4. Makklellan Dg., Reider Ch. Theory of number application in digital signals processing : Transl. from English. M. : Radio and communication, 1983. P. 376
5. Sverdlik M. B. Optimal discrete signals. M. : Sov. Radio, 1975. P. 200.
6. Radio systems of information transfer / B. A. Borisov [et al.]. M. : Radio and connection, 1990. P. 456.
7. Modern telemetry in the theory and in practice : A training course. SPb. : Science and Technics, 2007. P. 672.



U. V. Zakharov
Siberian State Technological University, Russia, Krasnoyarsk
A. U. Vlasov, R. V. Avakumov
Siberian State Aerospace University named after academician M. F. Reshetnev,
Russia, Krasnoyarsk

## MAGNETIZATION OF MULTILAYER FERROMAGNETIC FILM WITH A NONMAGNETIC INTERLAYER

*The magnetization of magnetic film, consisting out of two ferromagnetic layers with a nonmagnetic interlayer applied to antiferromagnetic substrate is considered in this paper.*

*Keywords: ferromagnetic, antiferromagnetic, interlayer interaction.*

Currently, the issue we consider is of great significance because there is an international interest in multilayer magnetic systems. Such systems are used in magnetoresistive sensors, components of magnetic send-receive, spin diodes. The magnetization of a multilayer magnetic system consisting of two ferromagnetic layers with a nonmagnetic interlayer applied to antiferromagnetic substrate is considered in the presented paper. The theoretical model used for the investigation of the stated process had been introduced in paper [1]. Inhomogeneous distribution takes place in this system because of substrate influence. This is why we use a stated model.

Physical properties of the film are defined by its boundaries. A bilayer system with the ferromagnetic-on-antiferromagnetic-substrate type is investigated in research paper [1] where the boundary condition of clamped magnetization vector type had been considered.

In paper [2] it had been demonstrated that the magnetization process of such a system has a threshold type. The author [2] points out an analogue between the magnetization process and the bending of elastic rod considered in paper [3]. It is also necessary to refer to paper [4].

Later the boundary condition of the clamped vector type was replaced by condition of elastically restrained vector type by introducing an effective interlayer at the ferromagnetic-antiferromagnetic boundary [5]. In later research of bilayer ferromagnetic film, the layers of which were rigidly bond with an antiferromagnetic substrate was overlooked in [6]; using an analog of a two-part elastic shank clamped at one edge and free at the other [7].

Today the presence of a nonmagnetic interlayer in ferromagnetic systems and its influence on threshold fields and the distribution of magnetization aren't being studied.

The potential energy of the magnetic system is given as expression [8]:

$$F\left(\mathbf{M},\frac{\partial M_i}{\partial x_k}\right)=\frac{1}{2}\alpha_{ik}\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_k}+w_a\left(\mathbf{M}\right)+f\left(M^2\right), \quad (1)$$

where, the first summand represents the quadratic form of derivatives, $w_a(\mathbf{M})$ is the magnetic anisotropy energy, $f(M^2)$ is some function of $M^2$. We only overlook isotropic ferromagnetic films where magnetization inhomogeneous is present along thickness of the object. Thereby, the second summand turns into zero and the first summand turns into the following expression:

$$\frac{1}{2}\alpha\left(\frac{d\mathbf{M}}{dz}\right)^2.$$

Axis $z$ is directed perpendicularly to the layers. A variation of the third summand results in zero because the magnetization vector length doesn't change; therefore we shall not consider it. Thereby, the energy of the ferromagnetic layer may be represented as:

$$U=\int_0^{d_1}\left(\frac{1}{2}\alpha_1\left(\frac{d\mathbf{M}_1}{dz}\right)^2-\mathbf{M}_1\mathbf{H}\right)dz+\int_{d_1+d_s}^{d_1+d_s+d_2}\left(\frac{1}{2}\alpha_1\left(\frac{d\mathbf{M}_1}{dz}\right)^2-\mathbf{M}_1\mathbf{H}\right)dz, \quad (2)$$

where $d_1$, $d_2$ are the thicknesses, $\mathbf{M}_1$, $\mathbf{M}_2$ are the magnetization densities, $\alpha_1$, $\alpha_2$ are the exchange constants of first and second layers respectively; $d_s$ is the interlayer thickness, $\mathbf{H}$ is the applied magnetic field.

