Disadvantages:
– algorithms are complicated;
– there are high system requirements.

Except for the algorithm of segmentation itself in this work there were described some other new ideas:
– the method of multi scale analysis with extraction of frequency information;
– the algorithm of two-dimensional function minimization which uses morphological filling;
– combination of areas growing and areas uniting;
– the criterion of the definition of the optimal moment to stop uniting.

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RESULTS OF COMPUTING EXPERIMENTS FOR WATER ECOLOGICAL SYSTEM MATHEMATICAL MODELING

The point-wise imitation and one-dimensional mathematical models of aquatic ecosystems have been overlooked. The developed models are intended for studying ecosystems in the Krasnoyarsk aquatic basin and in separate locations on the Yenisei River. The results of the computing experiments are presented.

Keywords: mathematical model, mathematical modeling of aquatic ecosystems, computing experiment.

Environmental issues have a designated place in the general list of issued for which mathematical modeling is used. The increase of the anthropogenic environmental impact, caused by intense exploitation of natural resources and the growth of industry leads to an ecological balance disruption. This is happening both on local (in separate areas of globe) and on planetary scale. The importance of struggling against anthropogenic eutrophication of reservoirs and their pollution is understood everywhere in the world. There had been a great amount of researches in limnology, mathematical modeling, and economy, connected with problems of preservation, restoration, and the effective exploitation of natural resources, such as lakes and manmade reservoirs. The ecological condition of the water bodies depends on a number of various factors and processes: hydrophysical, hydrobiological, hydrochemical, meteorological, and anthropogenic. Hydrophysical processes appreciably form a habitat of hydrobionts, define the transferred and sedimentation of substances, the intensity of pollution, and the self-cleaning of reservoirs.

The problem of water quality is complicated. Water bodies are complex physical, biochemical and ecological systems. To be able to predict the consequence of one decision or another, the corresponding tool by dint of which it is possible to analyze the sufficiency of information is required. Such a tool is the computing experiment based on mathematical modeling and numerical methods. An effective means of the arising problem objective analysis in the field of hydrobiology problems are the methods based on constructing and studying mathematical models of water ecosystems. The using of mathematical modeling and carrying out computing experiments allows us to predict the dynamics of water ecosystem development, and also to estimate the consequences of realizing various projects, connected with influence on the ecosystem.

A number of general claims to each mathematical model are known: the corresponding system of the equations should be closed and consistent; the model should describe a variety of physical phenomena and suppose the designing of realized numerical algorithm.

In the given work, some results of the calculations, carried out with a mathematical model of the water ecosystem (being an improvement of the model considered in [1]) are presented. The model is modified by the separation of green algae as independent components of a mathematical model and the introduction of an additional equation, describing the change in algae concentration.

As dynamic variables of model, the concentrations of green algae (CA0), of blue-green algae (CA1), of diatoms (CA2), of zooplankton (CZ), of bacteria (CB), of detritus (CD), of the inorganic phosphorus dissolved in water (PS), of the inorganic nitrogen dissolved in water (NS), of the organic matter dissolved in water (POB), and of the oxygen dissolved in water (O2) are taken.

In model the following processes are considered:
– growth of microorganisms;
– outflow of products of a metabolism;
– death rate of microorganisms;
– processes of settling;
– transitions on a trophic chain;
– decomposition processes;
– atmospheric reaeration (isolation of oxygen from water);
– denitrification (process of restoration of nitrates to the molecular nitrogen, caused by bacteria);

– limiting factors (illumination, temperature);

– water aeration (saturation of water by oxygen of air).

The main feature of the given model is the division of blue-green algae into two species: greens and blue-greens; this isn't presented in many models, but is of great importance for the research of the reservoirs' ecology, for their development is various.

The model allows predicting the dynamics of water ecosystem development; including the transformations of nitrogen and phosphorus, as basic biogenous elements, defining the efficiency and water quality in reservoirs.

