operator to translate the codes of WD values read from the IML.

Introduction results of the automation procedures. The developed software has completely solved the tasks in view of independent working IML, and thanks to the automation procedures, allowing:

– to automate the process of independent working IMLs to 90 %, leaving the operator only the performance and the analysis of specific checks;

– to reduce time spent for working out concrete IMLs, from several weeks to 1–2 working days;

– to spend simultaneous working to 8 IMLs as a part of a workplace, connected among each other on interblock sockets and connected to the CPM;

– to check the working IML capacity during irregular situations, by their modelling;

– to independently fulfill each complete the IML set (basic/reserve) connected to each complete the CPM set (basic/reserve);

– to fulfill the BM in gathering, with use regular cables as the IML connections;

– to use the PM at any stage of REE tests thanks to flexibility and universality.

Currently, the given software – the independent working IML is used in space vehicle management blocks – “Monsoon”, “Glonass-to”, “Amos-5”; and is used at the working SV “Luch-5”. During the tests, the correctness of the construction of the software and correctness of the approach connected with design of the automated procedures have been confirmed.

Thus, the developed software has proved its reliability, universality, and simplicity in use, thanks to what it is applicable for the working of the subsequent IML management block of perspective SVs. The procedures of the automated software and their algorithms are applicable for designing the software of workings REEs.

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A MULTIDIMENSIONAL ANALOG OF THE COOLEY-TUKEY FFT ALGORITHM

In this article a recurring sequence of orthogonal basis in the n-dimensional case has been applied to derive formulas of n-dimensional fast Fourier transform algorithm, which uses $\frac{2^n-1}{2^n}N^n\log_2 N$ *complex multiplication and* $nN^n\log_2 N$ *complex addition; where* $N=2^s$ *– is a number of counts on one of the axes.*

Keywords: space of signals, orthogonal basis sequence, multidimensional discrete Fourier transform.

Recurrent sequence of orthogonal bases in space of signals is well studied [1] and has numerous applications, including the derivation of Fourier’s formulas of fast transformation.

In this article the recurrent sequence of orthogonal bases to a *n*-dimensional case is applied in order to derive formulas of a fast *n*-dimensional Fourier transformation variant, using $\frac{2^n-1}{2^n}N^n\log_2 N$ complex multiplication and $nN^n\log_2 N$ complex addition, where $N=2^s$ – is a number of counts on one of the axes (known in studies as in [2]). This variant n FFT contains a smaller number of complex multiplication operations than other algorithms, where the multidimensional Fourier transformation is carried out by repeated application of one-dimensional FFT (for example, see [3; 4]).

Furthermore, we give definitions and basic statements from the theory of multidimensional signals, which are used in the article.

To construct *n*-dimensional recurrent sequence of orthogonal bases we use the scheme of the statement, given in [1] for a one-dimensional case.

1. The space of periodic n-dimensional signals.

Definition 1. With a fixed N, the n-dimensional periodic signal shall be a periodic complex function of integer argument, with the period N on each variable.

Define operations of adding the two signals x_1, x_2 and multiplying the signal x by a complex number c:

$$y(j) = x_1(j) + x_2(j);$$
$$y(j) = c \cdot x(j),$$

where $x(j)$ – is the count of a signal x at point $j \in \mathbb{Z}^n$. Then, a set of signals $\mathbb{C}_N^n$ becomes a linear complex space. A zero element in $\mathbb{C}_N^n$ is the signal O such, that $\mathrm{O}(j) = 0$ for all is $j \in \mathbb{Z}^n$. Scalar produce and norm of space $\mathbb{C}_N^n$ are:

$$\langle x, y\rangle = \sum_{j \in B_n(N)} x(j)\overline{y(j)},$$
$$\| x \| = \langle x, x\rangle^{1/2},$$

where $B_n(N)$ – is a set of integer vectors from $[0, N-1]^n$.

