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TO NONPARAMETRIC IDENTIFICATION OF DYNAMIC SYSTEMS UNDER NORMAL OPERATION

M. E. Kornet, A. V. Shishkina*

Siberian Federal University
Space and Information Technology Institute
26b, Kirensky Str., Krasnoyarsk, 660074, Russian Federation
*E-mail: nastya.shishkina9666@mail.ru

The problem of nonparametric identification of linear dynamic objects is being investigated. In contrast with parametric identification, the case is analyzed when equations describing a dynamic object are not specified according to the parameters. Moreover, the identification problem is analyzed under normal object operation, opposite to the previously known nonparametric approach based on Heaviside function input to the object and further Duhamel integral application. An arbitrary signal is inputted to the object during normal operation and weight function realizations are represented by observations of input-output object variables measured with random interferences. As a result, we have a sample of input-output variables. As linear dynamical system can be described by the Duhamel integral, with known input and output object variables, corresponding values of the weight function can be found. This is achieved by discrete representation of the latter. Having such realization, nonparametric estimate of the weight function in the form of the nonparametric Nadaraya–Watson estimate is used later. Substituting this into the Duhamel integral, we obtain a nonparametric model of a linear dynamical system of unknown order.

The article also describes the case of nonparametric model constructing when a delta-shaped function is inputted to the object. It was interesting to find out how delta-shaped function might differ from the delta function. The weight function was determined in the class of nonparametric Nadaraya–Watson estimates. Nonparametric models were investigated by means of statistical modeling. In general, nonparametric models have shown sufficient efficiency in terms of accuracy prediction by nonparametric model in relation to the actually measured output of the object. Evidentally, the accuracy of nonparametric models reduces with the growing influence of interference from the measurement of input-output variables or the discreteness of their measurement. Previously proposed nonparametric algorithms consider the case when Heaviside function was applied to the object, which narrows the scope of nonparametric identification practical use. It is important to construct nonparametric model of a dynamic object in conditions of normal operation.

Keywords: duhamel integral, transient function, weight function, delta-shaped input, Nadarya–Watson estimate, nonparametric model.

О НЕПАРАМЕТРИЧЕСКОЙ ИДЕНТИФИКАЦИИ ДИНАМИЧЕСКИХ СИСТЕМ В УСЛОВИЯХ НОРМАЛЬНОГО ФУНКЦИОНИРОВАНИЯ

М. Е. Корнет, А. В. Шишкина*

Сибирский федеральный университет
Институт космических и информационных технологий
Российская Федерация, 660074, г. Красноярск, ул. Академика Киренского, 26б
*E-mail: nastya.shishkina9666@mail.ru

Рассматривается задача непараметрической идентификации линейных динамических объектов. В отличие от параметрической идентификации, вид уравнения, описывающего динамический объект, не задан с точностью до параметров. Более того, задача идентификации рассматривается в условиях нормального функционирования объекта, в отличие от ранее известного непараметрического подхода, основанного на подаче на *вход объекта функции Хевисайда и дальнейшем применении интеграла Дюамеля. В условиях нормального функционирования на вход объекта подают сигнал произвольного вида. При этом на выходе объекта наблюдается соответствующий отклик. Следует заметить, что измерения входной и выходной переменных осуществляются со случайными помехами. В итоге, имеем реализацию (выборку) входных-выходных переменных. Поскольку линейная динамическая система может быть описана интегралом Дюамеля, то при известных входных и выходных переменных объекта могут быть найдены соответствующие значения весовой функции. Это достигается при дискретной записи последнего. Располагая подобной реализацией, в дальнейшем используется непараметрическая оценка весовой функции в виде непараметрической оценки Надарая–Ватсона. Подставляя ее в интеграл Дюамеля, получаем тем самым непараметрическую модель линейной динамической системы неизвестного порядка.*