The structure of model describing the functioning of an ecosystem is given in the flow chart (fig. 1).

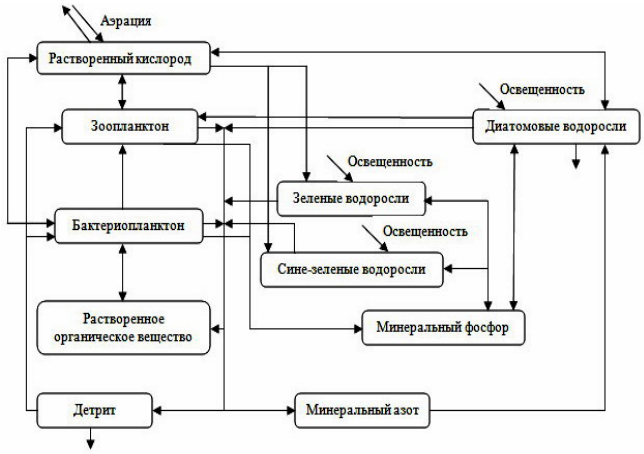

Fig. 1. Flow chart of the model. Arrows designate directions of substance streams between ecosystem components

On the basis of the flow chart the imitating model, describing the considered processes in the ecosystem is constructed. The mathematical model represents the following system of the ordinary differential equations with corresponding initial data:

$$\frac{dCA0}{dt} = (mA0 - RA0 - MA0)\cdot CA0 + \alpha_0 \cdot CA1 \cdot CA0,$$

$$\frac{dCA1}{dt} = (mA1 - RA1 - MA1)\cdot CA1 - \alpha_1 \cdot CA1 \cdot CA0,$$

$$\frac{dCA2}{dt} = (mA2 - RA2 - SA2 - MA2)\cdot CA2 - \frac{mZ \cdot CZ}{Y1},$$

$$\frac{dCZ}{dt} = (mZ - RZ - MZ)\cdot CZ,$$

$$\frac{dCB}{dt} = (mB - RB - MB)\cdot CB - \frac{mZ \cdot CZ}{Y2},$$

$$\frac{dCD}{dt} = MA0\cdot CA0 + MA1\cdot CA1 + MA2\cdot CA2 + MZ\cdot CZ +$$

$$+ MB\cdot CB \ - SA3 * CD - \frac{mB\cdot CB}{Y3} - \frac{mZ\cdot CZ}{Y4}, \qquad (1)$$

$$\frac{dPS}{dt} = -(mA0 - RA0)\cdot PP0\cdot CA0 - (mA1 - RA1)\times$$

$$\times PP1\cdot CA1 \ - (mA2 - RA2)\cdot PP2\cdot CA2 +$$

$$+ RZ\cdot CZ\cdot PP3 + RB\cdot CB\cdot PP4,$$

$$\frac{dNS}{dt} = RA0\cdot PN0\cdot CA0 + RA1\cdot PN1\cdot CA1 - (mA2 - RA2)\times$$

$$\times PN2\cdot CA2 + RZ\cdot CZ\cdot PN3 + RB\cdot CB\cdot PN4,$$

$$\frac{dPOB}{dt} = -\frac{mB\cdot CB}{Y5} + h0\cdot RA0\cdot CA0 + h1\cdot RA1\cdot CA1 +$$

$$+ h2\cdot RA2\cdot CA2 + h3\cdot RZ\cdot CZ + h4\cdot RB\cdot CB,$$

$$\frac{dO2}{dt} = K1\cdot(O2O - O2) + K_{acc}\cdot(mA0\cdot CA0 + mA1\times$$

$$\times CA1 + mA2\cdot CA2) \ - alf\cdot(RA1\cdot CA1 + RA2\cdot CA2 +$$

$$+ \ RZ\cdot CZ + RB\cdot CB) - B1\cdot mZ\cdot CZ.$$

where mAi are functions describing growth; coefficients RAi are breath; MAi are the death rate; SAi is the settling; Yi are proportionality coefficients; T in temperature in C^0; t is time.

