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MODELING OF PLASTIC FLOW BETWEEN RIGID PLATES APPROACHING TO A CONSTANT ACCELERATION

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In this paper we study equations describing a slow plastic flow of material. In this case the material is in the flat state of stress. The paper presents the equations that can be used to simulate slow plastic material flows compressed between rigid plates, converging with constant acceleration. In the above equations we neglect convective terms, which greatly simplifies all calculations. The Lie algebra of point symmetries admitted by these equations is calculated for reduced equations. It has dimension eight. The optimal system of one-dimensional subalgebras is constructed for this algebra. It allows to give a view of all the different invariant solutions of rank two. That means such solutions depend only on two independent variables. To demonstrate this we offer a table of switches of all basis operators, as well as a table of all internal automorphisms functioning. One of the solutions, which simulates the slow plastic flow of the material compressed between rigid plates, converging with constant acceleration, built in. Among the most popular solutions in the flat theory of ideal plasticity is the Prandtl's solution, which describes the compression of a plastic layer between rigid plates. In this case, the plates approach at a constant speed. The popularity of the solution is explained by its simplicity, as well as the fact that it can be used to describe various technological processes. The analogue of such a solution for the plane stress state cannot be constructed. In general, there are big problems with finding analytical solutions for the plane state of stress. It is caused by the fact that the equations describing this state are quite complex, even in spite of their linearization. In one of the previous works, one of the authors of the present article managed to find a solution that describes compression of a plastic layer between rigid plates which converge with constant acceleration. In this work the analogue of such a solution is found for the plane stress state. The authors hope that the suggested solution can also be used for the analysis of real technological processes.

Keywords: plane stress, the exact solution, non-stationary process.

МОДЕЛИРОВАНИЕ ПЛАСТИЧЕСКОГО ТЕЧЕНИЯ МЕЖДУ ЖЕСТКИМИ ПЛИТАМИ, СБЛИЖАЮЩИМИСЯ С ПОСТОЯННЫМ УСКОРЕНИЕМ

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Изучаются уравнения, описывающие медленные пластические течения материала. При этом материал находится в плоском напряженном состоянии. Приведены уравнения, которые можно использовать для моделирования медленных пластических течений материала, сжимаемого жесткими плитами, сближающимися с постоянным ускорением. В приведенных уравнениях мы пренебрегаем конвективными членами. Это позволят значительно упростить все вычисления. Для уравнений вычислена алгебра Ли точечных симметрий, допускаемая этими уравнениями. Она имеет размерность восемь. Для этой алгебры построена оптимальная система одномерных подалгебр. Это позволяет привести вид всех различных инвариантных решений ранга два, т. е. таких решений, которые зависят только от двух независимых переменных. Для этого приведена таблица коммутаторов всех базисных операторов, а также таблица действия всех внутренних автоморфизмов. Одно из таких решений, которое моделирует медленные пластические течения материала, сжимаемого жесткими плитами, сближающимися с постоянным ускорением, и построено в статье. Самое популярное решение в плоской теории идеальной пластичности – это решение Прандтля, которое описывает сжатие пластического *слоя жесткими плитами. При этом плиты сближаются с постоянной скоростью. Популярность решения объясняется его простотой, а также тем, что его можно использовать для описания различных технологических процессов. Аналог такого решения для плоского напряженного состояния построить не удается. Да и вообще с построением аналитических решений для плоского напряженного состояния большие проблемы. Это связано с тем, что уравнения, описывающие это состояние, достаточно сложные, даже несмотря на их линеаризацию. В одной из предыдущих работ одному из авторов этой статьи удалось построить решение, описывающее сжатие пластического слоя жесткими плитами, которые сближаются с постоянным ускорением. В этой статье аналог такого решения построен и для плоского напряженного состояния. Авторы надеются, что построенное решение тоже удастся использовать для анализа реальных технологических процессов.*

Ключевые слова: плоское напряженное состояние, точное решение, нестационарный процесс.