The energy of interlayer is given in the expression:

$$U_s=-\frac{\alpha_s}{d_s}\mathbf{M}_1\mathbf{M}_2, \quad (3)$$

where $\alpha_s$ is the interlayer exchange constant.

The total energy of the ferromagnetic system is:

$$U = \int_0^{d_1}\left(\frac{1}{2}\alpha_1\left(\frac{d\mathbf{M}_1}{dz}\right)^2 - \mathbf{M}_1\mathbf{H}\right)dz + \int_{d_1}^{d_2}\left(\frac{1}{2}\alpha_2\left(\frac{d\mathbf{M}_2}{dz}\right)^2 - \mathbf{M}_2\mathbf{H}\right)dz - \frac{\alpha_s}{d_s}\mathbf{M}_1\mathbf{M}_2\Bigg|_{d_1}. \qquad (4)$$

The value of thickness $d_s$ is negligible in comparison to thicknesses $d_1$ and $d_2$. It's rather convenient to proceed to the generalized coordinates, which represent the rotation angles between the magnetization vectors and $x$ axis directed along the applied field. Then,

$$U = \int_0^{d_1}\left(\frac{1}{2}\alpha_1 M_1^2\left(\frac{d\varphi_1}{dz}\right)^2 + M_1 H\cos\varphi_1\right)dz + \int_{d_1}^{d_2}\left(\frac{1}{2}\alpha_2 M_2^2\left(\frac{d\varphi_2}{dz}\right)^2 + M_2 H\cos\varphi_2\right)dz - \frac{\alpha_s}{d_s}M_1 M_2\cos\left(\varphi_2 - \varphi_1\right)\Bigg|_{d_1}. \qquad (5)$$

The minimum condition of potential energy gives:

$$\delta U = \alpha_1 M_1^2\frac{d\varphi_1}{dz}\delta\varphi_1\Bigg|_0^{d_1} + \alpha_2 M_2^2\frac{d\varphi_2}{dz}\delta\varphi_2\Bigg|_{d_1}^{d_2} - \int_0^{d_1}\left(\alpha_1 M_1^2\frac{d^2\varphi_1}{dz^2} + M_1 H\sin\varphi_1\right)\delta\varphi_1 dz - \int_{d_1}^{d_2}\left(\alpha_2 M_2^2\frac{d^2\varphi_2}{dz^2} + M_2 H\sin\varphi_2\right)\delta\varphi_2 dz + \frac{\alpha_s}{d_s}M_1 M_2\sin\left(\varphi_2-\varphi_1\right)\delta\left(\varphi_2-\varphi_1\right)\Bigg|_{d_1} = 0. \qquad (6)$$

Differential equations set follows from (6) as long as variations $\delta\varphi_1$ and $\delta\varphi_2$ contained in integral are arbitrary:

$$\begin{cases}\alpha_1 M_1\dfrac{d^2\varphi_1}{dz^2} + H\sin\varphi_1 = 0,\\ \alpha_2 M_2\dfrac{d^2\varphi_2}{dz^2} + H\sin\varphi_2 = 0.\end{cases} \qquad (7)$$

Matching of condition at point $d_1$ follows from (6):

$$\begin{cases}\dfrac{\alpha_1}{d_1}\dfrac{d\varphi_1}{dz} = \dfrac{\alpha_s}{d_s}\dfrac{M_2}{M_1}\sin\left(\varphi_2-\varphi_1\right),\\ \dfrac{\alpha_1 M_1^2}{d_1}\dfrac{d\varphi_1}{dz} = \dfrac{\alpha_2 M_2^2}{d_2}\dfrac{d\varphi_2}{dz}.\end{cases} \qquad (8)$$

Here $z$ is a normalized variable. Then we will use designation $C = (\alpha_s M_2)/(d_s M_1)$. This quantity characterizes the degree of the layer's fixity. The infinitely large value of $C$ gives $\varphi_1 = \varphi_2$, substituting for first equation of (8).