In the live description the incoming and proceeding streams' components are taken into account. Also is included the share of received resources (food) spent for growth and reproduction; proceeding is the consumption of species from given components; predators and death rate depending on every other possible reason. Meanwhile, the influence on the stream speed of the environment (temperature, etc.) is considered.

In microbiological systems as a rule, the growth rate is limited by a concentration of substrates. We have applied the hyperbolic dependence offered by Z mono for the description of the limitation process.

It is supposed, that the growth of green and blue-green algae is limited by phosphorus, while the growth of diatoms – by nitrogen and phosphorus. The growth functions, death rate, illumination, and temperature dependence, as well as all entrance data are included according to researches [1–3].

The constructed mathematical model represents the Cauchy problem for a system of ten ordinary differential equations. For the numerical solution of the Cauchy problem, the Runge–Kutta method of the fourth approximation order is applied:

$$\vec{y}_{n+1} = \vec{y}_n + \frac{1}{6}\tau(\vec{K}_1 + 2\vec{K}_2 + 2\vec{K}_3 + \vec{K}_4),$$

$$\vec{K}_1 = \vec{F}(t_n, \vec{y}_n),$$

$$\vec{K}_2 = \vec{F}(t_n + \frac{\tau}{2}, \vec{y}_n + \tau\frac{\vec{K}_1}{2}),$$

$$\vec{K}_3 = \vec{F}(t_n + \frac{\tau}{2}, \vec{y}_n + \tau\frac{\vec{K}_2}{2}),$$

$$\vec{K}_4 = \vec{F}(t_n + \tau, \vec{y}_n + \tau\vec{K}_3), \ n = 0, 1, \ldots,$$

where $\vec{y}$ is a vector function of unknown; $\vec{F}$ is the right part of system (1); τ is a step in time; $\vec{y}_0$ is specified.

Let's note that the set of components in the model considerably complicates the problem, both the modeling, and in studying the model; as it is required to specify its value for each coefficient (fig. 2).

A complex of the programs is written, allowing the inputting of entrance data in an interactive mode. The calculation results can be received numerically, presented

graphically, and transferred outside for subsequent processing. For the management of graphic representation of calculation results, a corresponding menu is provided.

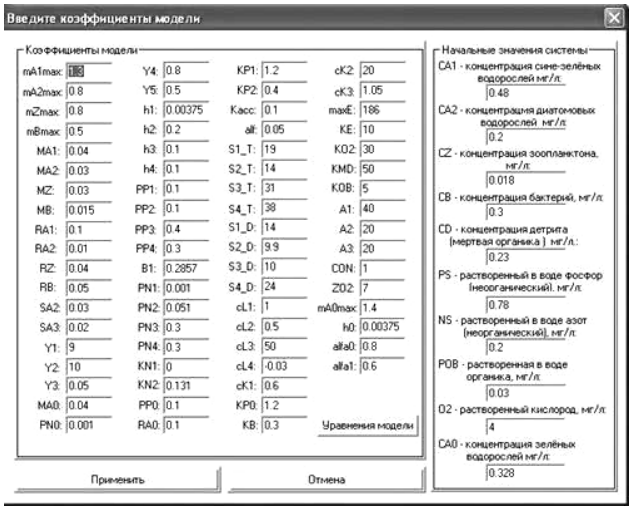

Fig. 2. Coefficients of modeling and initial system values

The program's complex is realized in Visual C ++ 6,0 with the use of MFC (Microsoft Foundation Classes) which is one of the most convenient and powerful tools among Windows' applications. The software product has a friendly interface, it is convenient in work, and is intended not only for mathematicians, but also for researchers who are not experts in programming.

The first numerical experiments have been devoted to the comparative analysis of results, obtained by means of a working mathematical model [1] and by its updating means (1) with same input data [4]. The calculation results have shown that the received concentrations of diatoms, bacteria, and detritus for a working model [1] poorly correspond to experimental data, in comparison with the results obtained from the model aforementioned.

