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PARAMETRIC IDENTIFICATION OF THE HEAT CONDITION OF RADIO ELECTRONIC EQUIPMENTIN AIRPLANE COMPARTMENT

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A mathematical model of the aircraft avionics thermal state describing the heat exchange of the onboard equipment housing with a honeycomb structure made of a carbon fiber composite, the process of heat transfer of the onboard equipment elements and the air is developed. The considered heat transfer process in a heterogeneous medium is described by the boundary value problem for the heat equation with boundary conditions of the third kind. To solve the direct problem of the onboard equipment housing with a honeycomb structure thermal state, the Monte- Carlo method on the basis of the probabilistic representation of the solution in the form of an expectation of the functional of the diffusion process is used. The inverse problem of the honeycomb structure heat exchange is solved by minimizing the function of the squared residuals weighted sum using an iterative stochastic quasigradient algorithm. The developed mathematical model of the onboard equipment in the unpressurized compartment thermal state is used for optimizing the temperature and airflow of the thermal control system of the blown onboard equipment in the unpressurized compartment of the aircraft.

Keywords: mathematical model, thermal state, honeycomb structure, parabolic boundary value problem.

ПАРАМЕТРИЧЕСКАЯ ИДЕНТИФИКАЦИЯ ТЕПЛОВОГО СОСТОЯНИЯ РАДИОЭЛЕКТРОННОГО ОБОРУДОВАНИЯ В ПРИБОРНОМ ОТСЕКЕ САМОЛЁТА

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Разработана математическая модель теплового состояния бортового радиоэлектронного оборудования летательного аппарата, описывающая теплообмен корпуса бортового оборудования с сотовой конструкцией из углепластикового композита, процесс теплопередачи элементов бортового оборудования и воздушной среды. Рассматриваемый процесс переноса тепла в неоднородной среде описывается краевой задачей для уравнения теплопроводности с граничными условиями третьего рода. Для решения прямой задачи теплового состояния сотовой конструкции корпуса бортового оборудования использован метод Монте-Карло на основе вероятностного представления решения в виде математического ожидания функционала от диффузионного процесса. Обратная задачи теплообмена сотовой конструкции решена путем минимизации функции взвешенной суммы квадратов невязок с помощью итерационного стохастического квазиградиентного алгоритма. Разработанная математическая модель теплового состояния бортового оборудования в негерметизирован-

ном отсеке была применена для оптимизации температуры и расхода воздуха системы обеспечения теплового режима продуваемого бортового оборудования в негерметизированном отсеке летательного аппарата.

Ключевые слова: математическая модель, тепловое состояние, сотовая конструкция, гетерогенные структуры, параболическая краевая задача.

Introduction. To design the aircraft avionics with a given reliability is necessary to determine the required thermal characteristics of the insulated body of avionics and thermal control system. To do this, the study of the thermal state of avionics in the aircraft compartment, using a mathematical model of their thermal state is conducted. The model should take into account the unsteady heat transfer of the insulated housing, the transfer of heat energy from one part of the equipment to another by the air flow, the convective heat transfer of the equipment elements.

The mathematical model will be a system of partial differential equations and ordinary differential equations, the number of which can reach tens or hundreds for real equipment. Therefore, it is necessary to develop effective methods for solving direct and inverse problems in the study of heat transfer of elements of onboard equipment and estimation of the error of parametric identification.

A physical model of the thermal state of onboard equipment in the compartment of the aircraft. The thermal condition of the onboard equipment in the aircraft compartment is formed by external and internal factors. External factors include heat exchange between the outer surface of the avionics and the air environment, radiation heat exchange of the outer surface of avionics and other surfaces in the compartment. Inside the onboard equipment, thermal energy is released by the elements of the onboard equipment and is withdrawn or supplied by the heat supply system [1].