Definition 2. The unit n-dimensional periodic impulse, with the period N on each variable, is a signal δ_N^n such, that $\delta_N^n(j) = 1$, if each coordinate of a vector j divided by N and $\delta_N^n(j) = 0$ otherwise.

The Following statements are true for a unit impulse.

1) $\delta_N^n(j_1,\dots,j_n) = \delta_N^n(|j_1|,\dots,|j_n|)$;

2) $\delta_N^n(j_1,\dots,j_n) = \delta_N^1(j_1)\cdot\ldots\cdot\delta_N^1(j_n)$;

3) For $x \in \mathbb{C}_N^n$ the equality is true:

$$x(j) = \sum_{t \in B_n(N)} x(t)\delta_N^n(j-t), \tag{1}$$

for any $j \in B_n(N)$.

Let $w_N = \exp(\frac{2\pi i}{N})$.

Lemma 1. Then

$$\delta_N^n(j) = \frac{1}{N^n}\sum_{t \in B_n(N)} w_N^{(j,t)}, \tag{2}$$

where (j,t) – is the scalar product of vectors j and t.

Equation is checked (2) by direct calculation.

Definition 3. The n-dimensional discrete Fourier transform is called a depiction: $F_N : \mathbb{C}_N^n \to \mathbb{C}_N^n$ take each signal x to a signal X, where:

$$X(j) = \sum_{t \in B_n(N)} x(t) w_N^{-(j,t)}, j \in B_n(N).$$

Note that, for DFT the formula of inversion is true:

$$x(t) = \frac{1}{N^n}\sum_{j \in B_n(N)} X(j)\cdot w_N^{(j,t)}$$

and the Parseval identity:

$$\text{if } X = F_N(x),\ Y = F_N(y),$$
$$\langle x, y\rangle = \frac{1}{N^n}\langle X, Y\rangle.$$

2. The recurrent sequences of orthogonal bases.

Let $N = 2^s$, $N_\nu = 2^{s-\nu}$, $\Delta_\nu = 2^{\nu-1}$. We shall construct recurrent sequence of bases $f_0, f_1, \dots, f_s$, where f_t – t-th basis, consisting of N^n signals $f_t(k)$, $k \in B_n(N)$. We will denote a value of a signal $f_t(k)$ at count $j = (j_1,\dots,j_n)$, $j \in B_n(N)$ by $f_t(k;j)$.

Let $B_n^1(N)$ by a set of integer vectors from $[0, N_\nu - 1]^n$ and $B_n^2(N)$ by a set of integer vectors from $[0, \Delta_\nu - 1]^n$. We will define the sequence of orthogonal bases as:

$$f_0(k;j) = \delta_N^n(j-k) = \delta_N^1(j_1-k_1)\times$$
$$\times\delta_N^2(j_2-k_2)\dots\delta_N^n(j_n-k_n), k, j \in B_n^{(N)}.$$
$$f_\nu(l_1+\sigma_1\Delta_\nu+p_1\Delta_{\nu+1}, l_2+\sigma_2\Delta_\nu+p_2\Delta_{\nu+1},\dots,l_n+\sigma_n\Delta_\nu+p_n\Delta_{\nu+1}) =$$
$$= \sum_{\tau_1=0}^{1}\dots\sum_{\tau_n=0}^{1} w_{\Delta_{\nu+1}}^{\sum_{i=0}^{n}\tau_i(l_i+\sigma_i\Delta_\nu)} f_{\nu-1}\times$$
$$\times(l_1+2\Delta_\nu p_1+\tau_1\Delta_\nu,\dots,l_n+2\Delta_\nu p_n+\tau_n\Delta_\nu), \tag{3}$$

where $p = (p_1,\dots,p_n) \in B_n^1(N), l = (l_1,\dots,l_n) \in B_n^2(N), \sigma_i$ is equal to 0 or 1 for all $i = 1,\dots,n, \nu = 1,\dots,s$.

For studying the properties of recurrent sequence of bases, we can use reverse rearrangement [1].