Introduction. The most popular decision in the flat theory of ideal plasticity is the one of Prandtl which describes compression of a plastic layer between rigid plates. At the same time plates approach at a constant speed. The popularity of the decision is explained by its simplicity and also by the fact that it can be used for the description of various technological processes as well as rocks [1–7]. The analog of such decision for flat tension state cannot be constructed. It is connected with the fact that the group of symmetries allowed by equations in case of flat deformation, differs from group of pure shear yield stress in case of flat tension [8–15]. In general, there is a big problem with analytical decisions for flat tension, which is caused by the fact that equations describing this state are rather complex, even despite linearization. In work [8] one of authors managed to find a solution describing compression of a plastic layer between rigid plates which approach with continuous acceleration for the case of flat deformation. In this work the analog of such solution is also framed for flat tension state. Authors hope that the solution will be also used for the analysis of real technological processes.

We will consider the equations describing flat tension state flat state of stress

$$\frac{\partial u}{\partial t}=\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau}{\partial y},\quad \frac{\partial v}{\partial t}=\frac{\partial \tau}{\partial x}+\frac{\partial \sigma_y}{\partial y}, \tag{1}$$

$$\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau^2=3k^2, \tag{2}$$

$$\frac{\dfrac{\partial u}{\partial x}}{2\sigma_x-\sigma_y}=\frac{\dfrac{\partial v}{\partial y}}{2\sigma_y-\sigma_x}=\frac{\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}}{6\tau}, \tag{3}$$

where σ_x, σ_y, τ – components of stress tension tensor; u, v – velocity vector components; k – pure shear yield stress.

System (1)–(3) is a system of five equations for five unknown functions.

Applying the ratios (2)–(3), equations (1)–(3) may be written down only in terms of function u, v.

We obtain

$$2\sigma_x-\sigma_y=\lambda\frac{\partial u}{\partial x},\ 2\sigma_y-\sigma_x=\lambda\frac{\partial v}{\partial y},$$

$$\lambda\left(\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)=6\tau,$$

$$\lambda=3\sqrt{3}k\left[\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2+\left(2\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)^2-\right.$$
$$\left.-\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\left(2\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right)+\frac{3}{4}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]^{-2},$$

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\frac{\lambda}{3}\left(2\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial y}\frac{\lambda}{6}\left(2\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \tag{4}$$
$$\frac{\partial v}{\partial t}=\frac{\partial}{\partial x}\frac{\lambda}{6}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\frac{\lambda}{3}\left(2\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}\right).$$

Group properties of the equations flat state of stress. We will find the group of point symmetries allowed by the system of flat tension equations. We will construct the optimum system of one-dimensional subalgebras and also give a type of all invariant solutions of rank one.

Theorem. Equations (4) allow the group of continuous transformations generated by operators

$$X_0=\frac{\partial}{\partial t},\ X_1=\frac{\partial}{\partial x},\ X_2=\frac{\partial}{\partial y},$$
$$Y_1=\frac{\partial}{\partial u},\ Y_2=\frac{\partial}{\partial v},\ T=x\frac{\partial}{\partial v}-y\frac{\partial}{\partial u},$$
$$Z=y\frac{\partial}{\partial x}-x\frac{\partial}{\partial y}+v\frac{\partial}{\partial u}-u\frac{\partial}{\partial v}, \tag{5}$$
$$N=t\frac{\partial}{\partial t}+x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y},\ M=t\frac{\partial}{\partial t}+u\frac{\partial}{\partial u}+v\frac{\partial}{\partial v}.$$

We will find the optimum system of one-dimensional subalgebras. For this purpose we will calculate commutators of operators (5). They are presented in tab. 1.

Automorphism corresponding to X_i, acting on operator X_j according to formula

$$A_i\left(X_j\right)=X_j+a\left[X_i,X_j\right]+\frac{a^2}{2!}\left[X_i,\left[X_i,X_j\right]\right]+$$
$$+\frac{a^3}{3!}\left[X_i,\left[X_i,\left[X_i,X_j\right]\right]\right]+\ldots$$

Here a – some valid parameter.

It is convenient to collect the influence of automorphisms on operators (5) in tab. 2.

Lemma. Operators X_0, X_i, Y_i, T generate the ideal in Lie algebra.

Thus, automorphisms A_0, …, A_8 cannot change coefficients under Z, M, N operators.

