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RESEARCH OF THE SEMI-MARKOVIAN PROCESS IN CONDITIONS OF LIMITEDLY RARE CHANGES IN ITS STATE*

In this work the SM-flow in conditions of limitedly rare changes in its states is considered. In the proposed asymptotic condition there is a probable distribution of a number of events coming from the SM-flow in time t. We have shown that this distribution can be multimodal.

Keywords: SM-process, state of flow, limit rare changes of flow states, method of the additional variable, asymptotic analysis method.

In this work the SM-process has been considered [1]; it is the general flow of flow models with homogeneous events.

We shall give a definition of the Semi-Markovian (SM) process. For this purpose we have considered a two-dimensional homogeneous Markovian stochastic process $\{\xi(n), \tau(n)\}$ with a discrete time. Here $\xi(n)$ the ercodic Markov chain with a discrete time and matrix $P = [p_{vk}]$ accept the probabilities of transition for one step [2]; the process $\tau(n)$ accepts non-negative values from the continuous set.

Then we determine the Markovian transition function $F(v, x, k, y)$ for the process $\{\xi(n), \tau(n)\}$:

$$F(k, x; v, y) =$$
$$= P\{\xi(n+1) = k, \tau(n+1) < x | \xi(n) = v, \tau(n) = y\}.$$

We shall consider two-dimensional processes such as $\{\xi(n), \tau(n)\}$ for which the following equalities are correct:

$$F(v, x; k, y) = F(v, x; k),$$

that is $F(v, x, k, y)$ does not depend on the values of the y process $\tau(n)$.

Denote

$$F(k, x; v) = A_{vk}(x) =$$
$$= P\{\xi(n+1) = k, \tau(n+1) < x | \xi(n) = v\}. \quad (1)$$

Matrix $A(x)$ with elements $A_{vk}(x)$ can be called Semi-Markovian.

The stochastic process of homogeneous events:

$$t_1 < t_2 < ... < t_n < t_{n+1} < ...$$

is called the Semi-Markovian process or SM, set by matrix $A(x)$; if for moments t_n the approaching of its events is correct, the following equations are performed:

$$t_{n+1} = t_n + \tau(n+1).$$

In view of (1), thee transitive probability matrix of Markov's chain $\xi(n)$ is defined by equation:

$$P = A(\infty).$$

This chain for the Semi-Markovian process is called the embedded Markov chain.

In general case, the elements of the Semi-Markovian matrix have a place in the multiplicative form which can be written as:

$$A_{vk}(x) = P\{\xi(n+1) = k, \tau(n+1) < x | \xi(n) = v\} =$$
$$= P\{\tau(n+1) < x, | \xi(n) = v, \xi(n+1) = k\} \times$$
$$\times P\{\xi(n+1) = k, | \xi(n) = v\} = G_{vk}(x) p_{vk},$$

where $G_{vk}(x)$ – is the conditional distribution function of an interval length of the Semi-Markovian process in condition that at the beginning of this interval the embedded Markov chain has an accepted value v, and at the end of it will accept value k.

Note, due to equation:

$$A_{vk}(x) = G_{vk}(x) p_{vk}, \qquad (2)$$

matrix $A(x)$ can be written as a Hadamard produce:

$$A(x) = G(x) * P \qquad (3)$$

of two matrixes $G(x)$ and P, and it is possible to suppose, that the Semi-Markovian process is set by two matrixes $G(x)$ and P.

The state of the semi-markovian process at the moment of time $t_n < t \le t_{n+1}$ is called state k of its embedded Markov chain, accepted at the beginning of interval $(t_n, t_{n+1}]$.

The research of the Semi-Markovian process will be will carried out in conditions of limit rare changes of (LRCS) flow conditions; i. e. the transition from one state of the embedded Markov chain to another is realized extremely rarely.

The conditions of limit rare condition changes of the Semi-Markovian process are formalized by the following equation for matrix $P(\delta)$ transition probabilities of its embedded Markov chain:

$$P(\delta) = I + \delta \cdot Q, \qquad (4)$$

where δ – is some small parameter ($\delta \to 0$); I – is an identity diagonal matrix.