Thus, the computing experiments that have been carried out have shown the effectiveness of separating green algae as an independent variable of the mathematical model for the reservoir ecosystem.

The following calculations with an improved mathematical model are meant for researching general tendencies of seasonal dynamic variable change for a model, using field data from the Novoselovsky reach of the Krasnoyarsk impoundment for 1998–2000. Notice that for the comparison of calculations results, we have used only the field data, the time moments of which are precisely known. During other time periods, due to the incompleteness of existing information, the comparative analysis of average data also shows qualitative calculation coincidence.

Particularly seasonal dynamics of diatoms demonstrate a qualitative picture of two "flowering" peaks: summer – with the maximum biomass of 5.9 mg/l, and autumn with the maximum biomass of 2.27 mg/l; this corresponds with the supervision data [5]. The total biomass of diatoms according to supervisions in July and August decreases to 1.2–2 mg/l. Model calculation has also shown a falling in values of biomass during this period (fig. 3).

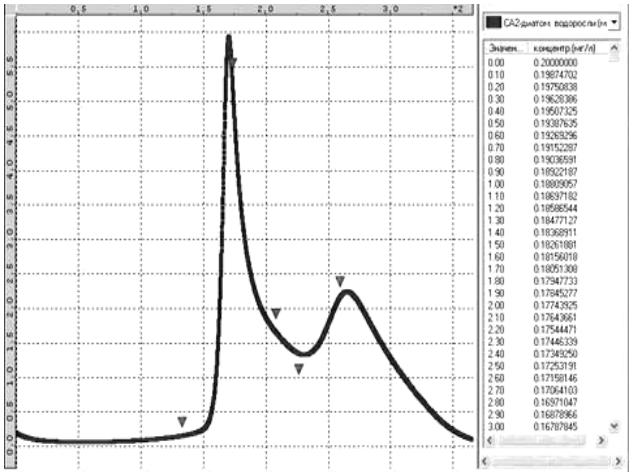

Fig. 3. Comparison of experimental data with numerical results for diatoms. Natural data is marked by triangles

The comparison of a seasonal course model for green and blue-green algae and experimental data [6], has shown that in a general understating of model concentration (approximately by 1.5 times) the relative time course had been precisely reconstructed (fig. 4). The annual course of biomass for zooplankton has a single peak and corresponds to the maximum values for green and blue-green algae biomasses, which also correspond with theoretical representations.

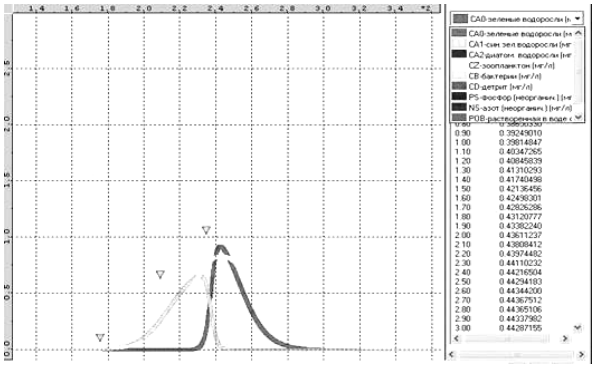

Fig. 4. Comparison of experimental data with numerical results for blue-green algae. Natural data are marked by triangles

For a seasonal course of chemical combinations of nitrogen and phosphorus concentrations, the calculation has shown the falling of values of nitrogen compound concentration during seasons, when intensive plankton growth occurs, and the maximum values at minima of plankton biomass.

The annual course of organic matter concentration has two expressed maxima with values of 0.29 mg/l in the beginning of the summer; and 0.54 mg/l in the autumn. These periods correspond to the maximum values of

phytoplankton and zooplankton biomasses. For seasonal dynamics of bacteria and detritus, two peaks of development are typical: the first falls in the middle of the summer, the second – in the beginning of autumn. Thus, detritus influence the growth of bacteria and stimulate their productivity, which also corresponds to natural data.