Let j by an integer from set $J = \{0, 1, \dots, 2^\nu - 1\}$ be equal to $j_{\nu-1}2^{\nu-1} + \dots + j_1 2 + j_0$ in a binary system, where $j_i = 0,1$ for all $i = 0,\dots,\nu-1$. A vector $(j_{\nu-1},\dots,j_1,j_0)_2$ is called a binary code of number j. We compare number $j_1 \in J$ with number j, which is set by a binary code $(j_0, j_1, \dots, j_{\nu-1})_2$. Rearrangement $rev_\nu(j) = j_1$ for set J is called reverse rearrangement. For reverse rearrangements the following equalities are true:

$$\begin{aligned} 2rev_{\nu-1}(q) &= rev_\nu(q); \\ 2rev_{\nu-1}(q)+1 &= rev_\nu(\Delta_\nu+q). \end{aligned} \tag{4}$$

Using a reverse rearrangement we can prove that:

$$f_\nu(l_1+p_1\Delta_{\nu+1},\dots,l_n+p_n\Delta_{\nu+1}) =$$
$$= \sum_{q_1=0}^{\Delta_{\nu+1}-1}\dots\sum_{q_n=0}^{\Delta_{\nu+1}-1} w_{\Delta_{\nu+1}}^{\sum_{i=0}^{n} l_i rev_\nu(q_i)} f_0(q_1+p_1\Delta_{\nu+1},\dots,q_n+p_n\Delta_{\nu+1}),$$

where $p = (p_1,\dots,p_n) \in B_n^1(N)$, $l = (l_1,\dots,l_n) \in B_n^2(N)$, $\nu = 1,\dots,s$.

In particular, if $\nu = s$ we have:

$$f_s(l;j) = \sum_{q_1=0}^{N-1}\dots\sum_{q_n=0}^{N-1} w_N^{\sum_{i=1}^{n} l_i rev_\nu(q_i)} \delta_N^n(j_1-q_1,\dots,j_n-q_n) = w_N^{\sum_{i=0}^{n} l_i rev_\nu(j_i)}.$$

Theorem 1. For all $\nu = 0,...,s$, $k \in B_n(N)$, a set of signals $f_\nu = f_\nu(k)$ is orthogonal and $\| f_\nu(k) \|^2 = 2^{n\nu}$.

The solution. Let $\nu = 0$. Then:

$$\langle f_0(k), f_0(k') \rangle = \sum_{j \in B_n(N)} \delta_N^n(j-k) \cdot \delta_N^n(j-k')$$

the last sum can be distinct from zero only when $k = k'$, in other case it is equal to 1 and the theorem is proved.

Let's now $\nu = 1,...,s$ and $k, k' \in B_n(N)$, which are presented in the following way:

$$k = (k_1,...,k_n) = (l_1 + p_1\Delta_{\nu+1},...,l_n + p_n\Delta_{\nu+1}),$$
$$k' = (k_1',...,k_n') = (l_1' + p_1'\Delta_{\nu+1},...,l_n' + p_n'\Delta_{\nu+1}),$$

where $l = (l_1,...,l_n)$, $l' = (l_1',...,l_n')$ belongs to $B_n^2(N)$, and $p = (p_1,...,p_n)$, $p' = (p_1',...,p_n')$ belongs to $B_n^1(N)$. Then:

$$\langle f_\nu(k), f_\nu(k') \rangle = \langle f_\nu(l_1 + p_1\Delta_{\nu+1},...,l_n + p_n\Delta_{\nu+1}), f_\nu(l_1' + p_1'\Delta_{\nu+1},...,l_n' + p_n'\Delta_{\nu+1}) \rangle =$$

$$= \langle \sum_{q_1=0}^{\Delta_{\nu+1}-1} ... \sum_{q_n=0}^{\Delta_{\nu+1}-1} w_{\Delta_{\nu+1}}^{\sum_{i=0}^{N} l_i rev_\nu(q_i)} f_0(q_1 + p_1\Delta_{\nu+1},...,q_n + p_n\Delta_{\nu+1}),$$