Matrix Q with elements q_{vk} is similar to a matrix with infinitesimal characteristics and has the same properties: $k \ne v$ matrix elements $q_{vk} > 0$, and also is correct:

$$\sum_k q_{vk} = 0, \ \sum_{k \ne v} q_{vk} = -q_{kk}.$$

The Semi-Markovian matrix for the SM process in condition of LRCS:

1. $k = v$

$$A_{kk}(x, \delta) = P\{\xi(n+1) = k, \tau(n+1) < x | \xi(n) = k\} =$$
$$= G_{kk}(x)\{1 + \delta \cdot q_{kk}\}.$$

2. $k \ne v$

$$A_{vk}(x, \delta) = P\{\xi(n+1) = k, \tau(n+1) < x | \xi(n) = v\} =$$
$$= G_{vk}(x) \cdot \delta \cdot q_{vk}.$$

On the other hand, in the multiplicate notation (3) of the Semi-Markovian matrix let's substitute (4) and get:

$$A(x, \delta) = G(x) * \{I + \delta \cdot Q\}. \qquad (5)$$

Let's state $m(t)$ – as the event number of the Semi-Markovian process, which appeares during t on the interval $[0, t)$.

The process $m(t)$ is non-Markovian, therefore it is necessary to make it Markovian by a method of additional variables. Let's define the process: $z(t)$ – is the length of an interval from time moment t till the moment of approach for the next event in the considered SM-process.

However, the two-dimensional process $\{m(t), z(t)\}$ is not Markovian, therefore let's consider one more

stochastic process $s(t)$ with piecewise constant continuous realizations on the left, defined by equation:

$$s(t) = \xi(n+1), \text{ if } t_n < t \le t_{n+1}.$$

This is on the interval $(t_n, t_{n+1}]$ process, $s(t)$ accepts and assures that value of the embedded Markov chain $\xi(n)$ accepts the beginning of the following interval [1].

The three-dimensional stochastic process is thus defined as $\{s(t), m(t), z(t)\}$ with two additional variables $s(t)$ and $z(t)$ – is Markovian with continuous time and with probability distribution:

$$P(s, m, z, t, \delta) = P\{s(t) = s, m(t) = m, z(t) < z\}, \quad (6)$$

it is simple to create a system of differential Kolmogorov equations:

$$\frac{\partial P(s, m, z, t, \delta)}{\partial t} = \frac{\partial P(s, m, z, t, \delta)}{\partial z} - \frac{\partial P(s, m, 0, t, \delta)}{\partial z} +$$
$$+ \sum_{v=1}^{\infty} \frac{\partial P(v, m-1, 0, t, \delta)}{\partial z} A_{vk}(z), \qquad (7)$$

at the set initial conditions:

$$\begin{cases} P(s, 0, z, 0, \delta) = R(s, z, \delta), \\ P(s, m, z, 0, \delta) = 0, \quad m \ge 1, \end{cases} \qquad (8)$$

where function $R(s, z, \delta)$ – is the stationary distribution of the two-dimensional Markovian process $\{s(t), z(t)\}$.

Let's denote the function:

$$H(s, u, z, t, \delta) = \sum_{m=0}^{\infty} e^{jum} P(s, m, z, t, \delta), \qquad (9)$$

where $j = \sqrt{-1}$ – is an imaginary unit.

For these functions it is possible to write down the following Cauchy problem from system (7) and initial condition (8):

$$\begin{cases} \dfrac{\partial H(s, u, z, t, \delta)}{\partial t} = \dfrac{\partial H(s, u, z, t, \delta)}{\partial z} - \dfrac{\partial H(s, u, 0, t, \delta)}{\partial z} + \\ + e^{ju} \sum_{v=1}^{\infty} \dfrac{\partial H(v, u, 0, t, \delta)}{\partial z} A_{vk}(z, \delta), \\ H(s, u, z, 0, \delta) = R(s, z, \delta). \end{cases} \qquad (10)$$

