

# MATHEMATICAL MODEL FOR GEOSTATIONARY SPACECRAFT DISTURBING TORQUES DETERMINATION

S. V. Latyntsev<sup>1\*</sup>, A. V. Murygin<sup>2</sup>

<sup>1</sup> JSC “Academician M. F. Reshetnev “Information Satellite Systems”  
52, Lenin Str., 52, Zheleznogorsk, Krasnoyarsk region, 662972, Russia Federation

<sup>2</sup> Reshetnev Siberian State University of Science and Technologies  
31, Krasnoyarsky Rabochy Av., Krasnoyarsk, 660037, Russian Federation

\* E-mail: lat.sv@mail.ru

Modern requirements to increase spacecraft active existence lead to the efficiency of all its resources use improvement. And one of the main spacecraft resources, which determines the period of active existence, is the orientation engines fuel. The fuel consumption of the orientation engines depends on the external disturbance torques affecting the spacecraft. The work is devoted to the development of a mathematical model that allows to determine external disturbance torques continuously affecting the spacecraft. The mathematical model is based on the assumption that the kinetic moment of the spacecraft remains unchanged in the inertial coordinate system. The use of an active flywheel orientation system makes it possible to measure a spacecraft kinetic and disturbance moments. A special feature of this measurement is the rigid connection of flywheels with the spacecraft body that rotates at an orbital speed. This feature makes it necessary to take into account the kinematic relationship of the flywheel kinetic moment with the kinetic and disturbance moments in the inertial space. Thus, according to the kinetic moment variation law, it was possible to obtain a mathematical model for the interrelation between the flywheel kinetic moment and external disturbance torques. To test the model, two of the most common methods of mean-square filter readings were examined: the Gaussian filter and the Kalman filter. Modeling systems of equations and coefficients of error matrices were determined for modeling. The model was tested in the GNU Octave mathematical computing environment using telemetry information received in 2017, from medium-sized spacecraft (based on the Express-1000H platform) and heavy (Express-2000) class. To compare the results, the graphs for calculating the kinetic moment from the model and the measured kinetic moment from the flywheels are given. The mean-square deviation of the compared values did not exceed 0.1 Nm for the Gaussian filter and 0.03 Nms for the Kalman filter. The graphs of disturbing torques estimation by a mathematical model are given. The mean-square deviation of the estimate of the disturbing torquess for the Gaussian filter did not exceed 0.9 % and for the Kalman filter it was 2 %. The convergence of the disturbing torques estimates shows the adequacy of the developed mathematical model.

Keywords: attitude determination and control system, spacecraft, disturbing torque.

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# МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОПРЕДЕЛЕНИЯ ВОЗМУЩАЮЩИХ МОМЕНТОВ ГЕОСТАЦИОНАРНОГО КОСМИЧЕСКОГО АППАРАТА

С. В. Латынцев<sup>1\*</sup>, А. В. Мурыгин<sup>2</sup>

<sup>1</sup> АО «Информационные спутниковые системы» имени академика М. Ф. Решетнёва»  
Российская Федерация, 662972, г. Железногорск Красноярского края, ул. Ленина, 52

<sup>2</sup> Сибирский государственный университет науки и технологий имени академика М. Ф. Решетнёва  
Российская Федерация, 660037, г. Красноярск, просп. им. газ. «Красноярский рабочий», 31

\* E-mail: lat.sv@mail.ru

Современные требования по увеличению срока активного существования космического аппарата приводят к повышению эффективности использования всех его ресурсов. И одним из главных ресурсов космического аппарата, который определяет срок активного существования, является топливо двигателей ориентации. Расход топлива двигателями ориентации зависит от внешних возмущающих моментов, действующих на космический аппарат. Представлена разработка математической модели, которая позволяет определять внешние возмущающие моменты, действующие непрерывно на космический аппарат. Математическая модель строится на предположении, что кинетический момент космического аппарата остается неизменным в инерциальной системе координат. Использование активной маховичной системы ориентации дает возможность измерять кинетический и возмущающие моменты космического аппарата. Особенностью такого измерения является жесткая связь маховиков с корпусом космического аппарата, который вращается с орбитальной скоростью. Эта особенность заставляет учитывать кинематическую взаимосвязь кинетического момента маховиков с кинетическим и возмущающими моментами в инерциальном пространстве. Таким обра-

зом, из закона изменения кинетического момента удалось получить математическую модель взаимосвязи кинетического момента маховиков и внешних возмущающих моментов. Для проверки модели были рассмотрены два наиболее распространенных метода среднеквадратичной фильтрации показаний – фильтр Гаусса и фильтр Калмана. Для моделирования были определены моделируемые системы уравнений и коэффициенты матриц ошибок. Проверка модели проводилась в среде математических вычислений GNU Octave по телеметрической информации, полученной в 2017 году с космических аппаратов среднего (на базе платформы «Экспресс-1000Н») и тяжелого («Экспресс-2000») класса. Для сравнения результатов приведены графики расчета кинетического момента по модели и измеренного кинетического момента с маховиков. Среднеквадратичное отклонение сравниваемых значений не превысило 0,1 Н·м/с для фильтра Гаусса и 0,03 Н·м/с для фильтра Калмана. Приведены графики оценки возмущающих моментов по математической модели. Среднеквадратичное отклонение оценки возмущающих моментов для фильтра Гаусса не превысило 0,9 % и для фильтра Калмана – 2 %. Сходимость оценок возмущающих моментов показывает адекватность разработанной математической модели.