Mathematical model of the thermal state of onboard equipment in the aircraft compartment. The body of the onboard equipment in the aircraft compartment is a structure that includes a heat-protective honeycomb panel made of carbon fiber composite filled with air. The process of heat transfer in honeycomb panels is described by the boundary value problem for the heat equation with discontinuous coefficients. To solve this boundary value problem, the Monte-Carlo method based on stochastic differential equations is used in combination with the method of wandering in moving spheres [2]. In general, the heat transfer process in the carbon fiber panel is described by the equations [1; 3]:

$$C_{cv}(x)T_{cv,t} = (\lambda_{cv}(x)T_{cv,x})_x, \quad 0<x<l,\ 0<t\le t_k; \qquad (1)$$

$$\lambda_{cv}(x)F_{cv}T_{cv,x} = $$
$$= \alpha_{cv,out}(t)F_{cv}(T_{cv}(t,x)-T_{air,out}(t))+Q_{cv,out}, \qquad x=0; \qquad (2)$$

$$\lambda_{cv}(x)F_{cv}T_{cv,x} = $$
$$= \alpha_{cv,in}(t)F_{cv}(T_{air,in}(t)-T_{cv}(t,x))+Q_{cv,in}, \qquad x=l; \qquad (3)$$

$$T_{cv}(0,x)=T_0(x), \quad 0<x<l, \qquad (4)$$

where

$$C_{cv}(x)=\begin{cases} C_{compo}, & x\in compo; \\ C_{air}, & x\in air, \end{cases}$$

$$\lambda_{cv}(x)=\begin{cases} \lambda_{compo}, & x\in compo; \\ \lambda_{air}, & x\in air, \end{cases}$$

It means, that the coefficients C_{cv}, λ_{cv} depend on which layer the heat transfer is considered.

In equations (1)–(4) the following notations are used: $T_{cv}(t,x)$ – the temperature of the honeycomb panel; $T_{cv,t}$ – the first derivative T_{cv} of t; $T_{cv,x}$ – the first derivative T_{cv} of x; $T_{cv,x,x}$ – the second derivative T_{cv} of x; $C_{cv}(x)$ – volumetric heat capacity of the case honeycomb panel, determined by the heat capacity of the composite C_{compo} and air capacity C_{air}; $\lambda_{cv}(l)$ – the thermal conductivity of the honeycomb panel, determined by the thermal conductivity of the composite λ_{compo} and air thermal conductivity λ_{air}; $\alpha_{cv,out}$ – heat transfer coefficient of the outer surface of the equipment housing; $\alpha_{cv,in}$ – heat transfer coefficient of the inner surface of the equipment housing; F_{cv} – the area of the equipment body for external and internal heat exchange; $Q_{cv,out}$ – heat energy of external sources; $T_{air,out}$ – the air temperature in the compartment; t – time; $T_{air,in}$ – air temperature in onboard equipment or its part; l – the thickness of the honeycomb panel.

The process of heat transfer of the elements of the onboard equipment is presented in the form of an ordinary differential equation describing the convective-radiation heat transfer with the surrounding structures:

$$T_{m,t}=\alpha_{air,m}(t)F_{air,m}/C_m(T_{air}(t)-T_m)+$$
$$+\sum_m g_{j,m}/C_m T_j^4(t)/T_{ms}^4 - c_0\varepsilon_m F_m/C_m T_m^4 + Q_m/C_m, \qquad (5)$$

where T_m – the temperature of *m*-element of onboard equipment; $T_{m,t}$ – the first derivative T_m of t; $\alpha_{air,m}$ – heat transfer coefficient of *m*-element of onboard equipment; $F_{air,m}$ – the area of *m*-element of onboard equipment in convective heat exchange; C_m – heat capacity of *m*-element of onboard equipment; $g_{j,m}$ – radiation heat transfer coefficient of the system "*j*-element – *m*-element of onboard equipment"; ε_m – emission black ratio of *m*-element; Q_m – heat dissipation or heat absorption energy of *m*-element by onboard equipment from the air conditioning system and converted from electrical energy.