The obtained calculation model coincides with experimental data, which testifies the adequacy of the examined model.

The one-dimensional model of the aquatic ecosystem. Along with point-wise model the mathematical model allowing mass transfer along the length of a reservoir (one-dimensional model in a horizontal plane) is examined. The mathematical model represents a differential equation system in partial derivatives of the first degree:

$$\frac{\partial \vec{U}}{\partial t} + V\frac{\partial \vec{U}}{\partial x} = \vec{F}(t, x, \vec{U}) \qquad (2)$$

with corresponding initial and edge conditions:

$$\vec{U}(t,0) = \vec{U}_0(t),$$

$$\vec{U}(0,x) = \vec{U}_1(x).$$

where V is the current speed of the reservoir; x is the spatial variable corresponding to the length of a reservoir; t is time; the right part of the equations (2) corresponds to the right part (1). In such modeling it is supposed, that the substance is evenly distributed along the width of the stream and moves with the average speed of the stream. So, the data is averaged for the depth and width of a reservoir. We will notice that the given model is expedient for using in the case when the length of a reservoir is greater than its width.

The introduced mathematical model is also realized numerically by means of an implicit difference scheme:

$$\frac{\vec{y}_j^{\,n+1} - \vec{y}_j^{\,n}}{\tau} + V\frac{\vec{y}_j^{\,n+1} - \vec{y}_{j-1}^{\,n+1}}{h} = \vec{F}_j^{\,n}.$$

For initial data, the data from the point-wise model is taken. For edge conditions – the solutions obtained from the point-wise model are used.

Initial distribution of all system components is considered uniform. The calculation was carried out for time from t = 0 to t = 365 (one year), for the Novoselovsky reach of the Krasnoyarsk impoundment, and also for sites on the Yenisei River downstream from the Krasnoyarsk Hydroelectric Power Plant with average current speed of the Yenisei of 1.2 km/h on a distance from ten to one hundred kilometers. In fig. 5 particularly, the results for bacteria calculations on a river site are given. It is visible, that under the specified conditions a concentration of bacteria in the chosen part of a reservoir changes considerably (we suppose that this difference of values is caused by the current).

Notice that the ecosystem of the Yenisei on the site adjoining the power plant is strongly impoverished because of the destroying action of the plant's turbines and the low water temperature. The self-cleaning process of water in this heavily polluted site is weakened. The zooplankton in process of substance decomposition plays an insignificant role. Bacterial mass cumulates which here, undergoes intense decomposition only at the inlet stream of the Angara.

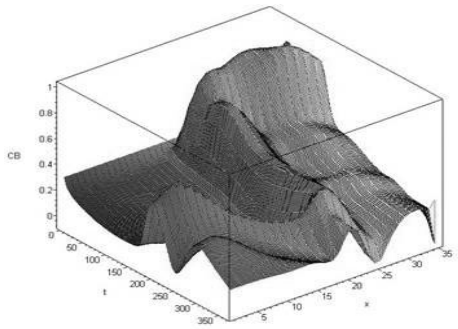

Fig. 5. Results of calculations for the bacteria carried out in a one-dimensional mathematical model

Calculations have shown that the further the distance is from the Krasnoyarsk Hydroelectric Power Plant, the more change occur in concentration, selected as a component; in particular, there is a shift of peaks for diatoms, bacteria, and detritus. At a distance from the power plant of over one hundred kilometers, the model depicts the dynamics of water ecosystem development in space less adequately. This is caused by the studying of quality water characteristics; it is necessary to take in account the more difficult and full processes: hydrodynamic (convective stirring, pressure, wind, and deep currents), heat transfer and illumination. The use of the models based on various variants of mechanic equations for liquid and heat transfer, and corresponding boundary conditions is necessary [7].