Let's denote:

$$H(u, z, t, \delta) = \{H(1, u, z, t, \delta), H(2, u, z, t, \delta), ...\},$$

also matrix $A(z, \delta)$ with elements $A_{kv}(z, \delta)$, then from (10) we receive the following:

$$\begin{cases} \dfrac{\partial H(u, z, t, \delta)}{\partial t} = \dfrac{\partial H(u, z, t, \delta)}{\partial z} + \dfrac{\partial H(u, 0, t, \delta)}{\partial z} \times \\ \times \{e^{ju} A(z, \delta) - I\}, \\ H(u, z, 0, \delta) = R(z, \delta), \end{cases} \qquad (11)$$

where I – is an identity matrix, and vector $R(z, \delta)$, is defining the initial condition of problem (11) with components $R(s, z, \delta)$, as shown in [3], given by:

$$R(z,\delta) = \kappa_1(\delta) r \int_0^z \big(P(\delta) - A(x,\delta)\big) dx,$$

where r – is the stationary value probability distribution of the embedded Markov chain $\xi(n)$; magnitude $\kappa_1(\delta)$ – is defined by equation:

$$\kappa_1(\delta) = \frac{1}{rA(\delta)E},$$

where matrix $A(\delta)$ is defined by equation:

$$A(\delta) = \int_0^\infty \big(P(\delta) - A(x,\delta)\big) dx.$$

Asymptotic probability distribution of event numbers, the arrival of which is in the SM-process in time t is found in a condition of limit rare changes for the conditions of process.

Let's denote:

$$\lim_{\delta \to 0} H(u,z,t,\delta) = H(u,z,t). \qquad (12)$$

In problem (11), considering (12) we shall execute a limiting transition at $\delta \to 0$, then $H(u, z, t)$ we will receive in the set of an independent Cauchy problems [3; 4]:

$$\begin{cases} \dfrac{\partial H(u,z,t)}{\partial t} = \dfrac{\partial H(u,z,t)}{\partial z} + \dfrac{\partial H(u,0,t)}{\partial z}\big\{e^{ju}A(z) - I\big\}, \\ H(u,z,0) = R(z), \end{cases} \quad (13)$$

where:

$$\lim_{\delta \to 0} P(\delta) = I, \ \lim_{\delta \to 0} A(z,\delta) = A(z), \lim_{\delta \to 0} A(\delta) = A,$$
$$\lim_{\delta \to 0} \kappa_1(\delta) = \kappa_1, \ \lim_{\delta \to 0} R(z,\delta) = R(z).$$

Considering the kind of matrix $A(z)$, from (12) are we get a set of independent differential equations:

$$\frac{\partial H(s,u,z,t)}{\partial t} = \frac{\partial H(s,u,z,t)}{\partial z} + \frac{\partial H(s,u,0,t)}{\partial z} \times$$
$$\times \big\{e^{ju} G_{ss}(z) - 1\big\}, \qquad (14)$$

the initial conditions are given in the following way:

$$H(s,u,z,0) = R(s,z). \qquad (15)$$

The solution of a problem (14–15) by applying of the transformation of Fourier–Stieltjes:

$$\varphi(s,u,\alpha,t) = \int_0^\infty e^{j\alpha z} d_z H(s,u,z,t). \qquad (16)$$

The function $\varphi(s, u, \alpha, t)$ is satisfied by the following equation:

$$\frac{\partial \varphi(s,u,\alpha,t)}{\partial t} = -j\alpha \varphi(s,u,\alpha,t) + \frac{\partial H(s,u,0,t)}{\partial z} \times$$
$$\times \big\{e^{ju} G_{ss}^*(\alpha) - 1\big\}, \qquad (17)$$

and the initial condition:

$$\varphi(s,u,\alpha,0) = \int_0^\infty e^{j\alpha z} d_z H(s,u,z,0) =$$
$$= \int_0^\infty e^{j\alpha z} d_z R(s,z) = R^*(s,\alpha), \qquad (18)$$

where

$$G_{ss}^*(\alpha) = \int_0^\infty e^{j\alpha z} d_z G_{ss}(z).$$

The solution of the differential equation (17) is:

$$\varphi(s,u,\alpha,t) = e^{-j\alpha t}\bigg\{R^*(s,\alpha) + \int_0^t e^{j\alpha \tau} \frac{\partial H(s,u,0,\tau)}{\partial z} \times$$
$$\times \bigg(e^{ju} G_{ss}^*(\alpha) - 1\bigg) d\tau\bigg\}. \qquad (19)$$