The boundary conditions are also given from (6):

$$\begin{cases}\alpha_1 M_1^2\dfrac{d\varphi_1}{dz}\delta\varphi_1\Bigg|_0 = 0,\\ \alpha_2 M_2^2\dfrac{d\varphi_2}{dz}\delta\varphi_2\Bigg|_{d_1+d_2} = 0.\end{cases} \qquad (9)$$

Angle $\varphi_1$ equals zero at the antiferromagnetic boundary because the applied magnetic field we consider is solidly joined to the antiferromagnetic anisotropy field. Where the vacuum border is angle $\varphi_2$ shall be subject to free magnetic momentums. Mathematically, this looks like:

$$\begin{cases}\varphi_1\big|_0 = 0,\\ \dfrac{d\varphi_2}{dz}\Bigg|_{d_1+d_2} = 0.\end{cases} \qquad (10)$$

For example let's examine a magnetic film consisting of two ferromagnetic with equal thickness $d$ and with a nonmagnetic interlayer:

$$\frac{d^2\varphi_i}{dz^2} + \frac{Hd^2}{\alpha_i M_i}\sin\varphi_i = 0, \quad i = 1, 2. \qquad (11)$$

In these equations $z$ is normalized variable. The solution of these equations could be represented as [1]:

$$\varphi_i\left(z\right) = 2\arcsin\left[k_i\,\mathrm{sn}\left(\sqrt{\frac{Hd^2}{\alpha_i M_i}}z + F_i, k_i\right)\right], \qquad (12)$$

where $\mathrm{sn}(u, k)$ is the Jacobi sine. Equations (11) amplified with the matching condition:

$$\begin{cases}\alpha_1 M_1\dfrac{d\varphi_1\left(1\right)}{dz} = \alpha_2 M_2\dfrac{d\varphi_2\left(0\right)}{dz},\\ \dfrac{\alpha_1}{d}\dfrac{d\varphi_1\left(1\right)}{dz} = C\sin\left(\varphi_2\left(0\right)-\varphi_1\left(1\right)\right),\end{cases}$$

and the border conditions:

$$\begin{cases}\dfrac{d\varphi_2\left(1\right)}{dz} = 0,\\ \varphi_1\left(0\right) = 0.\end{cases} \qquad (13)$$

Fig. 1 represents the system of coordinates. A new coordinate system is introduced for each layer. The spiral emphasizes the interlayer interaction.

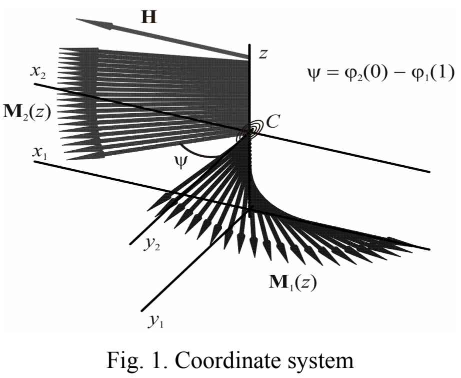


Fig. 1. Coordinate system

Applying the border conditions and matching the condition to the solution gives:

$$\begin{cases}k_1\,\mathrm{cn}\,u_1 = \gamma k_2\,\mathrm{cn}\,u_2,\\ \rho u_1 k_1\,\mathrm{cn}\,u_1 = \left(k_2\,\mathrm{sn}\,u_2\,\mathrm{dn}\,u_1 - k_1\,\mathrm{sn}\,u_1\,\mathrm{dn}\,u_2\right)\\ \left(\mathrm{dn}\,u_1\,\mathrm{dn}\,u_2 + k_1 k_2\,\mathrm{sn}\,u_2\,\mathrm{sn}\,u_1\right),\end{cases} \qquad (14)$$

where:

$$u_1 = \frac{\pi}{4}\sqrt{\frac{h}{h_c}},\ u_2 = F_2 = K(k_2) - \frac{\pi}{4\gamma}\frac{M_2}{M_1}\sqrt{\frac{h}{h_c}},$$

$$h_c = \left(\frac{\pi}{2}\right)^2 \frac{\alpha_1}{d^2},\ \gamma = \sqrt{\frac{\alpha_2 M_2^3}{\alpha_1 M_1^3}},\ \rho = \frac{\alpha_1}{Cd},$$

cn($u$, $k$), sn($u$, $k$), dn($u$, $k$) are the Jacobi cosine, sine and delta-function respectively. Constant $F_1$ equals zero. To define the threshold fields, elliptic modules $k$ must be turned to zero in (14). Then, after some transformations, the transcendental equation derives from (14):

$$\rho\frac{\pi}{4}\sqrt{\frac{h^{tr}}{h_c}} + \mathrm{tg}\left(\frac{\pi}{4}\sqrt{\frac{h^{tr}}{h_c}}\right) = \frac{1}{\gamma}\mathrm{ctg}\left(\frac{\pi}{4\gamma}\sqrt{\frac{h^{tr}}{h_c}}\right), \qquad (15)$$

where $h^{tr} = H^{tr}/M$ is the threshold field. The solution of (15) relative to $h^{tr}/h_c$ by different values of parameters $\gamma$ and $\rho$ gives surface shown in fig. 2.