$$\sum_{q_1'=0}^{\Delta_{\nu+1}-1} ... \sum_{q_n'=0}^{\Delta_{\nu+1}-1} w_{\Delta_{\nu+1}}^{\sum_{i=0}^{N} l_i' rev_\nu(q_i')} f_0(q_1' + p_1'\Delta_{\nu+1},...,q_n' + p_n'\Delta_{\nu+1}) \rangle =$$

$$= \sum_{q_1=0}^{\Delta_{\nu+1}-1} ... \sum_{q_n=0}^{\Delta_{\nu+1}-1} \sum_{q_1'=0}^{\Delta_{\nu+1}-1} ... \sum_{q_n'=0}^{\Delta_{\nu+1}-1} w_{\Delta_{\nu+1}}^{\sum_{i=1}^{n} l_i rev_\nu(q_i) - l_i' rev_\nu(q_i')} \times$$
$$\times \delta_N^n(q_1 - q_1' + (p_1 - p_1')\Delta_{\nu+1},...,q_n - q_n' + (p_n - p_n')\Delta_{\nu+1}).$$

Arguments of a unit impulse δ_N^n on the module do not exceed $N-1$. For $p_t = p_t'$ and at the some t arguments are distinct from zero for all $q_i, q_i' \in 0 : \Delta_{\nu+1} - 1$, $i = 1,...,N$, as $|q_i - q_i'| \le \Delta_{\nu+1} - 1$. Therefore $\langle f_\nu(k), f_\nu'(k') \rangle = 0$, if $p_j \ne p_j'$.

Let $p_j = p_j'$. For all $j = 1,...,n$ then:

$$\langle f_\nu(k), f_\nu(k') \rangle = \sum_{q_1=0}^{\Delta_{\nu+1}-1} ... \sum_{q_n=0}^{\Delta_{\nu+1}-1} w_{\Delta_{\nu+1}}^{\sum_{i=1}^{n} (l_i - l_i') rev_\nu(q_i)} =$$

$$= \sum_{q_1=0}^{\Delta_{\nu+1}-1} ... \sum_{q_n=0}^{\Delta_{\nu+1}-1} w_{\Delta_{\nu+1}}^{\sum_{i=1}^{n} q_i'} = \Delta_{\nu+1}...\Delta_{\nu+1}\delta_{\Delta_{\nu+1}}^n(l_1 - l_1',...,l_n - l_n').$$

From the last formula it is concluded, that the scalar product $\langle f_\nu(k, f_\nu(k')) \rangle$ is distinct from zero only, if $p_j = p_j'$, $l_i = l_i'$, where $i, j = 1,...,n$. In the last case $\| f_\nu(k_1,...,k_n) \| = \Delta_{\nu+1}...\Delta_{\nu+1} = 2^{n\nu}$ for all $k_1,...,k_n = 0 : N-1$. The theorem is now proved.

3. Sequence application of orthogonal bases to denote the fast discrete Fourier transform.

Let $x(j) = x(j_1,...,j_n) \in \mathrm{C}_N^n$, $j \in B_n(N)$. We compare a signal $x(j)$ a to signal $x_0(j) = x(rev_s(j_1),...,rev_s(j_n))$ and we will spread out $x_0(j)$ on basis f_ν:

$$x_0 = \frac{1}{2^{n\nu}} \sum_{k \in B_n(N)} x_\nu(k) f_\nu(k) \quad (5)$$

($\frac{1}{2^{n\nu}}$ – is a normalizing multiplier). Multiplying both parts (5) by $f_\nu(l)$ scalar $l \in B_n(N)$. Then

$$\langle x_0, f_\nu(l) \rangle = x_\nu(l),$$
$$x_\nu(k) = \sum_{j \in B_n(N)} x_0(j) f_\nu(k) =$$
$$= \sum_{j \in B_n(N)} x(rev_s(j_1),...,rev_s(j_n)) f_\nu(k; j),$$

and coefficients $x_\nu(k)$ in (5) are determined.