As

$$\lim_{t \to \infty} \varphi(s,u,\alpha,t) = \int_0^\infty e^{j\alpha z} d_z H(s,u,z,\infty) = 0,$$

that, having directed t to infinity in equation (19) we get:

$$0 = R^*(s,\alpha) + \int_0^\infty e^{j\alpha \tau} \frac{\partial H(s,u,0,\tau)}{\partial z}\big(e^{ju} G_{ss}^*(\alpha) - 1\big) d\tau.$$

From this equality we find the Fourier transformation in τ from function $\dfrac{\partial H(s,u,0,\tau)}{\partial z}$:

$$\int_0^\infty e^{j\alpha \tau} \frac{\partial H(s,u,0,\tau)}{\partial z} d\tau = R^*(s,\alpha)\big(1 - e^{ju} G_{ss}^*(\alpha)\big)^{-1}.$$

Having executed the reverse Fourier transformation we identify:

$$\frac{\partial H(s,u,0,\tau)}{\partial z} = \frac{1}{2\pi} \int_{-\infty}^\infty e^{-j\alpha \tau} R^*(s,\alpha)\big(1 - e^{ju} G_{ss}^*(\alpha)\big)^{-1} d\alpha.$$

Equation (19) considering the received transformation will be:

$$\varphi(s,u,\alpha,t) = e^{-j\alpha t}\bigg\{R^*(s,\alpha) + \int_0^t e^{j\alpha \tau} \frac{1}{2\pi} \times$$
$$\times \int_{-\infty}^\infty e^{-jy\tau} R^*(s,y)\big(1 - e^{ju} G_{ss}^*(y)\big)^{-1} dy\big(e^{ju} G_{ss}^*(\alpha) - 1\big) d\tau\bigg\}. \quad (20)$$

Knowing, that $H(s, u, \infty, t) = H(s, u, t) = \varphi(s, u, 0, t)$, $R^*(s, 0) = \kappa_1 r_s A_{ss}$, $G_{ss}^*(0) = 1$, we get an expression for function $H(s, u, t)$:

$$H(s,u,t) = \kappa_1 r_s A_{ss} + \frac{1}{2\pi} \int_{-\infty}^\infty \int_0^t e^{-jy\tau} d\tau R^*(s,y) \times$$
$$\times \big(1 - e^{ju} G_{ss}^*(y)\big)^{-1} dy\big(e^{ju} - 1\big). \qquad (21)$$

Considering:

$$\int_0^t e^{-jy\tau} d\tau = \frac{1}{jy}\big(1 - e^{-jyt}\big),$$

and:

$$R^*(s,y) = \int_0^\infty e^{jyz} d_z R(s,z) =$$

$$= \kappa_1 r_s \int_0^\infty e^{jyz} d \int_0^z (1 - G_{ss}(x)) dx = \frac{\kappa_1 r_s (G_{ss}^*(y) - 1)}{jy}.$$

We shall denote the equation (21) as:

$$H(s,u,t) = \kappa_1 r_s A_{ss} + \frac{\kappa_1 r_s}{2\pi} \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \times$$

$$\times (1 - G_{ss}^*(y))(1 - e^{ju} G_{ss}^*(y))^{-1} dy (e^{ju} - 1). \qquad (22)$$

Let's denote $h(u,t) = \sum_s H(s,u,t)$ as:

$$h(u,t) = \sum_s H(s,u,t) = 1 + \frac{(e^{ju} - 1)\kappa_1}{2\pi} \times$$

$$\times \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \sum_s r_s (1 - G_{ss}^*(y))(1 - e^{ju} G_{ss}^*(y))^{-1} dy.$$