Table 1

Commutator table

	X₀	**X₁**	**X₂**	**Y₁**	**Y₂**	**T**	**Z**	**N**	**M**
$\mathbf{X_0}$	0	0	0	0	0	0	0	X_0	X_0
$\mathbf{X_1}$	0	0	0	0	0	Y_2	$-X_2$	X_1	0
$\mathbf{X_2}$	0	0	0	0	0	$-Y_1$	X_1	X_2	0
$\mathbf{Y_1}$	0	0	0	0	0	0	$-Y_2$	0	Y_1
$\mathbf{Y_2}$	0	0	0	0	0	0	Y_1	0	Y_2
T	0	$-Y_2$	Y_1	0	0	0	0	$-T$	T
Z	0	X_2	$-X_1$	Y_2	$-Y_1$	0	0	0	0
N	$-X_0$	$-X_1$	$-X_2$	0	0	T	0	0	0
M	$-X_0$	0	0	$-Y_1$	$-Y_2$	$-T$	0	0	0

Table 2

Table action of automorphisms

	$\mathbf{X_0}$	$\mathbf{X_1}$	$\mathbf{X_2}$	$\mathbf{Y_1}$	$\mathbf{Y_2}$	**T**	**Z**	**N**	**M**
$\mathbf{A_0}$	X_0	X_1	X_2	Y_1	Y_2	T	Z	$N-a_0X_0$	$M-a_0X_0$
$\mathbf{A_1}$	X_0	X_1	X_2	Y_1	Y_2	$T-a_1Y_2$	$Z+a_1X_2$	$N-a_1X_1$	M
$\mathbf{A_2}$	X_0	X_1	X_2	Y_1	Y_2	$T+a_2Y_1$	$Z-a_2X_1$	$N-a_2X_2$	M
$\mathbf{A_3}$	X_0	X_1	X_2	Y_1	Y_2	T	$Z+a_3Y_2$	N	$M-a_3Y_1$
$\mathbf{A_4}$	X_0	X_1	X_2	Y_1	Y_2	T	$Z-a_4Y_1$	N	$M-a_4Y_2$
$\mathbf{A_5}$	X_0	$X_1+a_5Y_2$	$X_2-a_5Y_1$	Y_1	Y_2	T	Z	$N+a_5T$	$M-a_5T$
$\mathbf{A_6}$	X_0	$X_1-a_6Y_2$	$X_2+a_6X_1$	$Y_1-a_6Y_2$	$Y_2+a_6Y_1$	T	Z	N	M
$\mathbf{A_7}$	$X_0\exp a_7$	$X_1\exp a_7$	$X_2\exp a_7$	Y_1	Y_2	$T\exp(-a_7)$	Z	N	M
$\mathbf{A_8}$	$X_0\exp a_8$	X_1	X_2	$Y_1\exp a_8$	$Y_2\exp a_8$	$T\exp a_8$	Z	N	M

In this respect it is necessary to consider one-dimensional subalgebras of four types

$$D_1 = Z + \alpha M + \beta N + S,$$

$$D_2 = M + \alpha N + S,$$

$$D_3 = N + S,$$

where $S = \sum_{i=1}^{2}\left(\alpha_i X_i + \beta_i Y_i\right) + \gamma T + \alpha_0 X_0$; α, β, γ, α_i, β_i – constants.

Let us consider subalgebra $D_1 = Z + \alpha M + \beta N + S$.

Under the influence of automorphisms A_3, A_4, A_1, A_2 we have

$$A_1, A_2, A_3, A_4\left(D_1\right) = Z + \alpha' M + \beta' N + \gamma' T + \alpha_0' X_0.$$

Here constants with strokes above are received as a result of action of automorphisms in tab. 2. In this case there are only two types of disconjugate subalgebras

$$Z + \alpha\left(M - N\right) + \alpha_0 X_0 + \gamma T,\ Z + \alpha M + \beta N,$$

where α, β, α_0, γ – arbitrary constants.

We will consider $D_2 = M + \alpha N + S$. In this case there are three disconjugate subalgebras

$$\left(M - N\right) + \alpha_0 X_0 + \gamma T,\ M + \alpha N,\ M + \alpha X_1.$$

We will consider subalgebra $D_3 = N + S$.

