Математический сборник. 2010. Т. 201, № 5. С. 27–40.

13. Колесников С. Г. О рациональности и разрешимости сплетения конечных групп // Математические заметки. 2006. Т. 80, № 3. С. 395–402.

14. Baumslag G. Wreath products and *p*-groups // Proceeding Cambridge Philosophical Society. 1959. Vol. 55. P. 224–231.

15. Ремесленников В. Н. Конечная аппроксимируемость групп относительно сопряженности // Сибирский математический журнал. 1971. Т. 12, № 5. Р. 1085–1099.

16. Каргаполов М. И., Мерзляков Ю. И. Основы теории групп. 2-е изд. М. : Наука, 1977. 240 с.