**Ключевые слова:** система ориентации и стабилизации, космический аппарат, возмущающие моменты.

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**Introduction.** Since 2011 in Russia on the basis of modern platforms of medium ("Express 1000H") and heavy ("Express-2000") classes several geostationary spacecrafts (SC) of communication have been launched: "Luch 5A", "Luch 5B", "Luch 5B", "Express AM5", "Express AM6", "Express AT1", "Express AT2", "Yamal-300K", "Yamal-401", etc. In the course of a SC normal operation it experiences different disturbance torques which are compensated by orientation and stabilization active control system with flywheels application. Such system is very widespread among relay satellites [1–8]. The main advantage of flywheels systems is an opportunity to create the operating moments in external force fields absence or when this field is the disturbing factor. Also, flywheels do not expend a working body as jet engines do. But at disturbance torques long influence, flywheels rotation speed increases. Flywheels rotation maximal speed restriction leads to unloading due to jet engines inclusion [2]. Therefore it is necessary to have a fuel reserve for flywheels unloading during the whole term of active existence on modern SCs. The authors of works [9–13] presented the methods of geostationary SC flywheels unloading by solar pressure force use that allows to save the fuel. At the same time one of the specified methods first problems is the definition of a system kinetic moment which consists of a SC kinetic moment and a flywheels kinetic moment. The SC kinetic moment is calculated by multiplication of a SC angular velocity of rotation by SC inertia moments, the flywheels kinetic

moment is calculated by multiplication of flywheels rotation speed by a flywheel inertia moment. Using this method of SC kinetic moment determination an internal kinetic moment (for example, from the thermal regulation system [14]) and an error of SC inertia moments knowledge are not considered. As shown in [15], one more error source is the influence of unknown external disturbing torques. All these errors, at accurate pointing and stabilization of a SC, are compensated by a flywheel control system. And then it turns out that a flywheel kinetic moment consists of the cumulated kinetic moment from external disturbing forces and errors of SC total kinetic moment knowledge. In a control system which does not consider these errors unloading is performed less effectively.

The purpose of this work is to improve unloading effectiveness by solving the following tasks:

- 1) the development of a mathematical model which will allow to define authentically a cumulated kinetic moment from the external disturbing forces on flywheels;
- 2) to confirm mathematical model adequacy according to telemetric data from the SCs in flight.

**A system model.** Let us introduce the following right orthogonal systems of coordinates:

- 1) inertial coordinate systems (ICS), which corresponds to the international celestial coordinate system [16] and is indicated by  $O_I X_I Y_I Z_I$  (fig. 1);
- 2) coupled coordinate system (CCS) of  $OXYZ$ , its origin is in the spacecraft cog, the axes are its principal central axes of inertia (fig. 1).

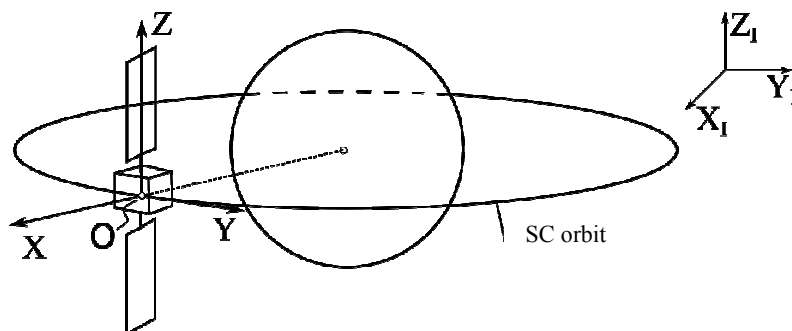


Fig. 1. Inertial and coupled coordinate systems

Рис. 1. Инерциальная и связанная системы координат

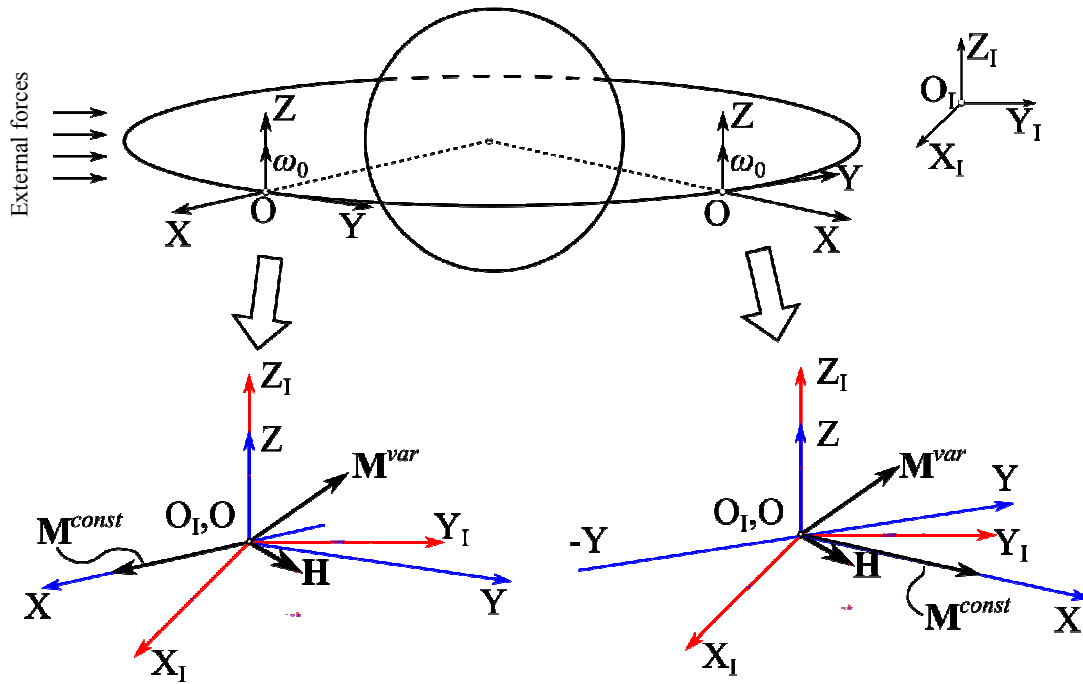


Fig. 2. The vector of the kinetic and disturbing torques in the inertial and coupled coordinate systems