Приведен также случай построения непараметрической модели при подаче на вход дельтообразной функции. Было интересно выяснить, насколько дельтообразная функция может отличаться от дельта-функции. Оценка весовой функции и в этом случае определялась в классе непараметрических оценок Надарая–Ватсона. Предложенные непараметрические модели был подробно исследованы средствами статистического моделирования. В основном непараметрические модели показали достаточно высокую эффективность с точки зрения точности прогноза непараметрической модели по отношению к реально измеренному выходу объекта. Естественно, точность непараметрических моделей уменьшается из-за роста влияния помех измерения входных-выходных переменных или дискретности их измерения. Ранее были предложены непараметрические алгоритмы идентификации для случая, когда на вход объекта подавалась функция Хевисайда, однако это несколько сужает рамки практического использования самой идеи непараметрической идентификации. Естественно, важным является случай построения непараметрической модели динамического объекта, находящегося в условиях нормальной эксплуатации. Эта особенность является наиболее важной из рассматриваемых приемов идентификации в условиях непараметрической неопределенности.

Ключевые слова: интеграл Дюамеля, переходная функция, весовая функция, дельтообразное входное воздействие, оценка Надарая–Ватсона, непараметрическая модель.

Introduction. The main objective of identification theory is the model construction based on input and output process variables' observations while the data about the object is incomplete [1–3]. The article is devoted to dynamic objects identification under nonparametric uncertainty [4; 5], when the dynamical model cannot be identified up to parameters vector due to the lack of priori data. In this case receiving of transient response and following estimation of an object weight function are reasonable.

The basis of this paper is Duhamel integral use, due to the principle of superposition [6; 7]. Identification algorithms of the object in normal operation conditions are described. Three methods of obtaining weight function estimation using Heaviside function [8; 9], delta-shaped input and arbitrary input are analyzed.

Problem formulation. Suppose that object is a dynamic system and described by the equation [1] $x_t = f(x_{t-1}, x_{t-2}, u_t)$, where $f(\cdot)$ – is unknown function; u_t – control input variable; x_t – output variable.

In fig. 1, a block diagram of the dynamic process is illustrated [2], with following notations: $\hat{x}_t$ – output of model; u_t – control variable; (t) – continuous time; t – discrete time; ξ_t, h_t – random noise acting on the object and output variable measuring channel, with zero mathematical expectation and limited dispersion.

Variables control is carried out through time interval Δt. Thus, it is possible to obtain initial input – output variables sample $\{x_i, u_i, i = \overline{1, s}\}$, where s – sample size.

Non-parametric identification algorithm when standard signals can be inputted to the object. Suppose that the object is described by a linear differential equation of unknown order. In this case, for zero initial conditions, $x(t)$ is found as

$$x(t) = \int_0^t h(t-\tau)u(\tau)d\tau\,, \qquad (1)$$

where $h(t-\tau)$ – weight function, that is derivative of transition function $h(t) = k'(t)$.

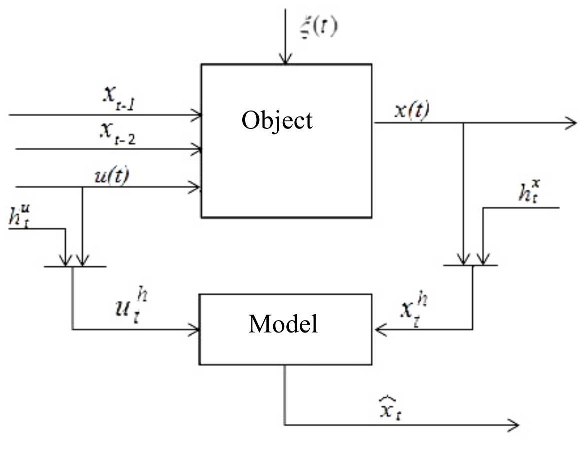

Fig. 1. Identification scheme

Рис. 1. Блок-схема системы идентификации

This problem reduces to the weight function estimation, so, firstly, it is needed to obtain the transition function.

As it was mentioned, weight function can be obtained by various means.