Knowing, that $h(u,t) = \sum_s H(s,u,t) \approx \sum_s H(s,u,t,\delta) =$

$$= \sum_{n=0}^\infty e^{jun} P(n,t),$$ and sorting it to the right part of the received equation of the exponent levels e^{ju}, it is possible to write down the following asymptotic equation as:

$$h(u,t) \approx \sum_{m=0}^\infty e^{jum} P(m,t) = 1 + \frac{(e^{ju} - 1)\kappa_1}{2\pi} \times$$

$$\times \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \sum_s r_s (1 - G_{ss}^*(y)) \sum_{m=0}^\infty e^{jum} G_{ss}^{*\,n}(y) dy =$$

$$= 1 - \frac{\kappa_1}{2\pi} \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \sum_s r_s (1 - G_{ss}^*(y)) dy +$$

$$+ \frac{\kappa_1}{2\pi} \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \sum_s r_s (1 - G_{ss}^*(y))^2 \sum_{m=1}^\infty e^{jum} G_{ss}^{*\,m-1}(y) dy.$$

Then the asymptotic probabilities distribution of possibility $P_1(m,t)$, number of events m which start in the SM time process t, look the following way:

$$\begin{cases} P_1(0,t) = 1 - \frac{\kappa_1}{2\pi} \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \sum_s r_s (1 - G_{ss}^*(y)) dy, \\ P_1(m,t) = \frac{\kappa_1}{2\pi} \int_{-\infty}^\infty \frac{1}{y^2} (1 - e^{-jyt}) \sum_s r_s (1 - G_{ss}^*(y))^2 \times \\ \times \sum_{m=1}^\infty e^{jum} G_{ss}^{*\,m-1}(y) dy, \end{cases} \qquad (23)$$

where

$$P_1(m,t) \approx P(m,t).$$

Above were obtained formulas that allow us to find the asymptotic probability distribution of the number of events, which happen in the SM-process in time t. This distribution was also found in prelimit situations [1], by

using methods of integrated transformation. It is necessary to find out how close the results received by the asymptotic method of analysis are to the results received in prelimit situations. For this purpose there is a magnitude $\Delta = \max_n \left| \hat{F}(n,t) - F(n,t) \right|$, where $\hat{F}(n,t)$ – is the function of distribution, obtained from the asymptotic analysis; $F(n,t)$ – is the function of distribution for prelimit situations, received in [1].

Let:

$$Q = \begin{cases} -3 & 1 & 2 \\ 3 & -4 & 1 \\ 4 & 2 & -6 \end{cases},$$

$$G(x) = \begin{cases} 1 - e^{-5x} & 1 - e^{-2x} & 1 - e^{-10x} \\ 1 - e^{-4x} & 1 - e^{-x} & 1 - e^{-9x} \\ 1 - e^{-6x} & 1 - e^{-3x} & 1 - e^{-8x} \end{cases}, \quad t = 6.$$

From table it can be seen that at the given parameters of values the magnitude Δ are:

Deviation results of the asymptotic from prelimited analysis

δ	0,001	0,0005	0,0001	0,00005	0,00001
Δ	0,0427	0,0216	0,0042	0,0021	0,0003

On fig. 1–3 are the probability distributions of the number of events in the SM-process, which occurred in time $t = 6$, obtained by using the asymptotic analysis and the prelimit situation.
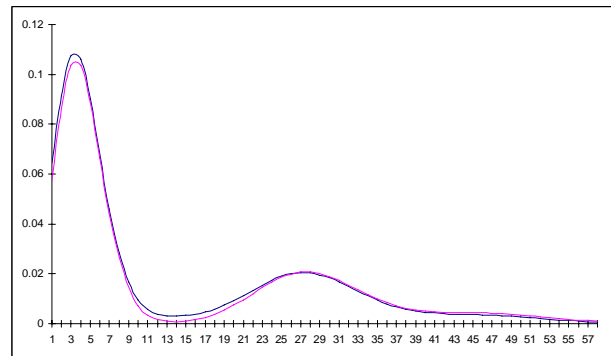

