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RESTORATION OF INFORMATION ON THE GROUP BY THE BOTTOM LAYER

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The question of the possibility of restoring information on the group by its bottom layer is considered. The problem is classical for mathematical modeling: restoration of missing information on the object employing part of the saved data. This problem will be solved in the class of layer-finite groups. A group is said to be layer-finite if it has a finite number of elements of every order. This concept was first introduced by S. N. Chernikov. It appeared in connection with the study of infinite locally finite p-groups in the case when the center of the group has a finite index in it. The bottom layer of the group G is the set of its prime order elements. By the bottom layer of the group, you can sometimes restore the group or judge about the properties of such a group. Among these results one can name those that completely describe the structure of the group by its bottom layer, for example: if the bottom layer of the group G consists of elements of order 2 and there are no non-unit elements of other orders in the group, then G is the elementary Abelian 2-group. V. P. Shunkov proved that if the bottom layer in an infinite layer-finite group consists of one element of order 2, then the group G is either a quasicyclic or an infinite generalized quaternion group. We will restore the information on the group by its bottom layer. This problem will be solved in the class of layer-finite groups. Group G is said to be recognizable by the bottom layer if it is uniquely recovered by the bottom layer. Group G is said to be almost recognizable over the bottom layer if there is a finite number of pairwise nonisomorphic groups with the same bottom layer as in group G. Group G is said to be unrecognizable by the bottom layer if there is an infinite number of pairwise nonisomorphic groups with the same bottom layer such as in group G. In this work conditions under which the group is recognized align the bottom layer have been established.

Keywords: group, layer finiteness, bottom layer, complete group, order of the group.

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ВОССТАНОВЛЕНИЕ ИНФОРМАЦИИ О ГРУППЕ ПО НИЖНЕМУ СЛОЮ

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Рассматривается вопрос о возможности восстановления информации о группе по ее нижнему слою. Вопрос является классическим для математического моделирования: восстановление недостающей информации об объекте по части сохранившихся данных. Этот вопрос будем решать в классе слойно конечных групп. Группа называется слойно конечной, если она имеет конечное число элементов каждого порядка. Это понятие впервые было введено С. Н. Черниковым. Оно появилось в связи с изучением бесконечных локально конечных p-групп в случае, когда центр группы имеет конечный индекс в ней. Нижним слоем группы G называется множество её элементов простых порядков. По нижнему слою группы иногда можно восстановить группу, иногда можно что-то сказать о свойствах такой группы. Среди этих результатов можно назвать те, которые описывают полностью строение группы по ее нижнему слою, например, если нижний слой группы G состоит из элементов порядка 2 и в группе нет неединичных элементов других порядков, то G – элементарная абелева 2-группа. В. П. Шунковым доказано, что если нижний слой в бесконечной слойно конечной группе состоит из одного элемента порядка 2, то группа G либо квазициклическая, либо бесконечная обобщенная группа кватернионов. Мы будем восстанавливать информацию о группе по ее нижнему слою. Эту задачу будем решать в классе слойно конечных групп. Группу G назовем распознаваемой по нижнему слою, если она однозначно восстанавливается по нижнему слою. Группу G назовем почти распознаваемой по нижнему *слою, если существует конечное число попарно неизоморфных групп с одинаковым нижним слоем таким же, как у группы G. Группу G назовем нераспознаваемой по нижнему слою, если существует бесконечное число попарно неизоморфных групп с одинаковым нижним слоем таким же, как у группы G. Установлены условия, при которых группа распознается по нижнему слою.*

Ключевые слова: группа, слойная конечность, нижний слой, полная группа, порядок группы.

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Introduction. A group is said to be layer-finite if it has a finite number of elements of every order. This concept was first introduced by S. N. Chernikov in his work [1] and appeared in connection with the study of infinite locally finite *p*-groups in the case when the center of the group has a finite index in it. In 1948 S. N. Chernikov [2] described construction of an arbitrary group in which number of elements of every order is infinite and the term layer-finite groups was used here for the first time. The main result describing the structure of layer-finite groups has been obtained by S. N. Chernikov: group is only layer-finite when can be presented in the form of product of two element-wise permutable subgroups from which the first is layer-finite complete Abelian group, and the second is a layer-finite group with finite Sylow subgroups. The results of layer-finite groups can be found in works [3–7].

The *bottom layer* of the group is a set of its elements of simple order. By the bottom layer of the group it is sometimes possible to restore the group, or to judge about some properties of such a group. Among these results can be named those which fully describe the structure of a group by its bottom layer, for example: if the bottom layer of a group consists of elements of order 2 and the group lacks nonidentity elements of other orders, then it is the elementary Abelian 2-group; if the bottom layer consists of the elements of orders 2, 3, 5 and in the group there are no nonidentity elements of other orders, then A. S. Kondratyev and V. D. Mazurov have proved that this is the group of even substitutions on five elements [8].

V. P. Shunkov has proved that if the bottom in the infinite layer-finite group consists of one element of order 2, the group is either the quasicyclic or infinite generalized group of quaternions [9]. Many results for groups with given bottom layer describe only some properties of groups. For example, V. D. Mazurov has proved that a group with the bottom layer consisting of elements 2, 3, 5 in which all other nonidentity elements have order 4, is locally finite [10].

We shall restore information on the group by its bottom layer. This problem will be solved in the class of layer-finite groups.

Main result. In the article the term bottom layer in the group is discussed and by means of obtained information the group structure is restored. We shall refer to the group *G* as to recognizable by the bottom layer if it can be definitely restored align the bottom layer. We shall refer to the group *G* as to almost recognizable by the bottom layer if there is a finite number of pairwise nonisomorphic groups with the identical to the group bottom layer. We shall refer to the group *G* as to unrecognizable by the bottom layer if there is an infinite number of pairwise nonisomorphic groups with the same bottom layer.