First case. Suppose that the object is described by linear differential equation of unknown order. In zero initial conditions, $x(t)$ is found as (1). Transition function is an object reaction on input impact, namely as Heaviside function $u(t)=1(t)$.

$$1(t)=\begin{cases}0, u(t)<0,\\ 1, u(t)\geq 0.\end{cases} \quad (2)$$

After obtaining transition function, it is needed to find its nonparametric estimation [10; 11]:

$$\bar{k}(t)=\frac{T}{sc_s}\sum_{i=0}^{s}k_i H\left(\frac{t-t_i}{c_s}\right), \quad (3)$$

where $\bar{k}_i$ – transition function estimate; k_i – transition function; t_i – discrete time of measurements; s – sample size; c_s – kernel smoothing; H – kernel function; T – time observation period [2].

We note that kernel function and kernel smoothing satisfy the following terms [10; 11]:

$$\frac{1}{c_s}\int_{-\infty}^{\infty}H\left(\frac{t-t_i}{c_s}\right)dt=1, \lim_{s\to 0}\frac{1}{c_s}\int_{-\infty}^{\infty}\varphi(t)H\left(\frac{t-t_i}{c_s}\right)dt=$$
$$=\varphi(t_i), H\left(\frac{t-t_i}{c_s}\right)\geq 0, \quad (4)$$
$$c_s>0, \lim_{c_s\to\infty}sc_s\to\infty, \lim_{s\to\infty}c_s\to 0,$$

where $\varphi(t_i)$ – an arbitrary function.

In particular, kernel function would be considered as Sobolev function (5):

$$H=\begin{cases}0, |t-t_i|>c_s\\ \dfrac{0.827}{c_s}e^{\left(\frac{-(t-t_i)^2}{(t-t_i)^2-c_s^2}\right)}, |t-t_i|\leq c_s.\end{cases} \quad (5)$$

Since weight function $h(t)$ is derivative of transition function $k(t)$, then

$$\bar{h}(t)=\frac{T}{sc_s}\sum_{i=0}^{s}\bar{k}_i H'\left(\frac{t-t_i}{c_s}\right). \quad (6)$$

Second case. The weight function could be obtained when a delta-shaped function is inputted. It has a step function type (7), Δt – discretization interval (fig. 2):

$$\delta^{\Delta}(t)=\left\{\frac{1}{\Delta t}, t\in\Delta t\right., \quad (7)$$

where Δt, for example, is an equation $\Delta t=t'-0$, or $\Delta t=t'-t''$.

Identification algorithm under normal object operation. Constructing an adaptive object model often requires identification of measuring channels under normal object operation [2; 12]. This means that inputted impacts must be small enough so that the effect on production would be minimal. This is necessary for keeping the process in acceptable limits [8].

Thus, the third case has the priority in solving the problem of nonparametric identification [4; 6]. The following algorithm when input impact has sinusoidal type function (as an example) is analyzed below.

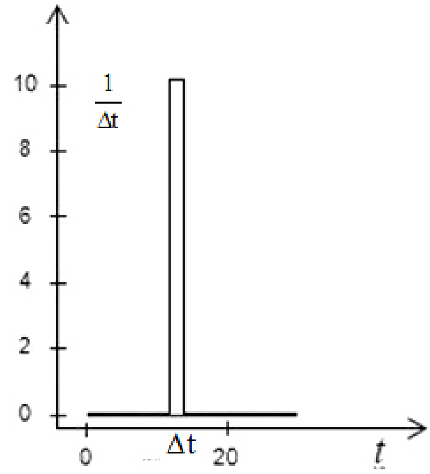

Fig. 2. Delta-shaped function example

Рис. 2. Пример дельтообразного входного воздействия

Third case. If control action and object output are known, weight function may be described by (1).

In a discrete form:

$$h_i=x_t-\left(\sum_{i=1}^{s}u_i\Delta\tau+\sum_{i=1}^{s}h_0\right), i=\overline{1,s}, \quad (8)$$

where s – sample size; $\Delta\tau$ – variables control time interval; u_i – control variable; x_t – object output; h_0 – value of the weight function on previous iteration steps.