In particular, for $\nu = 0$ we have from (1):

$$x_0(k) = \sum_{j \in B_n(N)} x(rev_s(j_1),...,rev_s(j_n)\delta_N^n(j-k) = x(rev_s(k_1),...,rev_s(k_n)). \quad (6)$$

From (3) we get the following:

$$x_\nu(l_1 + \delta_1\Delta_\nu + p_1\Delta_{\nu+1},...,l_n + \delta_n\Delta_\nu + p_n\Delta_{\nu+1}) =$$
$$= \langle x_0, f_\nu(l_1 + \delta_1\Delta_\nu + p_1\Delta_{\nu+1},...,l_n + \delta_n\Delta_\nu + p_n\Delta_{\nu+1}) \rangle =$$
$$= \sum_{\tau_1=0}^{1} ... \sum_{\tau_n=0}^{1} w_{\Delta_{\nu+1}}^{\sum_{i=1}^{n} \tau_i(l_i + \sigma_i\Delta_\nu)} \langle x_0, f_{\nu-1}(l_1 + 2\Delta_\nu p_1 + \tau_1\Delta_\nu,...,l_n +$$
$$+ 2\Delta_\nu p_n + \tau_n\Delta_\nu) \rangle = \sum_{\tau_1=0}^{1} ... \sum_{\tau_n=0}^{1} w_{\Delta_{\nu+1}}^{\sum_{i=1}^{n} \tau_i(l_i + \sigma_i\Delta_\nu)} \times$$
$$\times x_{\nu-1}(l_1 + 2\Delta_\nu p_1 + \tau_1\Delta_\nu,...,l_n + 2\Delta_\nu p_n + \tau_n\Delta_\nu), \quad (7)$$

where $p = (p_1,...,p_n) \in B_n^1(N)$, $l = (l_1,...,l_n) \in B_n^2(N)$, $\nu = 1,...,s$ and $\sigma_1,...,\sigma_n$ are equal to 0 or 1.

As

$$x_s(k) = x(k_1,...,k_n) =$$
$$= \sum_{j \in B_n(N)} x(rev_s(j_1),...,rev_s(j_n)) \cdot w_N^{-\sum_{i=0}^{n} k_i rev_s(j_i)} =$$
$$= \sum_{j \in B_n(N)} x(j) w_N^{-\sum_{i=0}^{n} k_i j_i} = X(k),$$

where $k \in B_n(N)$ and coefficients $x_s(k)$ define components of a spectrum for a signal x on a basic period.

From (6) and (7) we have received the recurrent scheme for the calculation of a spectrum for a signal $x \in \mathrm{C}_N^n$:

$$x_0(k) = x(rev_s(k_1),...,rev_s(k_n));$$
$$x_\nu(l_1+\sigma_1\Delta_\nu+p_1\Delta_{\nu+1},...,l_n+\sigma_n\Delta_\nu+p_n\Delta_{\nu+1}) = \quad (8)$$
$$= \sum_{\tau_1=0}^{1}...\sum_{\tau_n=0}^{1} w_{\Delta_{\nu+1}}^{\sum_{i=0}^{n}\tau_i(l_i+\sigma_i\Delta_\nu)} \cdot x_{\nu-1} \times$$
$$\times(l_1+2\Delta_\nu p_1+\tau_1\Delta_\nu,...,l_n+2\Delta_\nu p_n+\tau_n\Delta_\nu),$$

where $p=(p_1,...,p_n)\in B_n^1(N)$, $l=(l_1,...,l_n)\in B_n^2(N)$, $\nu=1,...,s$ and $\sigma_1,...,\sigma_n$ are equal to 0 or 1.

Let's find a number of complex addition and multiplication operations necessary for finding a spectrum of the signal for scheme (8).