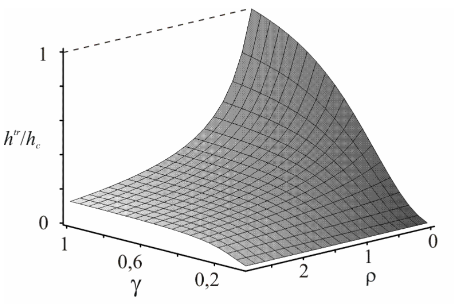


Fig. 2. Threshold field dependence on $\gamma$ and $\rho$

If there is no nonmagnetic interlayer ($d_s = 0$), and the ferromagnetic system consists of two equal layers ($\gamma = 1$), the equation (15) will depict the known expression:

$$h^{th} = \left(\frac{\pi}{2}\right)^2 \frac{\alpha}{4d^2}.$$

In other extreme cases ($\rho \to \infty$) the second equation of (14) gives:

$$\cos\frac{\pi}{4}\sqrt{\frac{h^{th}}{h_c}} = 0,$$

$$h^{th} = \left(\frac{\pi}{2}\right)^2 \frac{\alpha_1}{d^2}.$$

This result corresponds to the ferromagnetic layer the thickness of which equals $d$ as expected.

It's also necessary to investigate the behavior of magnetization curves. Let's examine a particular case where two identical ferromagnetic layers are separated by a nonmagnetic interlayer. Only variable $\rho$ varies. The average value of the magnetization vector projection is presented by the following expression:

$$\overline{m}_x = \int_0^1 m_x dz = -\int_0^1 \cos\varphi dz.$$

The integration gives:

$$\overline{m}_x = 2\left[1 - \frac{1}{u_1}E(\mathrm{am}\,u_1) - \frac{1}{u_1}E(\mathrm{am}(u_1 + F_2)) + \right.$$
$$\left. + E - [\mathrm{am}\,F_2,\ k_2]\right], \qquad (16)$$

where am($u$, $k$) is the Jacobi amplitude (fig. 3):

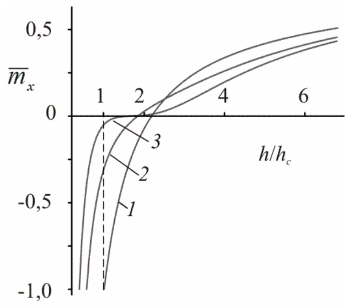


Fig. 3. The magnetization $x$-projection dependence on the applied field:
*1* – $\rho = 0$; *2* – $\rho = 1{,}4$; *3* – $\rho = 2{,}8$

Parabola *1* corresponds to the magnetic system consisting of one ferromagnetic layer with a thickness of $2d$ because $\rho$ and, consequently $d_s$ equals, zero in this case. Parabolas *2* and *3* correspond to the magnetic systems involving a non-magnetic interlayer and the curve shifted to less values of the applied magnetic field at the beginning of magnetization; corresponds to a greater thickness of the interlayer.

Currently, a research on interlayer influence on threshold fields in systems of ferromagnetic-interlayer-ferromagnetic-antiferromagnetic types has been conducted. Also, parabolas of magnetization with different values of effective parameter $\rho$, characterizing the interlayer exchange interaction have been plotted.

## References

1. Aharoni A., Frei E. H., Strinkman S. J. // Appl. Phys. 1959. Vol. 30, № 12. P. 1956–1961.

2. Zakharov Yu. V. Static and dynamic instabilities caused by switching of ferromagnetic layer // DAN. Vol. 344, № 3. P. 328–332.

3. Zakharov Yu. V., Okhotkin K. G. Nonlinear bending of thin elastic rod // PMTF. 2002. Vol. 43, № 5. P. 124–131.

4. Zakharov Yu. V., Isakova V. V., Okhotkin K. G. Analogy of magnetization reversal of a magnetic bilayer system and nonlinear bending of an elastic rod with compression // Vestnik SibSAU. 2009. Vol. 2 (23). P. 122–125.