Рис. 2. Векторы кинетического и возмущающих моментов в инерциальной и связанной системах координат

For a SC in the coupled coordinate system the law of a kinetic moment change is valid [1; 15]:

$$\dot{\mathbf{H}} + \boldsymbol{\omega} \times \mathbf{H} = \mathbf{M}^{dist} + \mathbf{M}^{ctrl}, \quad (1)$$

where  $\mathbf{H} = \mathbf{J} \cdot \boldsymbol{\omega}$  – SC total kinetic moment; a derivative is indicated by a point above  $\mathbf{H}$ , N·m/s;  $\mathbf{J}$  – matrix of SC inertia tensor, kg · m<sup>2</sup>;  $\boldsymbol{\omega}$  – SC angular velocity in an inertial coordinate system, rad;  $\mathbf{M}^{dist}$  and  $\mathbf{M}^{ctrl}$  – external forces moment and operating moment respectively, Nm.

Let us consider the geostationary SC of communication operating in a normal mode. Such SC characteristics are:

- 1) SC uniform circular orbiting with a day period;
- 2) SC stabilization and orientation to the given point of the Earth surface;
- 3) total absence of the atmosphere impact and considerably reduced values of gravitational and magnet moments in comparison with low-flying SCs [17].

The angular velocity of SC rotation around an  $OZ$  axis presence follows from the first and second characteristics, which is calculated as  $\omega_0 = 2\pi/86164$  c =  $7,27 \cdot 10^{-5}$  rad/s. It allows to rewrite the equation (1) as a set of equations:

$$\begin{cases} \dot{H}_X + \omega_0 H_Y = M_X^{dist} + M_X^{ctrl}, \\ \dot{H}_Y - \omega_0 H_X = M_Y^{dist} + M_Y^{ctrl}, \\ \dot{H}_Z = M_Z^{dist} + M_Z^{ctrl}, \end{cases} \quad (2)$$

where  $\omega_0$  – angular velocity of SC rotation, which is  $7,27 \cdot 10^{-5}$ , rad/s.

The third SC characteristic allows to draw a conclusion that the external disturbing torque will mainly occur due to the solar pressure. The solar pressure impact on a SC design is well described in [17; 18].

From the listed characteristics and system (2) it fairly follows:

1) angular velocity  $\omega_0$  on an  $OZ$  axis presence leads to the fact that the vector of a SC kinetic moment of  $\mathbf{H}$ , remaining invariable in an inertial coordinate system, makes a circle in the coupled coordinate system (fig. 2);

2) SC rotation in relation to the external disturbing forces in the coupled coordinate system leads to the division of the external disturbing torque in the coupled coordinate system into a variable and a constant component (fig. 2).

In fig. 2 two points of a SC orbit are presented. For each orbit point the mutual positioning of inertial ( $O_I X_I Y_I Z_I$ ) and coupled ( $OXYZ$ ) systems of coordinates is given. Projection of a vector of a SC kinetic moment  $\mathbf{H}$  on an inertial coordinate system axis in which it remains invariable is done. Also this vector is projected on the coupled coordinate system axis from which the change of a vector  $\mathbf{H}$  projection due to the angular SC turn, and respectively the coupled coordinate system, in an inertial coordinate system is visible. The similar situation is with the variable external disturbing torque, and is indicated as  $\mathbf{M}^{var}$ . The constant external disturbing torque, on the contrary, rotates together with the SC coupled system, and is indicated as  $\mathbf{M}^{const}$ . For the convenience the constant external disturbing torque is located on the  $OX$  axis.

The graphs of the projection of the kinetic moments  $\mathbf{H}$  vector on the axis  $OX$  and  $OY$  are presented in fig. 3.

The graphs of the projection of the kinetic moment vector on the axis  $OX$  and  $OY$  are well combined with formula (2). If we draw the graph with  $H_X$  on the abscissa axis and  $H_Y$  on the ordinates axis, the graph will resemble the spinning spiral that is well shown in work [9].