Lemma 2. For some r vectors $t=(t_1,...,t_r)$ and $\sigma=(\sigma_1,...,\sigma_r)$, where $t_i,\sigma_i\in\overline{0,1}$. Then the calculation of all the values of some function:

$$S(\sigma)=\sum_t f(t)(-1)^{\langle\sigma,t\rangle}$$

requires $r\cdot 2^r$ additions (subtractions).

The solution. To prove we apply an induction on r.

Let $r=2$. Then:

$$S(\sigma)=S(\sigma_1,\sigma_2)=\sum_{t_1=0}^{1}\sum_{t_2=0}^{1} f(t_1,t_2)\cdot(-1)^{\sigma_1t_1+\sigma_2t_2} =$$
$$= f(0,0)+f(1,0)(-1)^{\sigma_1}+f(0,1)(-1)^{\sigma_2}+$$
$$+f(1,1)(-1)^{\sigma_1+\sigma_2}.$$

Let's define:

$$S_1(\sigma)=S(0,0)=f(0,0)+f(1,0)+f(0,1)+f(1,1)=$$
$$=(f(0,0)+f(1,0))+(f(0,1)+f(1,1))=S_1^*+S_3^*;$$
$$S_2(\sigma)=S(1,0)=f(0,0)-f(1,0)+f(0,1)-f(1,1)=$$
$$=(f(0,0)-f(1,0))+(f(0,1)-f(1,1))=S_2^*+S_4^*;$$
$$S_3(\sigma)=S(0,0)=f(0,0)+f(1,0)-f(0,1)-f(1,1)=$$
$$=(f(0,0)+f(1,0))-(f(0,1)+f(1,1))=S_1^*-S_3^*;$$
$$S_1(\sigma)=S(0,0)=f(0,0)-f(1,0)-f(0,1)+f(1,1)=$$
$$=(f(0,0)-f(1,0))-(f(0,1)-f(1,1))=S_2^*-S_4^*,$$

where:

$$S_1^*=f(0,0)+f(1,0), S_2^*=f(0,0)-f(1,0),$$
$$S_3^*=f(0,1)+f(1,1), S_4^*=f(0,1)-f(1,1).$$

For calculating S_i^*, $i=1,2,3,4$ it is required to apply 4 additions (subtractions); to calculate all values S it requires 8 such operations and then the statement of the lemma is correct.

Let the statement of the lemma be correct, if $r=k$; for any function $g(t)$ i. e. all values of the function:

$$S(\sigma_1,...,\sigma_k)=\sum_t g(t)(-1)^{\langle\sigma,t\rangle}$$

are calculated by $2\cdot 2^k$ additions (subtractions), where $t=(t_1,...,t_k)$.

Let's consider $r=k+1$.

$$S(\sigma_1,...\sigma_{k+1})=\sum_{t_1=0}^{1}...\sum_{t_{k+1}=0}^{1} f(t_1,...,t_{k+1})\cdot(-1)^{\sigma_1t_1+...+\sigma_{k+1}t_{k+1}} =$$
$$=\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f(t_1,...,t_k,0)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k} +$$
$$+\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f(t_1,...,t_k,1)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k+\sigma_{k+1}}.$$

Let's denote:

$$S(\sigma_1,\sigma_2,...,\sigma_k,0)=S_1(\sigma_1,\sigma_2,...,\sigma_k)=$$
$$=\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f(t_1,...,t_k,0)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k} +$$
$$+\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f(t_1,...,t_k,1)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k} =$$
$$=\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1}(f(t_1,...,t_k,0)+f(t_1,...,t_k,1))\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k};$$
$$S(\sigma_1,\sigma_2,...,\sigma_k,1)=S_2(\sigma_1,\sigma_2,...,\sigma_k)=$$
$$=\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f(t_1,...,t_k,0)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k} -$$
$$-\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f(t_1,...,t_k,1)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k} =$$
$$=\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1}(f(t_1,...,t_k,0)-f(t_1,...,t_k,1))\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k}.$$

Let's define how many operations of addition (subtraction) are required for calculating S_1 and S_2.