Thus, nonparametric process model is following:

$$x_s(t)=\frac{T}{sc_s}\int_0^t\sum_{i=1}^{s}k_i H'\left(\frac{t-t_i}{c_s}\right)u(\tau)d\tau$$

or

$$x_s(t)=\frac{T}{sc_s}\int_0^t\sum_{i=1}^{s}h_i u(\tau)d\tau, \quad (9)$$

where k_i – transition function; h_i – weight function; c_s – kernel smoothing; s – sample size; T – observation period.

Computer experiment. Suppose that dynamical object is described by third-order differential equation. It can be represented as:

$$x_t=0.5x_{t-3}-x_{t-2}+x_{t-1}-0.5u_t. \quad (10)$$

Let us note that the equation (10) is used for obtaining sampling points. Nonparametric algorithm does not assume the known form of the differential equation, only information on the linearity of an object is known, in contrast with [13; 14].

The first method of obtaining weight function is to take the derivative of transition function (fig. 3), if Heaviside function is submitted to the object, then object output is a transitional feature: $x(t)=k(t)$, further it is necessary to find the value of transition function and weight function according to formulas (3) and (6):

In fig. 3: $k(t)$ – transition function, $h(t)$ – weight function.

Put known values of transition and weight functions into Duhamel integral (1) and get an object model, fig. 4.

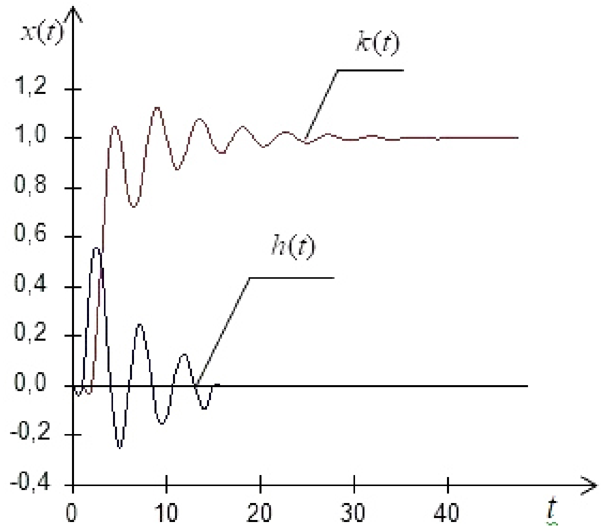

Fig. 3. Weight and transition response when $u(t) = 1(t)$

Рис. 3. Весовая и переходная характеристика процесса при $u(t) = 1(t)$

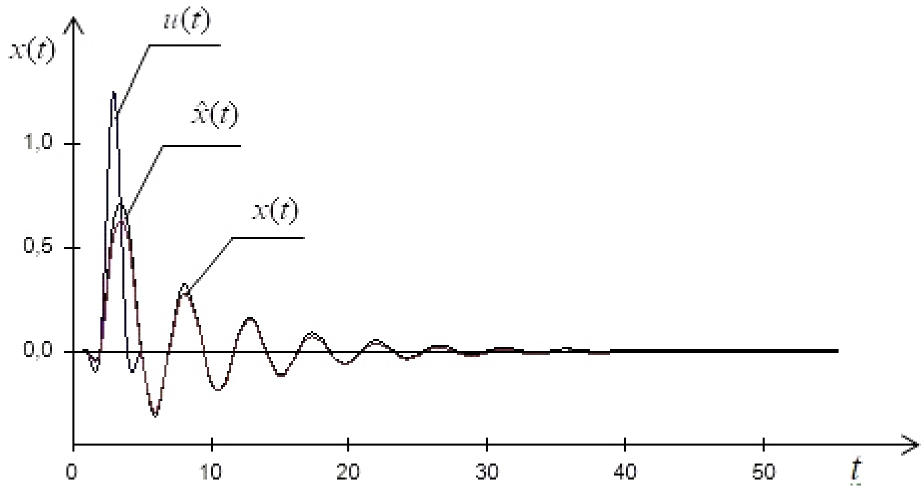

Fig. 4. Weight response when input is a delta-shaped function

Рис. 4. Весовая характеристика процесса при подаче
на вход объекта дельтообразного входного воздействия

Let us change the order of differential equation that describes the object and conduct computer experiments.