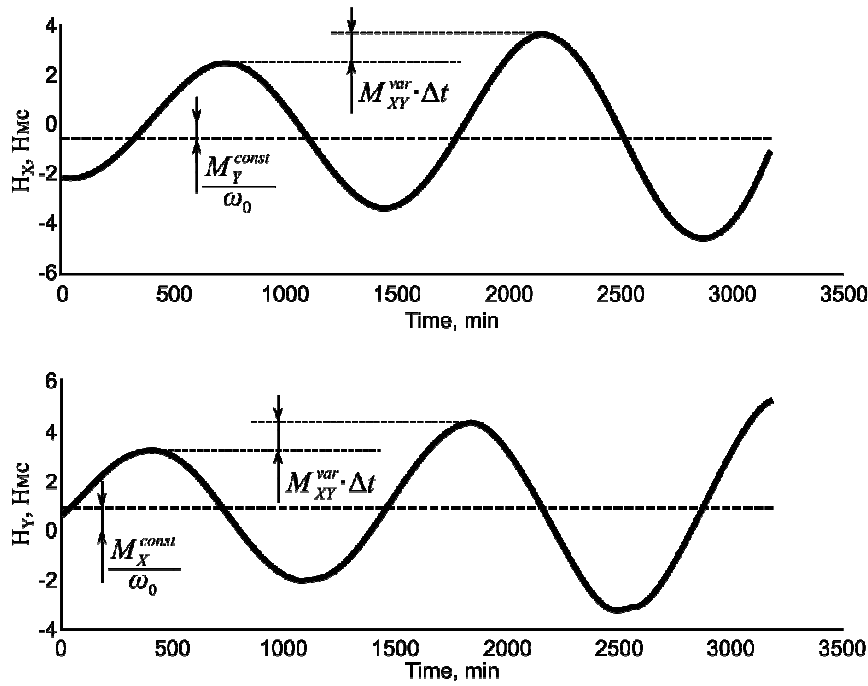

 Fig. 3. Graphs of the projection of the kinetic moment  $\mathbf{H}$  vector on the axis  $OX$  and  $OY$ 

 Рис. 3. Графики проекции вектора кинетического момента  $\mathbf{H}$  на оси  $OX$  и  $OY$ 

Considering that flywheels form the operating moment in the coupled frame [19–22], using (1) and the fact that the SC kinetic moment without flywheels remains constant ( $\dot{\mathbf{H}} + \boldsymbol{\omega} \times \mathbf{H} = \text{const}$ ), the flywheels kinetic moment is determined as:

$$\begin{aligned} \mathbf{H}(t) &= \mathbf{H}(t_0) + \int_{t_0}^{t_1} (\dot{\mathbf{H}} + \boldsymbol{\omega} \times \mathbf{H} - \mathbf{M}^{dist}) dt = \\ &= \mathbf{H}(t_0) - \mathbf{M}^{dist} \cdot \Delta t, \end{aligned} \quad (3)$$

where  $\mathbf{H}(t)$  – flywheel kinetic moment, N·m/s;  $t_0, t_1$  – initial and terminal time instant, s;  $\Delta t = t_1 - t_0$ .

To receive a set of equations of flywheels kinetic moment in axes of the coupled frame we will do the same [23] taking into consideration unperturbed motion:

$$\begin{cases} H_x(t) = H_x(t_0) \cos(\omega_0 t_1) + \\ + H_y(t_0) \sin(\omega_0 t_1) - M_x^{dist} \cdot \Delta t, \\ H_y(t) = -H_x(t_0) \sin(\omega_0 t_1) + \\ + H_y(t_0) \cos(\omega_0 t_1) - M_y^{dist} \cdot \Delta t, \\ H_z(t) = H_z(t_0) - M_z^{dist} \cdot \Delta t. \end{cases} \quad (4)$$

As it was shown above the external disturbing torque involves a variable and a constant component, and then proceeding from fig. 2 the external disturbing torque is calculated:

$$\begin{cases} M_x^{dist}(t) = M_{XY}^{var} \cdot \cos(\omega_0 t_1) + \frac{M_Y^{const}}{\omega_0 \cdot \Delta t}, \\ M_y^{dist}(t) = -M_{XY}^{var} \cdot \sin(\omega_0 t_1) + \frac{M_X^{const}}{\omega_0 \cdot \Delta t}, \\ M_z^{dist}(t) = M_Z^{var} + M_Z^{const}. \end{cases} \quad (5)$$

Then from formulas (4) and (5) taking into consideration replacement  $t_1 = t_0 + \Delta t$  follows:

$$\begin{cases} H_x(t) = (H_{XY}(t_0) + M_{XY}^{var} \cdot \Delta t) \times \\ \times \cos(\omega_0(t_0 + \Delta t)) + \frac{M_Y^{const}}{\omega_0}, \\ H_y(t) = (-H_{XY}(t_0) + M_{XY}^{var} \cdot \Delta t) \times \\ \times \sin(\omega_0(t_0 + \Delta t)) + \frac{M_X^{const}}{\omega_0}, \\ H_z(t) = H_z(t_0) - (M_Z^{var} + M_Z^{const}) \cdot \Delta t, \end{cases} \quad (6)$$

where  $H_{XY}(t_0)$  – kinetic moment vector projection in the  $XOY$  plane, N·m/s.

As it was already emphasized in practice the external disturbing torques knowledge is unknown, but flywheels kinetic moments are known. Therefore the inverse task – determination of the external disturbing torques on flywheels kinetic moments on mathematical model (6) is considered further.

**External disturbing torques determination.** The flywheels kinetic moment on an  $OZ$  axis does not depend on a kinetic moment of two other axes that allows to divide a problem of external disturbing torques determination into two. At the same time external disturbing torques determination on an  $OZ$  axis comes down to the problem of approximation which solution can be found in [24; 25] and does not represent complexity and interest.

The detailed consideration of the mathematical model (6) allows to reveal the known kinetic moment dependence on unknown parameters:  $M_X^{const}$ ,  $M_Y^{const}$ ,  $M_{XY}^{var}$ ,  $H_{XY}$  and  $t_0$ .

Let us write down unknown parameters in the matrix:

$$\alpha_k = \begin{bmatrix} M_X^{\text{const}} & M_Y^{\text{const}} & M_{XY}^{\text{var}} & H_{XY} & t_0 \end{bmatrix}. \quad (7)$$

There is no analytical solution of parameters (7) determination from the mathematical model (6), therefore the solution of a task comes down to numerical methods application and the problem of parameters (7) identification is considered with the use of mean-square indications processing by a filtration method. The Gaussian and the Kalman filters are widespread methods of filtration. The Gaussian filter is a recursive (iterative) filter for the linear estimation of system constant parameters. For the problem with variable defined parameters solving the most known method is the Kalman filter.