Let's denote $f^+(t_1,...,t_k)=f(t_1,...,t_k,0)+$, $+f(t_1,...,t_k,1)$ $f^-(t_1,...,t_k)=f(t_1,...,t_k,0)-f(t_1,...,t_k,1)$. For calculating all values f^+ required are 2^k additions and to calculate $S_1=\sum_{t_1=0}^{1}...\sum_{t_k=0}^{1} f^+(t_1,...,t_k)\cdot(-1)^{\sigma_1t_1+...+\sigma_kt_k}$ required are $k2^k$ according to the induction assumption.

Then the total sum of addition (subtraction) operations for calculation S_1 requires $k2^k+2^k$. Such a number of operations it is necessary for S_2.

Therefore, to calculate of all values S $(k+1)2^{k+1}$ additions (subtractions) are required; this needed to be proved.

Theorem 2. To calculate the recurrent scheme for a signal spectrum $x\in C_N^n$ ($N=2^s$)

$$x_0(k)=x(rev_s(k_1),...,rev_s(k_n));$$
$$x_\nu(l_1+\sigma_1\Delta_\nu+p_1\Delta_{\nu+1},...,l_n+\sigma_n\Delta_\nu+p_n\Delta_{\nu+1}) = \quad (9)$$
$$= \sum_{\tau_1=0}^{1}...\sum_{\tau_n=0}^{1} w_{\Delta_{\nu+1}}^{\sum_{i=0}^{n}\tau_i(l_i+\sigma_i\Delta_\nu)} \cdot x_{\nu-1} \times$$
$$\times(l_1+2\Delta_\nu p_1+\tau_1\Delta_\nu,...,l_n+2\Delta_\nu p_n+\tau_n\Delta_\nu),$$

demands $\frac{2^n-1}{2^n}N^n\log_2 N$ complex multiplications and $nN^n\log_2 N$ complex additions.

The solution. First, we will find a number of complex multiplications. Complex multiplication is multiplication only by $w_N^{\sum_{i=0}^{n}\tau_i(l_i+\sigma_i\Delta_\nu)}$. We shall define the number of products required in (9) for all parameters $\nu^*, l^* = (l_1^*,...,l_n^*)$ and $p^* = (p_1^*,...,p_n^*)$. For this purpose we shall consider products:

$$w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i(l_i^*+\sigma_i\Delta_{\nu^*})}\cdot x_{\nu^*-1}\times$$
$$\times(l_1^*+2p_1^*\Delta_\nu+\tau_1\Delta_{\nu^*},...,l_n^*+2p_n^*\Delta_\nu+\tau_n\Delta_{\nu^*}),$$

where $\sigma_i \in \overline{0,1}$.

We have to notice that:

$$w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i(l_i^*+\sigma_i\Delta_{\nu^*})} = w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i l_i^*+\tau_i\sigma_i\Delta_{\nu^*}} =$$
$$= w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i l_i^*}\cdot w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i\sigma_i\Delta_{\nu^*}} = (-1)^{\sum_{i=0}^{n}\tau_i\sigma_i}\cdot w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i l_i^*}. \quad (10)$$

That is product:

$$w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i(l_i^*+\sigma_i\Delta_{\nu^*})}\cdot x_{\nu^*-1}(l_1^*+2p_1^*\Delta_\nu+\tau_1\Delta_{\nu^*},...,l_n^*+2p_n^*\Delta_\nu+\tau_n\Delta_{\nu^*}).$$

Let parameters $\nu^*, l^* = (l_1^*,...,l_n^*)$ and $p^* = (p_1^*,...,p_n^*)$ by fixed; then product (10) can possibly be replaced with complex product:

$$w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i l_i^*}\cdot x_{\nu^*-1}(l_1^*+2p_1^*\Delta_\nu+\tau_1\Delta_{\nu^*},...,l_n^*+2p_n^*\Delta_\nu+\tau_n\Delta_{\nu^*}) \quad (11)$$

as $(-1)^{\sum_{i=0}^{n}\tau_i\sigma_i}$ can accept values only ± 1.