Suppose that the object is described by differential equation of the second order represented as follows:

$$x_t = 0.25x_{t-1} - 0.33x_{t-2} + 0.33u_t . \qquad (11)$$

Suppose the integral of delta-shaped function differs from 1.

Fig. 5 illustrates the experiment when the integral of delta-shaped $\delta^{\Delta}(t)$ equals 1, $u(t) = \frac{1}{\Delta t}$ delta-shaped input, $x(t)$ – object output, Δt – discretization interval, $\hat{x}(t)$ – output object model.

Note that when $\Delta t \in [0.1; 1]$ $\Delta t \in [0.1; 1]$, input $u(t)$ takes values from 1 to 10, it can conform to the technological requirements.

Consider the case when delta-shaped function integral $\delta^{\Delta}(t)$ differs from 1. As a result, delta-shaped function becomes "pseudo-delta-shaped", in particular integral of delta function does not equal 1 (fig. 6).

Fig. 5 illustrates discretization interval Δt = 0.1, integral of delta-shaped function $\delta^{\Delta}(t)$ equals 1, recovery error w = 4.2 %.

In fig. 6 discretization interval Δt = 0.1, integral of delta-shaped function $\delta^{\Delta}(t) > 1$, recovery error w = 40 %.

Hence, in order to construct the appropriate model, the following term should be kept – integral of delta-shaped function must be equal 1.

In conditions of normal object operation as an arbitrary input signal we take the following function:

$$u_t = t - t/2 - A * \sin(0.5t) , \qquad (12)$$

where A – oscillation amplitude.

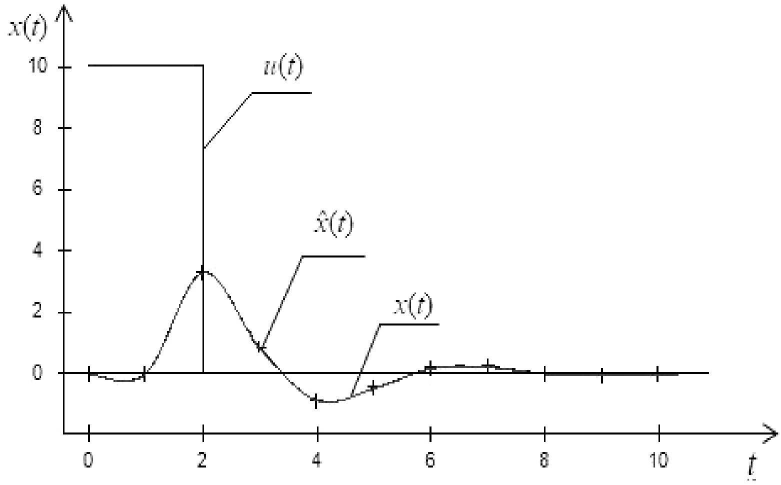

Fig. 5. Algorithm work with delta-shaped input

Рис. 5. Результат работы алгоритма
при дельтообразном входном воздействии

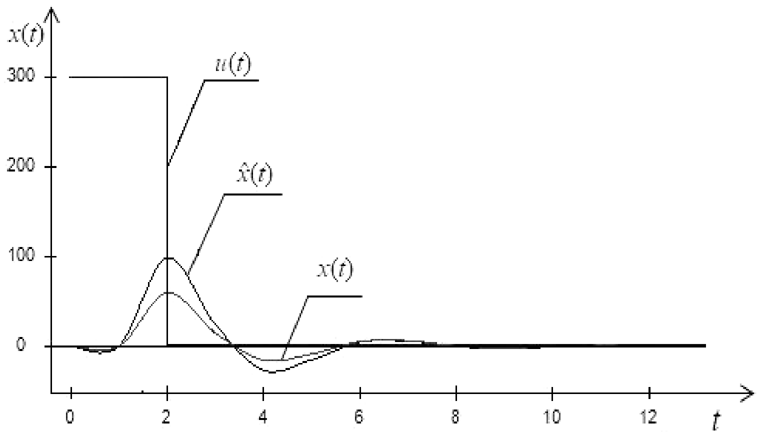

Fig. 6. Algorithm work with “pseudo-delta-shaped” input

Рис. 6. Результат работы алгоритма
при «псевдодельтообразном» входном воздействии