After the automorphisms' influence A_0, A_1, A_2, A_5, A_6 we obtain not only disconjugate subalgebras

$$N + \beta_1 Y_1.$$

Now we have to make calculations with the ideal. The X_0 operator forms the center of the ideal. Therefore it can be excluded from consideration for now.

Suppose

$$S_1 = T + \alpha_i X_i + \beta_i Y_i,\ i = 1, 2.$$

We have

$$A_1 A_2 S_1 = T + \alpha_i' X_i,$$

$$A_6\left(T + \alpha_i' X_i\right) = T + \alpha X_1,$$

Suppose

$$S_2 = X_i + \alpha X_2 + \beta_i Y_i,$$

$$A_5 A_6 S_2 = X_1 + \alpha Y_1.$$

The last disconjugate subalgebra from ideal has appearance Y_1.

Now, taking into account that X_0 is the centre of the ideal we will write down the final optimum system of one-dimensional subalgebras

$$Z + \alpha\left(M - N\right) + \beta X_0 + \gamma T,\ Z + \alpha M + \beta N,$$

$$\left(M - N\right) + \alpha X_0 + \gamma T,\ M + \alpha N,\ M + \alpha X_1,\ N + \alpha Y_1,$$

$$T + \alpha X_1 + \beta X_0,\ X_1 + \alpha Y_1 + \beta X_0,\ Y_1 + \alpha X_0,\ X_0.$$

Remark 1. Dissimilar subalgebras correspond to various values of constants α, β, γ.

Remark 2. With automorphisms A_7, A_8, as well as with external automorphisms of the system (4), the number of constants in optimum system can be reduced.

1. We will give a type of all invariant solutions of rank 2 which can be constructed on one-dimensional subalgebras:

$$Z + \alpha\left(M - N\right) + \beta X_0 + \gamma T,$$

if $\alpha \neq 0$, then

$$u_r = r^{-1} f\left(\xi, \eta\right),\ v_\theta = r^{-1} g\left(\xi, \eta\right) - \frac{3\gamma}{\alpha} r^2,$$

$$\xi = \beta\theta - t,\ \alpha\theta - \ln r = \eta,$$

if $\alpha = 0$, then

$$u_r = f(\xi,\eta),\; v_\theta = g(\xi,\eta) - \gamma r\theta,\; \xi = \beta\theta - t,\; \eta = r .$$

Here and further r, θ – polar coordinates; u_r, v_θ – velocity vector components in polar coordinates; f, g – some differentiable function of two variables.

2. $Z + \alpha M + \beta N,$

if $\beta \neq 0$, then

$$\ln u_r - \beta\theta = f(\xi,\eta),\; \ln v_\theta - \beta\theta = g(\xi,\eta),$$

$$\xi = (\alpha+\beta)\theta - \ln t,\; \eta = \alpha\theta - \ln r ,$$

if $\beta = 0$, then

$$u_r = f(\xi,\eta),\; v_\theta = g(\xi,\eta) - \gamma r\theta,$$

$$\xi = \alpha\theta - \ln t,\; \eta = \alpha\theta - \ln r .$$

3. $M - N + \alpha X_0 + \gamma T$,

$$u = r^{-1} f(\xi,\eta),\; v_\theta = r^{-1} g(\xi,\eta) - 3\gamma r^2,$$

$$\xi = \alpha \ln r - t,\; \eta = \theta .$$

4. $M + \alpha N$,

if $\alpha \neq -1$, then

$$u = t^{-(1+\alpha)} f(\xi,\eta) = t^{-(1+\alpha)} g(\xi,\eta),\; \xi = x/y,\; \eta = t^{(1+\alpha)} x ,$$

if $\alpha = -1$, then

$$u = tf(x,y),\; v = tg(x,y) .$$

5. $M + \alpha X_1$,

$$u = \alpha x_1 + f(x,y),\; v = \alpha x_1 + g(x,y) .$$

6. $N + \alpha Y_1$,

$$u = -\alpha \ln t + f\left(x/t, y/t\right),\; v = g\left(x/t, y/t\right).$$

7. $T + \alpha X_1 + \beta X_0$,

if $\alpha \neq 0$, then

$$u = -xy/\alpha + f(y, \beta x - \alpha t),\; v = -\frac{1}{2\alpha}x^2 + g(y, \beta x - \alpha t) .$$

If $\alpha = 0, \beta \neq 0$, then

$$u = -yt/\beta + f(x,y),\; v = -xt/\beta + g(x,y) .$$

If $\alpha = 0,\; \beta = 0$, then there are no invariant solutions.