The equation of air heat exchange in the unpressurized blown onboard equipment is presented in the form of an ordinary differential equation describing the convective heat transfer of the inner surface of the housing of the on-board equipment, the elements of the on-board equipment and the enthalpy transfer from one part of the on-board equipment to another:

$$T_{air,k,t} = \alpha_{cv,in}(t) F_{cv} / C_{air,k} [T_{cv}(t,x) - T_{air,k} + \\ + \sum_j \alpha_{air,j} F_{air,j} / C_{air,k} (T_j - T_{air,k}) + \\ + c_p J_{air,k} F_k / C_{air,k} (T_{air,k-1} - T_{air,k}); \quad x = l, \qquad (6)$$

where $T_{air,k-1}, T_{air,k}$ – air flow temperatures respectively in ($k-1$) and k parts of onboard equipment; $J_{air,k}$ – the mass rate of the air flow in k part of onboard equipment; F_k – the total area of the air channels in k part of onboard equipment; c_p – specific heat capacity of air; $C_{air,k}$ – heat capacity of air in k part of onboard equipment.

Summation in equation (6) is carried out according to the *j*-element included in the *k*-part of the on-board equipment.

Heat capacity of air $C_{air,k}$ is calculated by the equation:

$$C_{air,k} = c_p \rho_{air,k} (W_{air,ent} F_{air,ent} \Delta t + V_{air,k}), \qquad (7)$$

where $\rho_{air,k}$ – air density in *k*-part of onboard equipment; $W_{air,ent}$ – air velocity at the inlet to the on-board equipment; $F_{air,ent}$ – the area of air channels at the inlet to the first part of the on-board equipment; Δt – time discretization interval in solving a system of differential equations; $V_{air,k}$ – air volume in *k*-part of the on-board equipment.

Heat transfer coefficients of surfaces $\alpha_{cv,out}$, $\alpha_{cv,in}$, $\alpha_{air,m}$ in equations (2)–(6) will be calculated using the methods described in [3; 4].

The coefficients of radiation heat transfer in equation (5) are determined by the Monte-Carlo method [5].

Application of the Monte Carlo method to solve the direct problem of the thermal state of the honeycomb structure of the housing of onboard equipment. In the case where the honeycomb panel (figure) the housing of the onboard equipment is considered as a homogeneous medium with averaged coefficients of volumetric heat capacity and thermal conductivity, heat transfer through the housing of the onboard equipment is described by equations (1)–(4). However, the averaged thermophysical properties of inhomogeneous medium can vary with a change in the direction of heat flow [6]. For this reason, we also consider the determination of the thermal state of a honeycomb panel as a solution to a three-dimensional boundary value problem for the thermal conductivity equation with a discontinuous thermal diffusivity. Due to the peculiarities of the method used, it is assumed that the thermal diffusivity coefficients of the composite and the air are constant.

There is a description of the boundary value problem below. The area in which the boundary value problem under consideration is defined is a rectangular parallelepiped $G = (-l_1, l_1) \cdot (-l_2, l_2) \cdot (0, l_3)$. Where *G* is the union of two disjoint subsets: $G = G_1 \cup G_2$, where G_1 – is a subset, corresponding to the frame and plates limiting the panel, G_2 is the union of subsets, corresponding to the cells with air. The considered heat transfer process takes place on the time interval [0,T] and is described by the following boundary value problem for the heat equation:

$$\frac{\partial T_{cv}}{\partial t} = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(a(x) \frac{\partial T_{cv}}{\partial x_i} \right), \qquad (8)$$

where

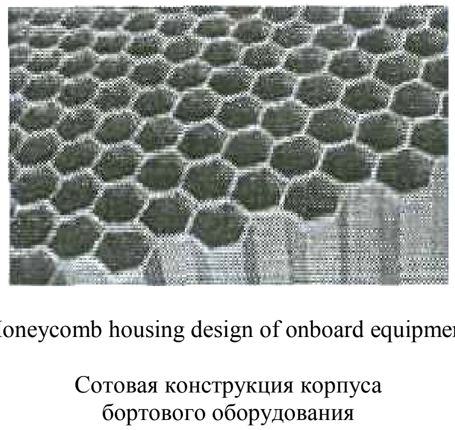
$$a(x) = \begin{cases} a_{compo}, x \in G_1 \\ a_{cair}, x \in G_2 \end{cases};$$

$$\left. \frac{\partial T_{cv}}{\partial x_1} \right|_{x_1 = -l_1} = 0, \quad \left. \frac{\partial T_{cv}}{\partial x_1} \right|_{x_1 = l_1} = 0; \qquad (9)$$