Fig. 1. Distribution probabilities of a number of events
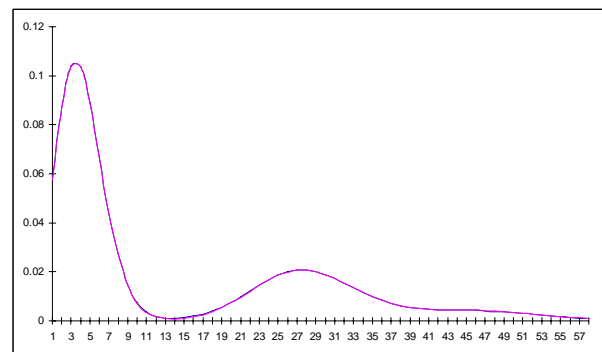for the SM-flow at $\delta = 0{,}001$

Fig. 2. Distribution probabilities of a number of events
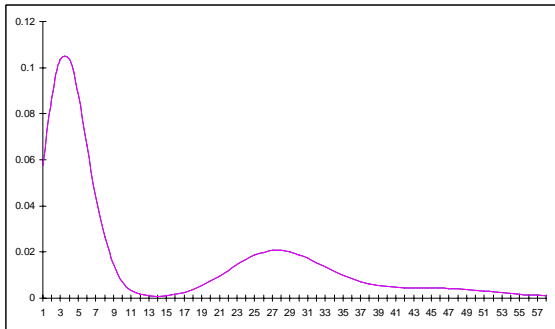for the SM-flow at $\delta = 0{,}0001$

Fig. 3. Distribution probabilities of a number of events
for the SM-flow at $\delta = 0,00001$

In this research we have found the distribution of the asymptotic probability for a number of events occurring in the SM-flow in time *t*. By reducing the parameters δ, the deviation results of the asymptotic analysis for the prelimited varies: at $\delta \leq 0,0001$ it is equal to less than 1 %. Also, it is necessary to notice, that the obtained distribution is multimodal.

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**MECHANICAL ANALYSIS OF ONBOARD EQUIPMENT AND THE ISSUE
OF FINITE-ELEMENT MODEL ADEQUACY**

In this article we have considered issues related to the applied software package, designed to conduct onboard mechanical analysis of spacecraft equipment using FEM, together with an algorithm that simplifies the aforementioned models and estimates their correctness.

Keywords: hardware and software complex, mechanical analysis, satellite-borne equipment, simplified model, finite-element model.

The design of a complicated high-tech product, which includes space onboard equipment, is based on the currently used analysis methods. This allows building complicated structures and analyzing them thoroughly. Today these methods are successfully used is computers and modern CAE systems (Computer-Aided Engineering – engineering analysis systems). By applying this method, difficulties during the design and the strength analysis could be resolved, by bringing design models much closer to the existing structures.

The Finite Element Method (FEM) is one of the most universal methods of mechanical analysis, which is performed for different structures. It is, in fact, a variation method used to calculate detailed irregular models. Its capabilities of constructing overall systems, which can calculate all structural parts without separating the elements of the structure, give this method more advantages. The design of onboard equipment is characterized by the tendency of its design model improvements; this can be explained in two ways:

– in some cases it is possible to significantly reduce the mass of the onboard equipment, owing to more a detailed analyses of voltage and the distorted construction;

– this analysis is often the only method to estimate structure behavior in space conditions and on the way to space after onboard equipment simulation; ground operations in most cases are extremely limited.

Nevertheless, together with a number of beneficial characteristics FEM also has some disadvantages. The continuous progress in computational technologies stipulates the building of more detailed models, in the size of which there could be approximately 10^5 equation members. It is easier to perform a dynamic analysis of a reduced model while calculating the detailed models; this is still restricted in terms of the obtained results validity.

To eliminate such difficulties, the created applied programs packages are used to perform mechanical analysis, including analysis of the reduced models. ASONIKA-TM is the most popular product used for these purposes. It is used for analyzing mechanical characteristics of the upper level structures (cabinet, rack, and unit). Printed circuit boards with lumped EEE parts and associated material data package are not taken in