It is necessary for filters operation:

1. To define interrelation of a required parameters vector and parameters measured on  $k$  iteration. In a general view this interrelation is defined from the ratio:

$$\psi_k = h(\alpha_k, t) + R_k, \quad (8)$$

where  $h(\alpha_k, t)$  – function defining the model of measurements.

Then from formula (6) formula (8) is written as:

$$\begin{bmatrix} H_X(t) \\ H_Y(t) \end{bmatrix}_k = \begin{bmatrix} (H_{XY}(t_0) + M_{XY}^{\text{var}} \cdot \Delta t) \times \\ \times \cos(\omega_0(t_0 + \Delta t)) + \frac{M_Y^{\text{const}}}{\omega_0} \\ (-H_{XY}(t_0) + M_{XY}^{\text{var}} \cdot \Delta t) \times \\ \times \sin(\omega_0(t_0 + \Delta t)) + \frac{M_X^{\text{const}}}{\omega_0} \end{bmatrix}_k + R_k. \quad (9)$$

The matrix of the previous estimation transfer to a new point will be single:  $\Phi_k = E_{2 \times 5}$ .

2. To linearize a set of equations (6). After linearization the system is as follows:

$$U_k = \frac{\partial h(\alpha_k, t)}{\partial \alpha} = \begin{bmatrix} 1/\omega_0 & 0 & t \cdot \cos(u) & \cos(u) & -(H_{XY} + M_{XY}^{\text{const}} \cdot \Delta t) \cdot \omega_0 \cdot \sin(u) \\ 0 & 1/\omega_0 & t \cdot \sin(u) & -\sin(u) & (-H_{XY} + M_{XY}^{\text{const}} \cdot \Delta t) \cdot \omega_0 \cdot \cos(u) \end{bmatrix}$$

where  $u = \omega_0(t_0 + \Delta t)$ .

3. To set errors matrixes. The Kalman filter accuracy depends on the chosen values of the inverse coefficients matrix, initial values of an error matrix, a motion model error matrix and a measurements noise matrix in a complex way. Analytically such dependence can not be obtained, therefore such matrixes are manually selected and

presented by diagonal matrixes, with the main diagonal values according to tab.1.

**Numerical modeling results.** Filters model operation was performed in mathematical computing environment GNU Octave (ver. 4.2.1, John W.Eaton research, etc.). The modeling method includes:

- 1) initial parameters setting;
- 2) SC telemetric information from the database processing;
- 3) averaging of kinetic moments from flywheels at the period of 1 minute;
- 4) calculation of filter parameters according to sense telemetric information;
- 5) filter operation results conclusion.

Modeling was performed according to the data of telemetric information from the flying SC of average ("Express-1000H" platform) and heavy ("Express 2000" platform) classes. For the middle class SC modeling was performed for the interval from 2 o'clock 23.04.2017 till 10 o'clock 28.04.2017, for the heavy SC – from 0 o'clock 9.10.2017 for 6 o'clock 11.10.2017.

In fig. 4 the graphs of calculation of the difference between the kinetic moment from flywheels and estimation of the kinetic moment for the middle class spacecraft on mathematical model are given. In fig. 5 similar graphs for heavy class SC are given. Mean-square deviation (MSD) –  $\sigma$  designation is introduced in all drawings.

The graphs analysis showed that with the increase in measurements the difference between flywheels kinetic moments and its estimation decreases and does not exceed 0.2 N·m/s for the Gaussian filter (fig. 4, a and 5, a) and 0.06 N·m/s for the Kalman filter (fig. 4, b) and 5, b).

In fig. 6 and 7 the graphs of calculation of the external disturbing torques respectively by the Gaussian and the Kalman filters for the middle class SC are presented. In fig. 8 and 9 similar graphs for the heavy class SC are presented.

The errors convergence confirms the accepted mathematical model adequacy (6). Convergence time equal to one turn should be noted, that corresponds to one day of a SC on the GSO.

Disturbing torques computing results are presented in tab. 2.

The disturbing torques error by the Gaussian filter is less (not more than 0,9 %) than by the Kalman filter (not more than 2 %) in some cases, however, the difference between the measured values of the kinetic moment and filters estimation is more. It is explained by smaller sensitivity of the Gaussian filter to the deviation of parameters measured from the predicted value unlike the Kalman filter.

Table 1

Values of error matrixes main diagonals

| Error matrix name              | Value                                                         |
|--------------------------------|---------------------------------------------------------------|
| Measurements ( $K_\psi$ )      | $0.0001^2; 0.0001^2$                                          |
| Required values ( $K_\omega$ ) | $0.01^2; 0.01^2; 0.01^2; (10^{-8})^2; 1^2$                    |
| System models ( $Q_k$ )*       | $(10^{-10})^2; (10^{-10})^2; (10^{-10})^2; (10^{-10})^2; 1^2$ |

\* Only for the Kalman filter.