Let's note the basic results obtained in the work:

Point-wise and one-dimensional mathematical models of water ecosystems consisting of ten differential equations had been constructed. The use of models allows revealing of processes progressing dynamics in difficult ecological water systems, to predict a system status in time and in space (the one-dimensional model in a horizontal plane) for a distance up to one hundred kilometers, and to analyze problematic situations. The models in particular, make it possible to describe the change of hydrobionts and the basic biogenic elements, and also to reproduce occurrence situations, both for one and for two peaks of phytoplankton flowering during the vegetative season, depending on external conditions. It is necessary to notice, however, that the offered mathematical models are very sensitive to changes in parameters and demand a meticulous selection of coefficients for each specific aquatic ecosystem.

The results obtained by means of the models described above, can as well be used as well for estimating ecological risks.

A complex of computer programs has been produced, and the numerical simulation of some processes progressing in the ecosystem of the Novoselovsky reach of the Krasnoyarsk impoundment and in the Yenisei River had been carried out.

The authors express their gratitude to Professor Z. G. Gold for his useful consultations.

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ON PROPERTIES OF KNAPSACK SYSTEMS OF INFORMATION PROTECTION WITH THE OPEN KEY IN Z_p

Properties of sequences of numbers expressed through components of a knapsack vector are investigated. Properties of isomorphic and similar knapsack systems of information protection are analyzed. Methods of increasing cryptographic security of knapsack systems of information protection with an open key are presented.

Keywords: a knapsack vector, isomorphism, cryptoanalysis, density, injectivity.

Let's express a set of natural numbers {0, 1, …, p–1} through Z_p and a set of all numerical sets of length n with components from Z_p. through Z_p^n .

A knapsack problem for set $w \in N$ and vector $A = (a_1, a_2, ..., a_n)$, where $a_i \in N$, $I = 1…n$, has the solution in Z_p if there is an equation solution

$$Ax^T = w, x \in Z_p^n \qquad (1)$$

we will call vector A of equations (1) a knapsack vector.

A knapsack vector $A = (a_1, a_2, ..., a_n)$ is an injective one if for any natural w the equation (1) has not more than one solution. A knapsack vector which has two elements $a_i = a_j$, $I \neq j$, is not injective. Injectivity of a knapsack vector allows to speak about uniqueness of restoration of the original text according to the cryptogram. Supergrowing knapsack vectors are the simplest injective knapsack vectors from the point of view of understanding and algorithmization. For their components in Z_p the following relationships are carried out:

$$a_j > \sum_{i=1}^{j-1}(p-1)a_i, j = 2...n \qquad (2)$$

A knapsack vector $A = (a_1, a_2, ..., a_n)$ is a nondecreasing one if its components are ordered according to the rule $a_{i-1} \le a_i$, $I = 2…n$. Accordingly, the vector is increasing if its components are ordered according to the rule $a_{i-1} < a_i$, $I = 2…n$.

Definition. Let's call vector $\Delta A = (\delta_1, \delta_2, ..., \delta_n)$ a variation of vector $A = (a_1, a_2, ..., a_n)$ ($a_i \in N$, $I = 1…n$) in Z_p, For its components the following correlations are carried out:

$$\delta_1 = a_1, \ \delta_j = a_j - \sum_{i=1}^{j-1}(p-1)a_i, j = 2...n. \qquad (3)$$

On the basis of vector ΔA it is possible to define a knapsack vector A in Z_p corresponding to it:

$$a_1 = \delta_1,$$
$$a_i = \delta_i + (p-1)\sum_{j=1}^{i-1} a_j = \delta_i + (p-1)\sum_{j=1}^{i-1} p^{i-j-1}\delta_j,$$
$$I = 2...n. \qquad (4)$$

Let's express a set of various values w for which equation (1) has the solution through $\mu(p, A)$. Capacity $\mu(p, A)$ does not exceed p^n since the quantity of various