8. $X_1 + \alpha Y_1 + \beta X_0,$

$$u = -\alpha x + f(\beta x - t, y),\; v = g(\beta x - t, y) .$$

9. $Y_1 + \alpha X_0,$

$$u = -\alpha t + f(x,y),\; v = g(x,y) .$$

10. X_0,

$$u = f(x,y),\; v = g(x,y) .$$

Solution describing compression of the plastic layer between rigid plates approaching with continuous acceleration. We will find the system of equations describing plastic currents with continuous acceleration. These solutions are invariant to subalgebra

$$M = t\frac{\partial}{\partial t} + u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v}$$

and are written as follows

$$u = tu(x,y),\; v = tv(x,y) . \tag{6}$$

Inserting (6) in (4) we obtain the system of two equations in function *u*, *v*:

$$\begin{aligned} u &= \frac{\partial}{\partial x}\frac{\lambda}{3}\left(2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial y}\frac{\lambda}{6}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\ v &= \frac{\partial}{\partial x}\frac{\lambda}{6}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\frac{\lambda}{3}\left(2\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right), \end{aligned} \tag{7}$$

where

$$\sigma_x = \frac{\lambda}{3}\left(2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),\; \sigma_y = \frac{\lambda}{3}\left(\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y}\right),$$

$$\tau = \frac{\lambda}{6}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$\begin{aligned} \lambda = 3\sqrt{3}k\Bigg[&\left(2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + \left(2\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right)^2 - \\ &-\left(2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\left(2\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right) + \frac{3}{4}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\Bigg]^{-2}, \end{aligned} \tag{8}$$

$$2f = \frac{d}{dy}\frac{kf'}{\sqrt{g'^2 + 1/4 f'^2}},$$

$$g = \frac{d}{dy}\frac{2kg'}{\sqrt{g'^2 + 1/4 f'^2}} .$$

By differentiating equation (8) at *y*, we have

$$\begin{aligned} 2f' &= \frac{d^2}{dy^2}\frac{kf'}{\sqrt{g'^2 + 1/4 f'^2}}, \\ 2g' &= \frac{d^2}{dy^2}\frac{2kg'}{\sqrt{g'^2 + 1/4 f'^2}}. \end{aligned} \tag{9}$$

Further, for simplicity we take $k = 1$ and enter new variables

$$f' = 2\omega(y)\cosh(y),\; g' = \omega(y)\sinh(y) .$$

The system (9) will be written as follows:

$$\begin{aligned} 2\omega\cosh(y) &= \left(\cosh(y)\right)''_{yy}, \\ \omega\sinh(y) &= \left(2\sinh(y)\right)''_{yy}. \end{aligned} \tag{10}$$

Dividing the second equation of the system (10) by the first, we receive

$$\frac{\sin(h)}{2\cos(h)} = \frac{2(\sin(h))''}{(\cos(h))''} .$$

After simple transformations we receive the ordinary differential equation of the second order which coefficients do not depend on *y*:

$$h''\left(4\cos^2(h) + \sin^2(h)\right) - 3h'^2\cos(h)\sin(h) = 0 .$$

After standard replacement $p = h'$, $h'' = pp'$, we obtain

$$\frac{p'}{p} = \frac{3\cos(h)\sin(h)}{4\cos^2(h) + \sin^2(h)}.$$

By integrating the equation, we have

$$\ln p = -\frac{3}{\sqrt{3}}\operatorname{arctg}\sqrt{3}\cos(h) + c.$$

After the second integration, we have an implicit dependency $h = h(y)$:

$$C\int_0^h \exp\left(\sqrt{3}\operatorname{arctg}\sqrt{3}\cos(h)\right)dh = y. \qquad (11)$$

Fig. 1 demonstrates dependence of function h on variable y.

Now we calculate the components of the tension tensor. The graphs are provided in fig. 2–4.