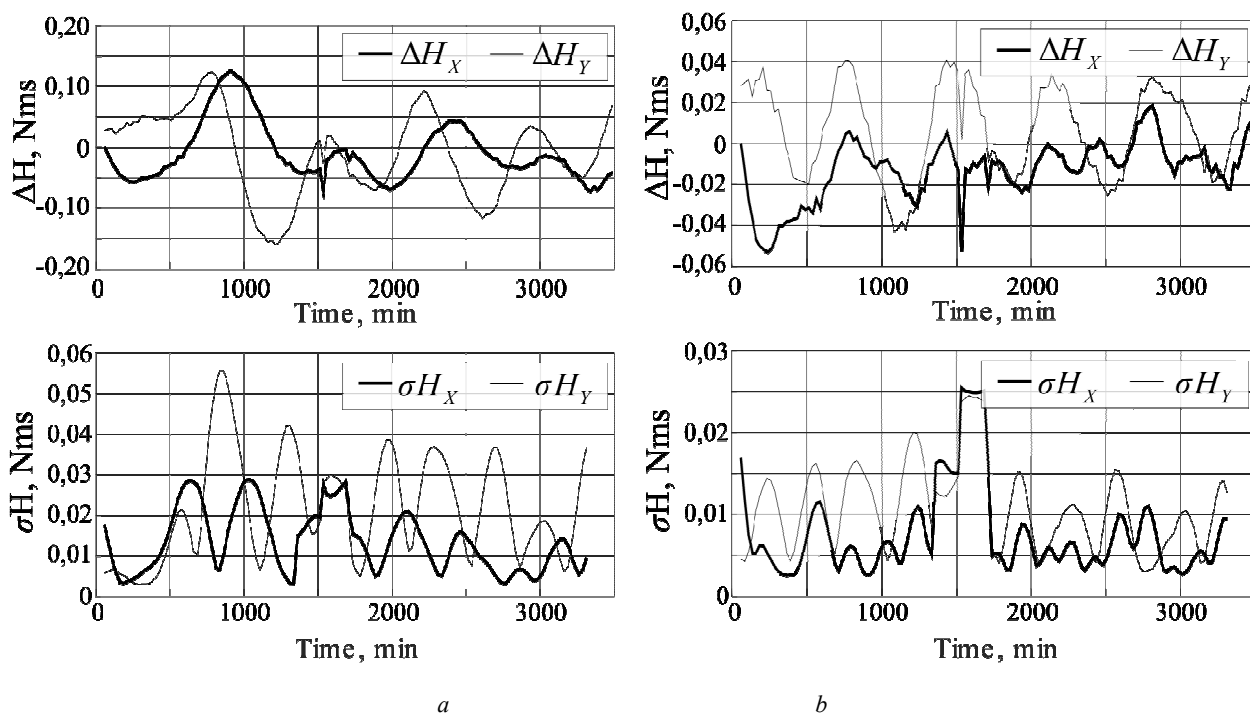


Fig. 4. The Graphs of the calculation of the difference between the kinetic moment from the flywheels and the estimation of the kinetic moment for the spacecraft of the middle class:  
a – the Gaussian filter; b – the Kalman filter

Рис. 4. Графики расчета разницы между кинетическим моментом с маховиков и оценкой кинетического момента для КА среднего класса: а – фильтр Гаусса; б – фильтр Калмана

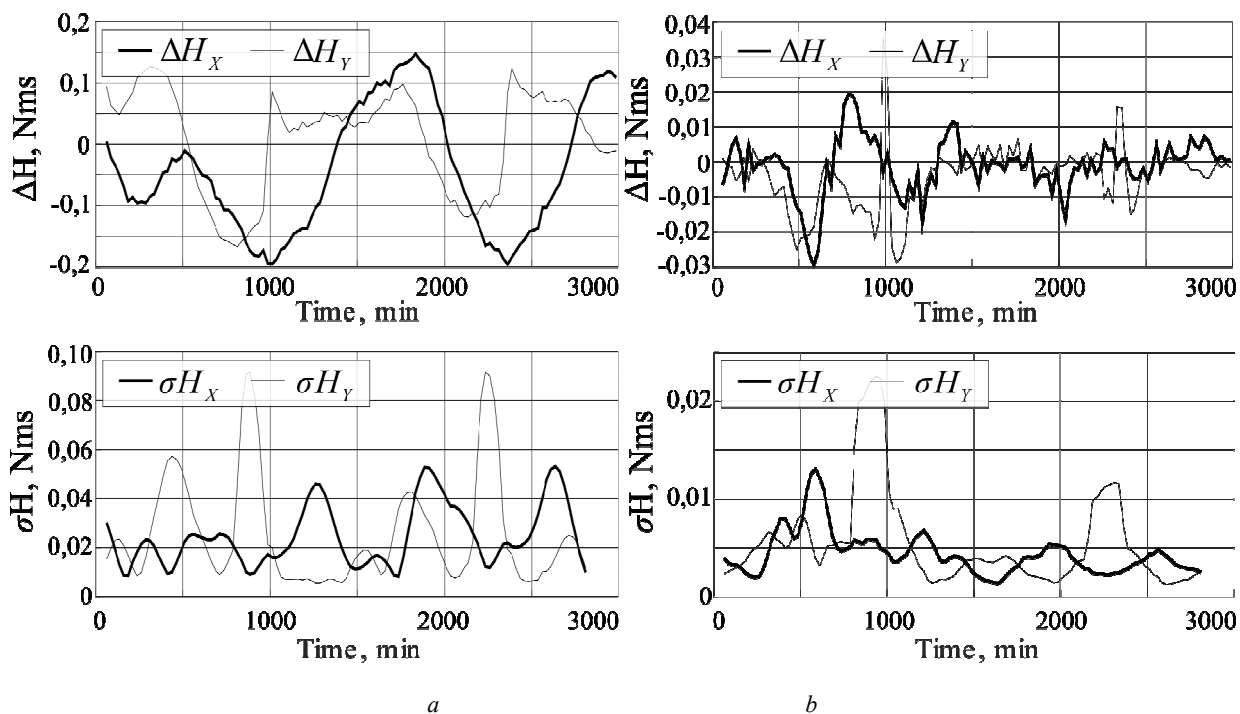


Fig. 5. The Graphs of the calculation of the difference between the kinetic moment from the flywheels and the estimate of the kinetic moment for the spacecraft of the heavy class:  
a – Gaussian filter; b – the Kalman filter