We shall consider finite non-Abelian simple groups with the bottom layer consisting of elements of orders 2, 3, 5. In case we add information on the group order, the group *G* will be recognizable by the bottom layer on condition that its order is $|G| = 2^2 \cdot 3 \cdot 5$ (this is A_5) and unrecognizable by the bottom layer on condition that its order is $|G| = 2^3 \cdot 3^2 \cdot 5$ (one of these groups becomes A_6) [11].

We shall consider the examples describing properties of groups with given bottom layer. N. D. Gupta and V. D. Mazurov have proved that for the group G, which without identity element coincides with the bottom layer consisting of elements of orders 3, 5, one of the statements is true: 1) $G = FT$; where F is normal nilpotent with no more than two 5-subgroup of a class and $|T| = 3$; 2) G contains normal nilpotent with no more than three 3 subgroup of a class and T is G/T – 5-group [12]. In the same work it is shown that the group without identity element coincides with the bottom layer consisting of elements of orders 2, 5 either contains the elementary Abelian 5-subgroup of the index 2, or the elementary Abelian normal Sylow 2-subgroup.

Let us prove that if G is the layer-finite complete group, on the bottom layer of which $p^n - 1$ element of order p, than the group G is a direct product of n-quasicyclic p-groups (i. e. it is definitely recognizable by the bottom layer.)

Let the group *G* satisfies given conditions. As according to the proposition 1 (the known theorems to which we refer to as to propositions with the corresponding numbers are listed at the end of article) each complete subgroup of the layer-finite group *G* is in the center of the group *G*, the group, being complete, is the Abelian one. According to the proposition 2 the complete Abelian group *G* is split into the direct sum of subgroups, isomorphic to additive group of rational numbers or quasicyclic groups, perhaps, on various prime numbers. In this splitting of groups, presence of rational numbers can't be possible as it is the layer-finite group *G* and, therefore no elements of infinite order are present. Since on the bottom layer of the group *G* only order $p^n - 1$ elements present, than quasicyclic groups can be only on one number p. Since on the bottom layer of the group G $p^n - 1$ element of order p, these are n quasicyclic factors in G (direct product has n quasicyclic p-group $p^n - 1$ element of order p). Thus, we have proved that random finite complete Abelian p-group can be detected align the bottom layer.

Let us prove that if in the locally solvable group G is subject to maximality and the bottom layer, consisting of $p-1$ element of order p, and $q-1$ element of order q, where there are elements of orders $p^m q^n$, for $m = 0, 1, 2, ..., n = 0, 1, 2, ...,$ and no elements of other orders, the group G is detectable align the bottom layer.

Objectively, according to the proposition *3* the locally solvable group G with minimality condition is extension of direct product of finite number of quasicyclic groups with help of a finite group. Due to its bottom layer structure, group G contains at list one quasicyclic p-group and one quasicyclic q-group. Since in the group G $p-1$ element of order p, it has a single subgroup of order p. Then, according to the proposition 4 any finite p-subgroup of the group G is cyclic. According to the proposition *5* it must be cyclic and quasicyclic [13]. Now, on account that in the group G there are elements with all degrees p, it can be concluded that Sylow p-subgroup of the group G is quasicyclic. Similarly, Sylow q-subgroup of the group G is also quasicyclic. Then, taking into account the structure of the group G, we conclude that all the Sylow subgroups of the group G are in direct product of quasicyclic groups, the group G itself coincides with direct product of its two Sylow p- and q-subgroups.

Similarly, it can be proved that if in the locally solvable group G with minimality condition with the bottom layer, consisting of $p^k - 1$ elements of order p, and $q^l - 1$ elements of order q, where there are elements of orders $p^m q^n$, under $m = 0, 1, 2, ..., n = 0, 1, 2, ...,$ with no elements of other orders, then, the group G is recognizable by the bottom layer (group G coincides with the direct product of k quasicyclic p-subgroups and l quasicyclic q-subgroups).

Hence, as additional restrictions to the tasks of the bottom group layer group completeness and its layer-finite structure can be used: if G is a complete layer-finite group with the bottom layer consisting of its elements orders $p_1, p_2, ..., p_n$ and the group G contains $p^{k_i} - 1$ elements of order p_i, $i = 1, 2, ..., n$, then, G is the direct product of quasicyclic p_i-groups, $i = 1, 2, ..., n$, taken under k_i factors for each p_i respectively. Thus G is recognizable by the bottom layer.

When proving the results given in the article the following theorems were employed, with reference to the propositions and their correspondent number.

Proposition 1. (Lemma 3.1. [14]). Each complete subgroup of locally normal (layer-finite in particular) group G is hold in the centre of the group G.

Proposition 2. (Theorem 9.1.6. [15]). Non-zero complete Abelian group G extends in direct sum of subgroups, isomorphic to additive group of rational numbers or quasicyclic groups, perhaps, on various prime numbers.

Proposition 3. (Theoreme 1.1 [14]). Infinite locally solvable group G subject to minimality is the extension of the direct product of finite number of quasicyclic groups applying the finite group.

Proposition 4. (Theorem 12.5.2 [16]). Finite p-group, containing only one subgroup of order *p*, is cyclic or generalized quaternion group.

Proposition 5. (Theorem 4.2 [13]). Sylow p-subgroups of the periodically local cyclic group *G* are cyclic or quasicyclic.

Conclusion. The work has determined conditions, under which the group is ditected align the bottom layer. The provement of the group restoration from the layer-finite group class has been considered. Restoration of the group under given bottom layer consisting of the elements of simple order p has been proved.

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