We have

$$\tau = \frac{kf'}{\sqrt{g'^2 + \frac{1}{4}f'^2}} = 2k\cos(h(y)),$$

$$\sigma_y = 2k\sin(h(y)),$$

$$\sigma_x = \sigma_y \pm \frac{\sqrt{2k^2 - 3\sigma_y^2 - 3\tau^2}}{2}.$$

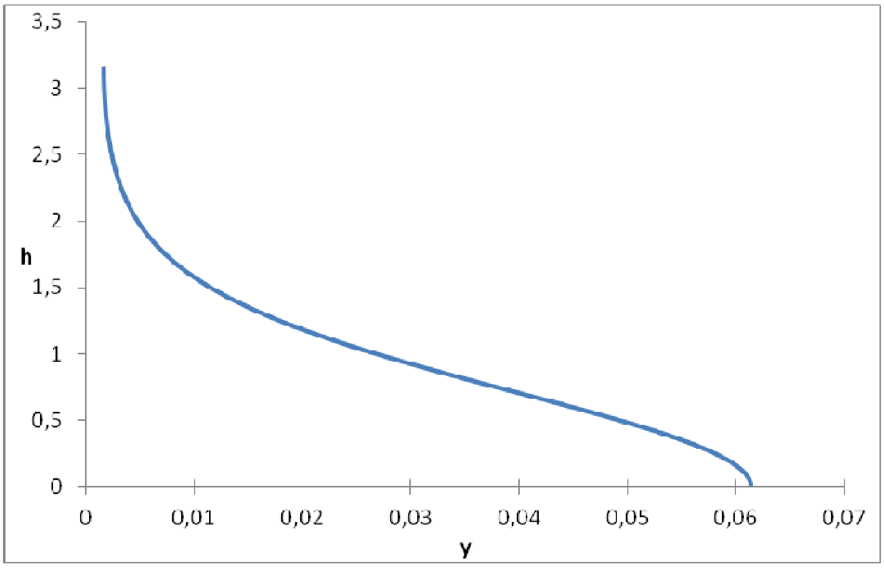

Fig. 1. Dependence of function h on variable y

Рис. 1. График зависимости функции h от переменной y

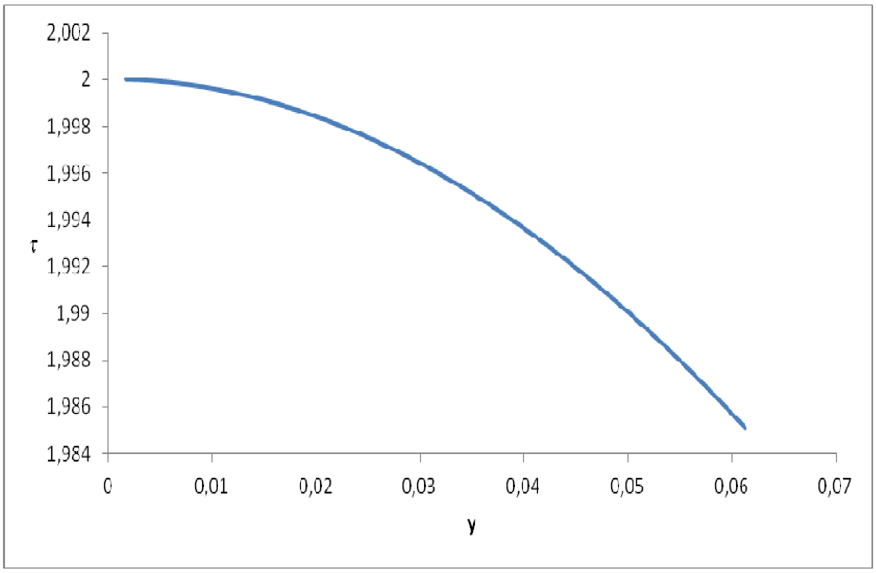

Fig. 2. Dependence of function τ on variable y

Рис. 2. График зависимости τ от переменной y

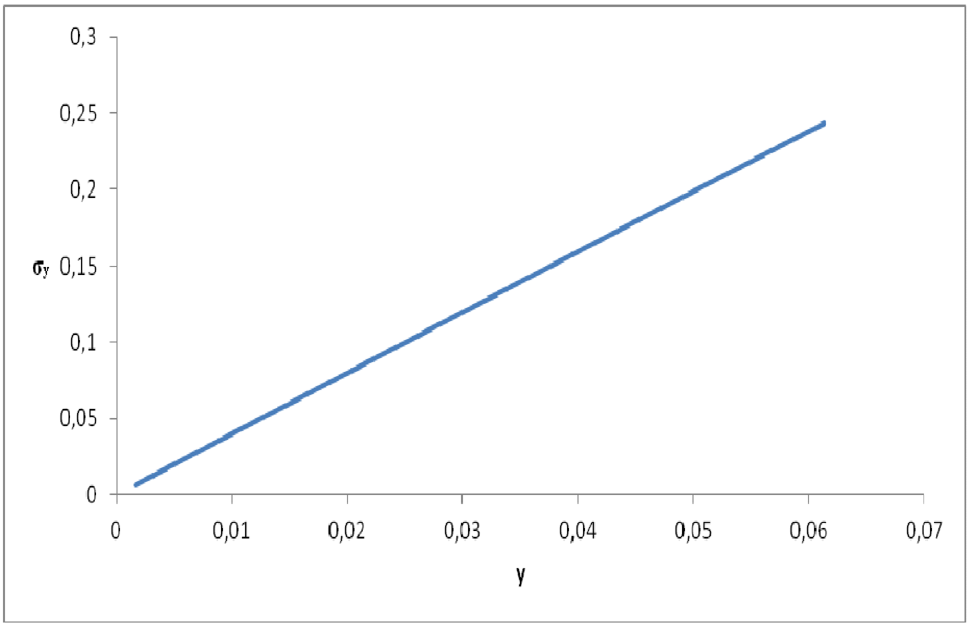

Fig. 3. Dependence of function σ_y on variable y