Рис. 5. Графики расчета разницы между кинетическим моментом с маховиков и оценкой кинетического момента по математической модели для КА тяжелого класса:  
а – фильтр Гаусса; б – фильтр Калмана

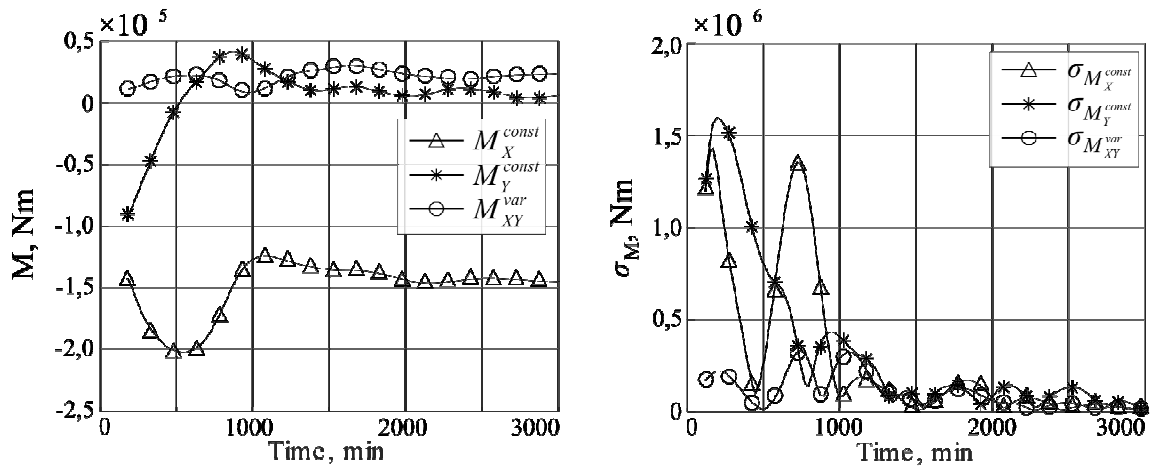


Fig. 6. The graphs of the disturbing torques calculation by the Gaussian filter for the middle class spacecraft

Рис. 6. Графики расчета возмущающих моментов фильтром Гаусса для КА среднего класса

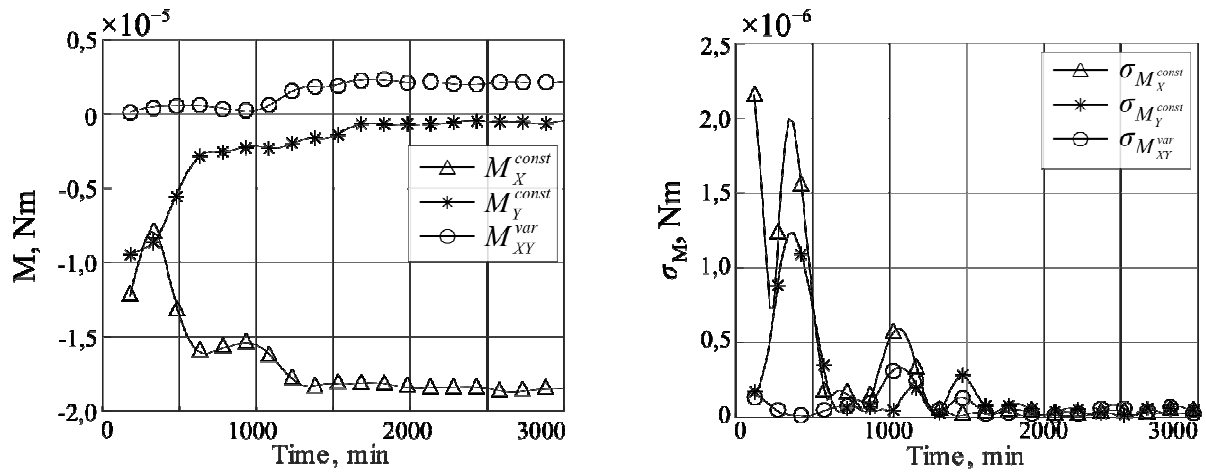


Fig. 7. The graphs of the disturbing torques calculation by the Kalman filter for the middle class spacecraft

Рис. 7. Графики расчета возмущающих моментов фильтром Калмана для КА среднего класса

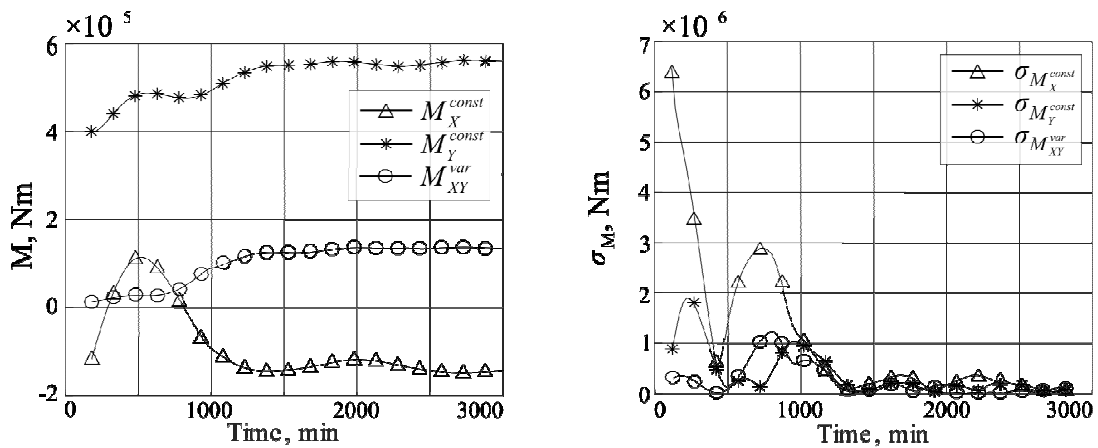


Fig. 8. The graphs of the disturbing torques calculation by the Gaussian filter for the heavy class spacecraft