$$\left. \frac{\partial T_{cv}}{\partial x_2} \right|_{x_2 = -l_2} = 0, \quad \left. \frac{\partial T_{cv}}{\partial x_2} \right|_{x_2 = l_2} = 0; \qquad (10)$$

$$-\lambda_{cv} \left. \frac{\partial T_{cv}}{\partial x_3} \right|_{x_3 = l} = \alpha_{cv,out}(t) (T_{cv} - T_{air,out}(t)); \qquad (11)$$

$$\lambda_{cv} \left. \frac{\partial T_{cv}}{\partial x_3} \right|_{x_3 = l} = \alpha_{cv,in}(t) (T_{cv} - T_{air,in}(t)). \qquad (12)$$



Honeycomb housing design of onboard equipment

Сотовая конструкция корпуса
бортового оборудования

In (8)–(12) equations the following designations are used: a_{compo}, a_{air} – thermal diffusivity coefficients of the composite and air, respectively; $\alpha_{cv,out}$, $\alpha_{cv,in}$ – heat transfer coefficients of the panel surface and the air environment outside and inside the onboard equipment, respectively; $T_{air,out}$, $T_{air,in}$ – air temperature at the outer side of the panel and the inner, respectively.

In [7] the existence of generalized solutions of boundary value problems with discontinuous coefficients is proved. Moreover, these solutions can be approximated by solutions of boundary value problems, in which the coefficients are the approximations of the initial discontinuous coefficients. For example, it is possible to obtain an approximate solution of the original problem by solving the problem with smoothed coefficients based on integral averaging [8]. In this paper we propose to determine the approximate solution of the problem as a problem with smoothed coefficients by the Monte-Carlo method based on the probability of representation of the solution in the form of a mathematical expectation of the functional of the diffusion process. Initially, in work [9] the estimates of the mathematical expectation of this functional were determined on the basis of the numerical solution of stochastic differential equations by the Euler method. The disadvantage of this method is its great complexity. A significant acceleration of the calculation was obtained using the combined method proposed in [2], in which the calculation of the trajectories of the diffusion process in air – filled cells (G2) was carried out by the method of wandering through moving spheres, and along the frame, bounding the plates (G1) and in their close area – by the Euler method. Note that the use of the combined method is possible only in the case of constant thermal properties of the substances that make up the honeycomb panel. A detailed description of the combined method is given in work [2].

Algorithm of parametric identification of mathematical model of thermal condition of onboard equipment. To determine the vector of the coefficients θ of the model of the thermal state of the honeycomb panel, the minimum of the function $\Phi(\theta)$ of the weighted sum of squares of residuals [10] using an iterative minimization algorithm with the derived functions $\Phi(\theta)$ should be defined. For this purpose, it is suggested to use a variant of the stochastic quasigradient algorithm with variable metric [8], in which approximations to the minimum point are constructed according to the rule:

$$\theta^{k+1} = \theta^k - \rho_k \, \mathrm{H}^k \nabla^k \Phi, \quad k = 0, 1, \dots, \qquad (13)$$

where H^k – a random square matrix of size $l \times l$; $\nabla^k \Phi$ – gradient of the objective function at a point θ^k; ρ_k – step parameter.

Matrix sequence H^k is calculated by the scheme:

$$\mathrm{H}^0 = I, \mathrm{H}^{k+1} + (I - \mu_k \nabla^{k+1}\Phi \cdot (\Delta^{k+1}\theta)^T)\mathrm{H}^k, \\ \Delta^{k+1}\theta = \theta^{k+1} - \theta^k. \qquad (14)$$

Parameter μ_k is chosen from the equality $\mu_k = \mu / (\left|\nabla^{k+1}\Phi\right| \cdot \left|\Delta^{k+1}\theta\right|$, where μ – is such a constant, that $0 < \mu < 1$.

At each step of the algorithm, the step parameter is automatically adjusted ρ_k. If $\Phi(\theta^{k+1}) > \Phi(\theta^k)$, so $\rho_{k+1} = \rho_k / \gamma$, where $\gamma > 1$ – is a fixed parameter. If $\Phi(\theta^{k+1}) < \Phi(\theta^k)$, so the following sequence of actions is performed: $\rho_{k,i} = \rho_k{}^i\gamma$, $\theta^{k+1,i} = \theta^k - \rho_{k,i}\,\mathrm{H}^k\nabla^k\Phi$, and the calculation $\Phi(\theta^{k+1,i})$, $i = 0, 1, \dots$.