Let us add a random noise that arising in the channel of output signal measurement $x(t)$

$$h_t = l x_t \xi_t, \qquad (13)$$

where $\xi_t \in [-1; 1]$, noise level $l = 5\,\%, 10\,\%$.

Calculate the recovery error – w according to the formula (14), where $\overline{x} = \frac{1}{s}\sum_{i=1}^{s} x_t$ – arithmetical mean, $\hat{x}(t)$ – object model output:

$$w = \frac{\sum_{i=1}^{s} | x_t - \hat{x}_t |}{\sum_{i=1}^{s} | x_t - \overline{x} |}, \qquad (14)$$

Fig. 7 appeals to the following definitions: $u(t)$ – input impact, $x(t)$ – object output, $\hat{x}(t)$ – model output. Noise level = 5 %, recovery error w – 0.067, according to the chart and recovery error, this model could be considered as satisfactory.

Thus, table illustrates that lowering oscillation amplitude leads to model accuracy decreasing.

Dependence between recovery error and oscillation amplitude

A	W
10.5	0.5 %
3.5	1.4 %
2.5	2 %
1.5	3.3 %
1	4.9 %
0.5	9.8 %
0.1	53.4 %

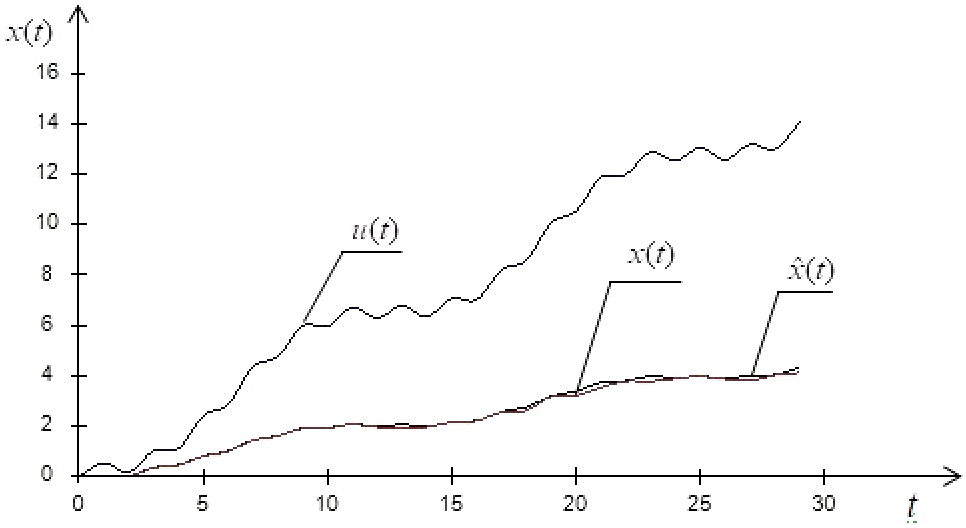

Fig. 7. Object output when input is an arbitrary signal

Рис. 7. Результаты выхода объекта при произвольном входном воздействии

Let us change the input signal and answer the question of how the quality of constructed model depends on the oscillation amplitude:

$$u_t = A * \sin(0.1t), \qquad (14)$$

where A – oscillation amplitude.

We conduct computer experiments, in table following descriptions are analyzed A – oscillation amplitude, w – recovery error.

Conclusion. The problem of nonparametric identification of linear dynamical objects in conditions of incomplete data is analyzed. The main result of this paper is resolving of identification problem in an object's normal operation conditions. The nonparametric linear dynamical system models that based on Duhamel integral estimation by means of Nadaraya–Watson statistics are submitted.