Рис. 3. График зависимости σ_y от переменной y

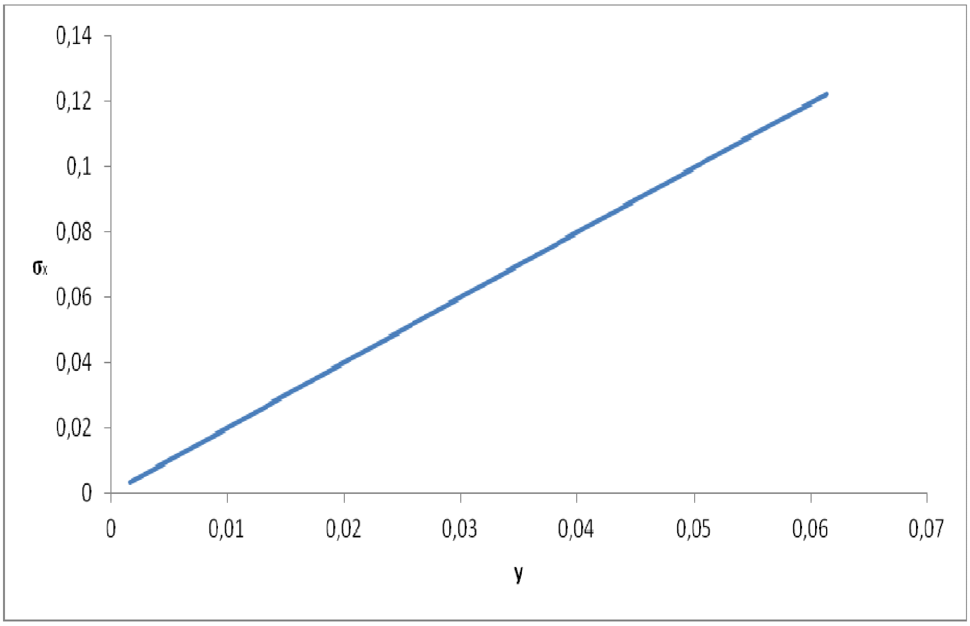

Fig. 4. Dependence of function σ_x on variable y

Рис. 4. График зависимости σ_x от переменной y

Conclusion. From the graphs and formulas it is clear that the solution provided can be used to describe compression of a plastic thin layer which is in conditions flat state of stress. Thickness of the layer does not exceed 0.06, however, the plates which compress the layer with continuous acceleration are characterized by constant pressure.

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