Рис. 8. Графики расчета возмущающих моментов фильтром Гаусса для КА тяжелого класса

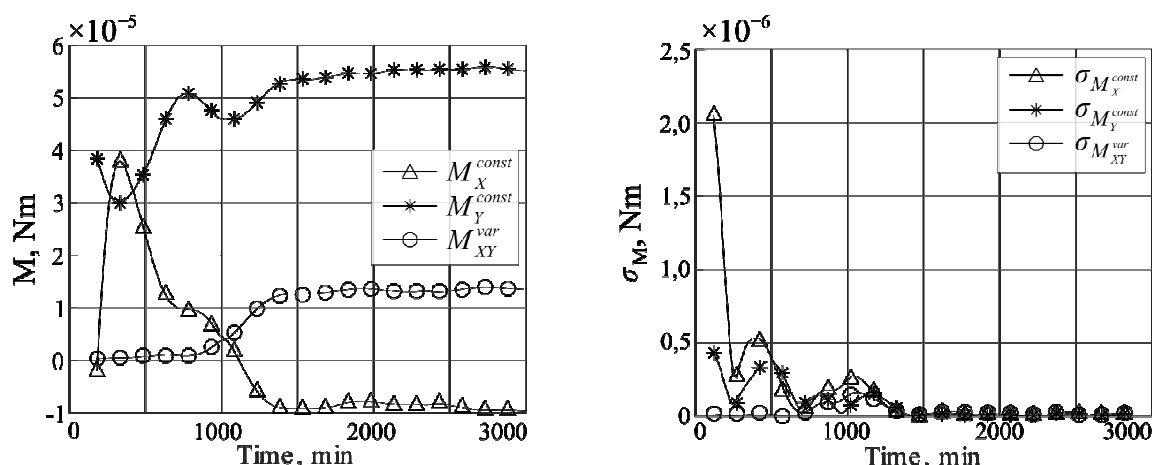


Fig. 9. The graphs of the disturbing torques calculation by the Kalman filter for the heavy class spacecraft

Рис. 9. Графики расчета возмущающих моментов фильтром Калмана для КА тяжелого класса

Table 2

Disturbing torques computing results, Nm

| Disturbing torque name                    | Gaussian filter      |                                | Kalman filter        |                                |
|-------------------------------------------|----------------------|--------------------------------|----------------------|--------------------------------|
|                                           | Value                | MSD                            | Value                | MSD                            |
| Middle class SC                           |                      |                                |                      |                                |
| Constant disturbing torque $M_x^{const}$  | $-1.5 \cdot 10^{-5}$ | $1.6 \cdot 10^{-8}$<br>(0.1 %) | $-1.8 \cdot 10^{-5}$ | $1.2 \cdot 10^{-8}$<br>(0.1 %) |
| Constant disturbing torque $M_y^{const}$  | $7.0 \cdot 10^{-7}$  | $6.6 \cdot 10^{-9}$<br>(0.9 %) | $-1.2 \cdot 10^{-7}$ | $2.4 \cdot 10^{-9}$<br>(2.0 %) |
| Variable disturbing torque $M_{xy}^{var}$ | $2.1 \cdot 10^{-6}$  | $1.3 \cdot 10^{-9}$<br>(0.1 %) | $2.1 \cdot 10^{-6}$  | $1.3 \cdot 10^{-8}$<br>(0.6 %) |
| Heavy class SC                            |                      |                                |                      |                                |
| Constant disturbing torque $M_x^{const}$  | $-1.4 \cdot 10^{-5}$ | $1.3 \cdot 10^{-7}$<br>(0.9 %) | $-6.8 \cdot 10^{-6}$ | $3.4 \cdot 10^{-8}$<br>(0.5 %) |
| Constant disturbing torque $M_y^{const}$  | $5.6 \cdot 10^{-5}$  | $5.5 \cdot 10^{-8}$<br>(0.1 %) | $5.4 \cdot 10^{-5}$  | $3.9 \cdot 10^{-8}$<br>(0.1 %) |
| Variable disturbing torque $M_{xy}^{var}$ | $1.4 \cdot 10^{-5}$  | $5.5 \cdot 10^{-8}$<br>(0.4 %) | $1.3 \cdot 10^{-5}$  | $6.7 \cdot 10^{-8}$<br>(0.5 %) |

**Conclusion.** As a result of the work done the mathematical model considering the spacecraft kinetic moment change, disclosing the interrelation of the flywheel control system kinetic moment and the external disturbing torque was developed. The model differs from the known models for the fact that the constant and variable component of the external disturbing torque is considered separately. A model adequacy check was performed according to telemetric information of middle and heavy class SC by mean-square indications processing methods – by the Gaussian and the Kallman filters.

Mean-square deviation of the disturbing torques assessment is less than 0,9 % for the Gaussian filter and less than 2 % for the Kalman filter.

The obtained mathematical model of the spacecraft external disturbing torques determination allows to predict the change of a flywheels kinetic moment control system that can be used for flywheels unloading effectiveness improvement and respectively to fuel consumption decrease.

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