The main conclusions that could be made on the basis of extensive numerical research of nonparametric models are as follows: although in practice delta function cannot be submitted to the object input, sometimes it is possible to submit delta-shaped input signal and then construct a satisfactory model. Certainly, noise increase in input-output variables measurement and increase in discreteness of input-output variables control, in natural way, worsen accuracy of nonparametric models [15–17].

In addition, it is important to note that the algorithm does not require particular object equation and known differential equation order, all equations that have been described are analyzed as the examples. Thus, algorithm is not dependent on the type of input impact, the main condition is observance of the superposition principle.

References

1. Tsypkin Ya. Z. *Informatsionnaya teoriya identifikatsii* [Information theory of identification]. Moscow, Nauka, Fizmatlit Publ., 1995, 336 p.

2. Raibman N. N. *Chto takoe identifikatsiya* [What is identification]. Moscow, Nauka Publ., 1970, 119 p.

3. Eykhoff P. *Osnovy identifikatsii sistem upravleniya* [Fundamentals of identification of control systems]. Moscow, Mir Publ., 1975, 681 p.

4. Medvedev A. V. *Neparametricheskie sistemy adaptatsii* [Nonparametric adaptation systems]. Novosibirsk, Nauka Publ., 1983, 174 p.

5. Medvedev A. V. [Adaptation under conditions of non-parametric uncertainty]. *Adaptivnye sistemy i ikh prilozheniya* [Adaptive systems and their applications]. Novosibirsk, Nauka Publ., 1978, P. 4–34.

6. Medvedev A. V. [The theory of nonparametric systems. Modeling]. *Vestnik SibGAU.* 2010, No. 4 (30), P. 4–10 (In Russ.).

7. Medvedev A. V. *Elementy teorii neparametricheskikh sistem upravleniya. Aktual'nye problemy informatiki, prikladnoy matematiki i mekhaniki. Informatika* [Elements of the theory of nonparametric control systems. Actual problems of computer science, applied mathematics and mechanics. Informatics]. Novosibirsk, Krasnoyarsk, Izd-vo Sib. otd-niya Ros. akad. Nauk Publ., 1996, P. 87–112.

8. *Metody klassicheskoy i sovremennoy teorii avtomaticheskogo upravleniya. T. 1: Matematicheskie modeli, dinamicheskie kharakteristiki i analiz sistem upravleniya* [Methods of classical and modern theory of automatic control. Vol. 1: Mathematical models, dynamic characteristics and analysis of control systems]. Ed. K. A. Pupkova, N. D. Egupova. Moscow, MSTU im. N. E. Bauman Publ., 2004, 656 p.

9. *Metody klassicheskoy i sovremennoy teorii avtomaticheskogo upravleniya. T. 2: Statisticheskaya dinamika i identifikatsiya sistem avtomaticheskogo upravleniya* [Methods of classical and modern theory of automatic control. Vol. 2: Statistical dynamics and identification of automatic control systems]. Ed. K. A. Pupkova, N. D. Egupova. Moscow, MSTU im. N. E. Bauman Publ., 2004, 640 p.

10. Nadaraya E. A. *Neparametricheskoe otsenivanie plotnosti veroyatnostey i krivoy regressii* [Nonparametric estimation of the probability density and the regression curve]. Tbilisi, Izd. Tbil. University Publ., 1983, 194 p.

11. Kotkinik V. Ya. *Neparametricheskaya identifikatsiya i sglazhivanie dannykh* [Nonparametric identifica-

tion and data smoothing]. Moscow, Nauka Publ., 1985, 336 p.

12. Grop D. *Metody identifikatsii system* [Methods of identification of systems]. Ed. E. I. Krinetskiy, A. Vasilyev, V. I. Lopatin. Moscow, Mir Publ., 1979, 304 p.

13. Tse E., Bar-Shalom Y. An actively adaptive control for linear systems with random parameters via the dual control. *Automatic Control, IEEE Trans.* 2003, Vol. 18, Iss. 2, P. 109–117.