These actions are performed until the value of the function Φ decreases and the conditions are met: $\rho_{\min} \le \rho_{k,i} \le \rho_{\max}$ ($\rho_{\min}, \rho_{\max}$ – minimum, maximum step length, respectively) and $i < i_{\max}$ ($i_{\max}$ – the specified maximum number of iterations to increase the step). The values θ^{k+1}, ρ_{k+1} are assumed to be equal to the values $\theta^{k+1,i}$, $\rho_{k,i}$ that are equal to the minimum of the obtained values $\Phi(\theta)$.

Parametric identification of a mathematical model of the thermal state of the other elements of onboard equipment proposed to carry out the composition method of the steepest descent method, Newton method and quasi-Newton method of Broyden – Fletcher – Goldfarb – Shanno [11].

When solving a rigid system of ordinary differential equations, it is proposed to use the implicit Rosenbrock method of the second order [12].

Estimation of the parameters of the mathematical model of the avionics compartment of the aircraft. Verification of the proposed theoretical method was performed for onboard equipment in the aircraft compartment, which is a block of onboard equipment in the body with a honeycomb design. The unit is blown with air from the thermal control system. The air cools or heats the elements of the onboard equipment located in the compartment. The elements of the block are separated by air layers. At the same time, the thickness of the honeycomb structure, temperature and air consumption of the system for ensuring the thermal regime of the onboard equipment unit were optimized.

The main criterion for optimizing the thickness of the honeycomb structure, temperature and air flow of the thermal control system is the air temperature in the equipment unit within the limits of 283.15–293.15 K.

The air temperature in the compartment is adjustable from 253.15 K to 328.15 K [13; 14]. At the same time, the values of the airflow in the thermal control system must be within the range of 1.5–2.0 kg/s.

The coefficient of thermal conductivity of the honeycomb structure of the block body is $\lambda_{cv} = 8 \cdot 10^{-2}$ W/(m·K).

The thickness of the honeycomb structure l_{cv} of the block body took $2 \cdot 10^{-2}$–$5 \cdot 10^{-2}$ m.

The vector of coefficients of the model

$$\Theta = [\, l_{\text{cv}} \quad T_{stm} \quad G_{stm} \,]^T \qquad (15)$$

includes the thickness of the honeycomb structure l_{cv} in m, the necessary characteristics of the system to ensure the thermal control (the values of air temperature T_{stm} in K and air flow consumption G_{stm} in kg/s).

Estimates of the coefficients of the model $\hat{\hat{\Theta}}$ thickness of the honeycomb structure, for temperature and airflow consumption, respectively, are equal:

$$\hat{\hat{\Theta}} = [0.003 \quad 287.4 \quad 1.9]^T .$$

Joint confidence intervals $\Delta\Theta^*$ of uncertainty of coefficient estimates (15) $\hat{\hat{\Theta}}$ with confidence probability $\beta = 0.99$ are, respectively, equal to

$$\Delta\Theta^* = [0.0004 \quad 5.0 \quad 0.08]^T .$$

Joint confidence intervals $\Delta\Theta^*$ of each of the sought coefficients are obtained by the method given in [15].

Conclusion. A theoretical method for determining the parameters of the housing of avionics with honeycomb structures and the system of providing the thermal control of avionics on the basis of the developed mathematical model of the thermal state of avionics of the aircraft is proposed.

The determination of the thermal state of the honeycomb structure is carried out by a combined method, in which the calculation of the trajectories of the diffusion process in the cells filled with air is based on the method of wandering through the moving spheres, and along the frame, limiting plates and in their close area – by the Euler method.

A stochastic quasigradient algorithm with a variable metric is used for parametric identification of the mathematical model of the thermal state of the honeycomb structure of the housing of the onboard equipment.

The proposed method makes it possible to optimize the thermal parameters of the housing of the onboard equipment and the system of ensuring the thermal control in the design of onboard equipment.

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