5. Magnetization thresholds of soft-magnetic layer in exchange coupled structure depends on surface exchange / U. V. Zakharov [et al.] // New magnetic materials of microelectronics : Collected papers of XX international seminar-school. M. : Physfac of MSU, 2006. P. 201–203.

6. Magnetization reversal of the multilayer magnetic film / Yu. V. Zakharov, K. G. Okhotkin, A. D. Skorobogatov, V. V. Isakova // Magnetism on a Nanoscale (EASTMAG-2007) : Brief outline reports of Euro-Asian symposium Kasan : KSU, 2007. P. 184.

7. Vlasov A. Yu., Okhotkin K. G. Non-linear bend of stiffness-variable compound rod with longitudinal and transversal stress // Physical-mathematical science. 2006. № 1. P. 26–28.

8. Akhiezer A. I., Bariakhtar V. G., Pelemitskii S. V. Spin waves. M. : Nauka, 1967.



V. V. Zolotarev, E. A. Danilova
Siberian State Aerospace University named after academician M. F. Reshetnev,
Russia, Krasnoyarsk

## ON APPLICATION OF FACTORIAL ANALYSIS IN PROBLEMS OF SECURITY ESTIMATION OF AUTOMATED SYSTEMS ELEMENTS

*The possibility of factorial analysis application in theestimation of the state of information systems security is considered. The procedure of selection and classification of factors as well as calculation of factors influence on the resultant indicator size are described.*

*Keywords: risk management, information risk, factorial analysis.*

Factorial analysis is one of the possible methods of automated systems security analysis. This method of analysis allows both to establish cause-and-effect relations between negative events and to characterize them quantitatively.

Let's consider the application peculiarities of a security estimation factorial model (further a factorial model) in the problem of security estimation of electronic document management system (EDMS). At the same time we will introduce universality elements into the offered model which will allow to use it for the estimation of various elements of both EDMS and other automated systems. We will especially note the applicability of the offered model and the solutions found on its basis for a human factor estimation.

*The model description.* Let there be an information system $IS$ which consists of $N$ numbers of $E$ elements, each of which in its turn consists of $K$ components. A component of each element in a certain period of time can accept $x$ of states $s$ with probability $r$:

$$IS = \{E_l\}, \qquad E_i = \begin{Bmatrix} s_1^1 & s_2^1 & \dots & s_{x1}^1 \\ s_1^2 & s_2^2 & \dots & s_{x2}^2 \\ \dots & \dots & \dots & \dots \\ s_1^k & s_2^k & \dots & s_{xk}^k \end{Bmatrix},$$

where

$$l \in [1 \dots N]. \tag{1}$$

Let $x$ be the quantity of degrees of a component freedom. It is obvious that in using a similar model of the system it is possible to use the method of a morphological box of Zwicky [1, p. 196] in various variants. At the same time it is possible to calculate the quantity of cause-and-effect relations between the states of information system elements if we calculate them as a number of placings with repetitions:

$$k_{\text{сост}} = \left( \sum_{i=1}^{N} \sum_{j=1}^{K} x_{s_{ij}} \right)^m, \tag{2}$$

where $m$ is the quantity of interlinks between system elements components. During such calculation a number of assumptions was made which is necessary to mention as these assumptions restrict the model application range:

– it is necessary to reduce the quantity of freedom degrees to some uniform value which assumes a standard set of states of system elements components;

– it is necessary to provide the completeness of an initial set of freedom degrees of each system element component which assumes a certain approach to the choice of indicators defining freedom degrees;

– the private function of utility should be calculated for each system element separately, thus resorting to simplification of calculations;

– it is necessary to possess the information about internal connections of analyzed system elements. Without updating such approach is inapplicable for a system with incomplete information about internal connections.

Let's consider basic elements of an applied factorial model:

– the private function of element $E$ utility for performing the main task of $IS$ system (further – private function of utility):

$$u^*(t) = f(E),$$

where

$$E = \begin{pmatrix} r_1^{(1)}(t) & \dots & r_1^{(x)}(t) \\ r_2^{(1)}(t) & \dots & r_2^{(x)}(t) \\ \dots & \dots & \dots \\ r_k^{(1)}(t) & \dots & r_k^{(x)}(t) \end{pmatrix}, \tag{3}$$