14. Wenk C. J., Bar-Shalom Y. A multiple model of an adaptive dual control algorithm for stochastic systems with unknown parameters. *Automatic Control*, *IEEE Trans.* 2003, Vol. 25, Iss. 4, P. 703–710.

15. Liung L. *Identifikatsiya sistem* [Identification of systems]. Moscow, Nauka Publ., 1991, 423 p.

16. *Metody klassicheskoy i sovremennoy teorii avtomaticheskogo upravleniya. T. 3: Sintez regulyatorov sistem avtomaticheskogo upravleniya* [Methods of classical and modern theory of automatic control. Vol. 3: Synthesis of regulators of automatic control systems]. Ed. K. A. Pupkova, N. D. Egupova. Moscow, MSTU im. N. E. Bauman, 2004, 656 p.

17. Agafonov E. D., Shishkina A. V. [Nonparametric control of a dynamical system]. *Siberian Journal of Science and Technology.* 2017, Vol. 18, Iss. 4, P. 711–718.

Библиографические ссылки

1. Цыпкин Я. З. Информационная теория идентификации. М. : Наука : Физматлит, 1995. 336 с.

2. Райбман Н. С. Что такое идентификация. М. : Наука, 1970. 119 с.

3. Эйкхофф П. Основы идентификации систем управления. М. : Мир, 1975. 681 с.

4. Медведев А. В. Непараметрические системы адаптации. Новосибирск : Наука, 1983. 174 с.

5. Медведев А. В. Адаптация в условиях непараметрической неопределенности // Адаптивные системы и их приложения / СО АН СССР. Новосибирск : Наука, 1978. С. 4–34.

6. Медведев А. В. Теория непараметрических систем. Моделирование // Вестник СибГАУ. 2010. № 4 (30). С. 4–10.

7. Медведев А. В. Элементы теории непараметрических систем управления // Актуальные проблемы информатики, прикладной математики и механики. Информатика. Новосибирск ; Красноярск : Изд-во Сиб. отд-ния Рос. акад. наук, 1996. С. 87–112.

8. Методы классической и современной теории автоматического управления. Т. 1. Математические модели, динамические характеристики и анализ систем управления / под ред. К. А. Пупкова, Н. Д. Егупова. М. : Изд-во МГТУ им. Н. Э. Баумана, 2004. 656 с.

9. Методы классической и современной теории автоматического управления. Т. 2. Статистическая динамика и идентификация систем автоматического управления / под ред. К. А. Пупкова, Н. Д. Егупова. М. : Изд-во МГТУ им. Н. Э. Баумана, 2004. 640 с.

10. Надарая Э. А. Непараметрическое оценивание плотности вероятностей и кривой регрессии. Тбилиси : Изд-во Тбил. ун-та, 1983. 194 с.

11. Катковник В. Я. Непараметрическая идентификация и сглаживание данных. М. : Наука, 1985. 336 с.

12. Гроп Д. Методы идентификации систем / пер. с англ. В. А. Васильева, В. И. Лопатина ; под ред. Е. И. Кринецкого. М. : Мир , 1979. 304 с.

13. Tse E., Bar-Shalom Y. An actively adaptive control for linear systems with random parameters via the dual control approach // Automatic Control, IEEE Transactions. 2003. Vol. 18, iss. 2. P. 109–117.

14. Wenk C. J., Bar-Shalom Y. A multiple model adaptive dual control algorithm for stochastic systems with unknown parameters // Automatic Control, IEEE Transactions. 2003. Vol. 25, iss. 4. P. 703–710.

15. Льюнг Л. Идентификация систем. М. : Наука, 1991. 423 с.

16. Методы классической и современной теории автоматического управления. Т. 3. Синтез регуляторов систем автоматического управления / под ред. К. А. Пупкова, Н. Д. Егупова. М. : Изд-во МГТУ им. Н. Э. Баумана, 2004. 656 с.

17. Агафонов Е. Д., Шишкина А. В. О непараметрическом управлении динамической системой // Сибирский журнал науки и технологий. 2017. Т. 18, № 4. С. 711–718.