The number of complex products of a kind (11) is equal to a number of every possible vector $\tau = (\tau_1,...,\tau_n)$, where $\tau_i \in \overline{0,1}$, i. e. $2^n - 1$, since we have a product of real numbers for $r = 0$. As parameter $\nu \in 1:s$, vectors $l = (l_1,...,l_n)$ and $p = (p_1,...,p_n)$ (where $p_i = 0,1,...,N^\nu - 1$ ($N^\nu = N/2^\nu$)), $l_i = 0,1,...,\Delta_\nu - 1$ ($\Delta_\nu = 2^{\nu-1}$), then the total complex multiplications is equal to: $s(\frac{N}{2^\nu}\cdot 2^{\nu-1})^n\cdot(2^n-1) = \frac{2^n-1}{2^n}N^n\log_2 N$.

Let's find the amount of complex addition in the algorithm. For fixed ν^* and $l^* = (l_1^*,...,l_n^*)$, $p^* = (p_1^*,...,p_n^*)$ we have:

$$x_\nu^*(l_1^*+\sigma_1\Delta_{\nu^*}+p_1\Delta_{\nu^*+1},...,l_n^*+\sigma_n\Delta_{\nu^*}+p_n^*\Delta_{\nu^*+1}) =$$
$$= \sum_{\tau_1=0}^{1}...\sum_{\tau_n=0}^{1} w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i(l_i^*+\sigma_i\Delta_\nu)}\cdot x_{\nu^*-1}\times \quad (12)$$
$$\times(l_1^*+2p_1^*\Delta_\nu+\tau_1\Delta_{\nu^*},...,l_n^*+2p_n^*\Delta_\nu+\tau_n\Delta_{\nu^*}).$$

From (10) follows, that for calculation (12) we need to calculate expressions:

$$f(\tau) = w_{\Delta_{\nu^*+1}}^{\sum_{i=0}^{n}\tau_i l_i^*} x_{\nu^*-1}(l_1^*+2p_1^*\Delta_\nu+\tau_1\Delta_{\nu^*},...,l_n^*+2p_n^*\Delta_\nu+\tau_n\Delta_{\nu^*})$$

which depend only on $\tau = (\tau_1,...,\tau_n)$, where $\tau_i \in \overline{0:1}$. Then (12) can be presented as:

$$x_\nu^*(l_1^*+\sigma_1\Delta_{\nu^*}+p_1\Delta_{\nu^*+1},...,l_n^*+\sigma_n\Delta_{\nu^*}+p_n^*\Delta_{\nu^*+1}) =$$
$$= \sum_{\tau_1=0}^{1}...\sum_{\tau_n=0}^{1}(-1)^{\sum_{i=0}^{n}\tau_i\sigma_i}\cdot f(\tau).$$

This way, complex addition is required from Lemma 2 to calculate:

$$x_\nu^*(l_1^*+\sigma_1\Delta_{\nu^*}+p_1\Delta_{\nu^*+1},...,l_n^*+\sigma_n\Delta_{\nu^*}+p_n^*\Delta_{\nu^*+1}) \;\; n2^n.$$

As parameter $\nu \in 1:s$, vectors $l = (l_1,...,l_n)$ and $p = (p_1,...,p_n)$, where $p_i = 0,1,...,N^\nu - 1$ ($N^\nu = N/2^\nu$), $l_i = 0,1,...,\Delta_\nu - 1$ ($\Delta_\nu = 2^{\nu-1}$), then the sum of complex additions are equal to: $s(\frac{N}{2^\nu}\cdot 2^{\nu-1})^n\cdot(n2^n) = nN^n\log_2 N$. The theorem is now proved.

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