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## ASYMMETRIC INVARIANT ECHO-JACK OF THE SECOND ORDER WITHOUT A PROTECTIVE TIME INTERVAL AND ITS CHARACTERISTICS

The structure of the invariant echo-jack of the second order without a protective time interval is synthesized. Control elements of such echo-jack are given. The calculation of the basic technical characteristics is presented.

Keywords: invariant, an echo-jack, asymmetric, echo-compensator, asymmetric, power of self-noise.

The main telecommunication network uses, in overwhelming majority of cases, fiber-optical transmission systems. However the standard streams often don't reach the consumer because of the «last mile», presented by a copper cable.

Existing modern high-speed technologies of access use xDSL adaptive echo-jacks in the equipment. The principle of work of adaptive echo-jacks is based on modelling of parameters of an unknown system. The main defect in the work of such systems is the big criticality of work to correlation communications of signals of two directions. In some cases it can lead to work failure.

Another approach in construction of adaptive echo-jacks is the use of invariants. So in works [1; 2] the structures of invariant echo-jacks with protective and without protective time intervals are presented. The basic difference between invariant echo-jacks and classical ones is that the former are controlled by transmission signals while the latter are controlled by an error signal.

The given article is devoted to the further research in the above-mentioned direction where the results of synthesis of the invariant echo-jack of the second order are presented.

We have a communication channel (echo-path) meeting the requirement of stationarity. The communication channel is limited by a pass-band with bottom  $f_e$  and top  $f_t$  frequencies. It is necessary to synthesize the algorithm of functioning of the invariant echo-jack of the second order.

If the communication channel (echo-path) meets the requirement of stationarity we can write the following equality:

$$H_{i-1}(z) = H_i(z) = H_{i+1}(z) = H_{i+2}(z), \qquad (1)$$

where  $H_i(z) - z$  - is the image of the transmission characteristics of an echo-path on i-block of processing.

In [3] it is proved that if the identical signal V(Z) arrives at an input of two parallel working linear digital filters then the correlation of relations of Z-images on the next blocks of processing on the exits of these digital filters is equal to

$$\frac{S_i(z)}{S_{i-1}(z)} : \frac{S_{i-1}(z)}{S_{i-2}(z)} = \frac{\Pi_i(z)}{\Pi_{i-1}(z)} : \frac{\Pi_{i-1}(z)}{\Pi_{i-1}(z)},$$
(2)

where  $S_i(z), S_{i-1}(z), S_{i-2}(z) - z$  – is the signal image on the exit of the additional digital filter on i, (i - 1) and (i - 2) processing blocks respectively under the entrance influence equal to  $V_i(Z); V_{i-1}(Z); V_{i-2}(Z); \Pi_i(z), \Pi_{i-1}(z), \Pi_{i-2}(z) - z$  is the echo-signal image on i, (i - 1) and (i - 2) processing blocks respectively under the entrance influence equal to  $V_i(Z); V_{i-1}(Z); V_{i-2}(Z)$ .

Let's consider the cascade connection of a sending device and an echo-path to be the digital filter forming  $\Pi$  (Z). The additional digital filter included in parallel to a sending device forms signals S(Z).

Equality (2) does not impose restrictions on a signal of transmission V(Z) and allows to transmit it without a protective time interval.

In its turn equality (2) falls apart into the following equalities [3]:

$$\hat{\Pi}_{i}(z) = \Pi_{i-1}(z) \cdot \frac{S_{i}(z)}{S_{i-1}(z)} 
\hat{\Pi}_{i}(z) = \Pi_{i-2}(z) \cdot \frac{S_{i}(z)}{S_{i-1}(z)} \cdot \frac{S_{i-1}(z)}{S_{i-2}(z)} 
\Pi_{i}(z) = \frac{1}{2} \hat{\Pi}_{i}(z) + \frac{1}{2} \hat{\Pi}_{i}(z)$$
(3)

Equality (3) is a basis for the synthesis of the invariant echo-jack of the second order. The estimations of echo-signals on the previous blocks of processing can be used for calculation of the size of an echo-signal on the current block of processing. In fig. 1 the structure of the invariant echo-jack of the second order with control elements is presented.

In figure 1 it is clearly seen, that unlike the classical echojack, the invariant one is included before the intake. The invariant echo-jack is controlled by the transmission signals S(z). It results in the fact that its technical characteristics do not depend on correlations of signals of two directions. For echosignals an invariant echo-jack is an original rejector, and for reception signals it is a two-port network bringing minimum AHI and FHI. The characteristic of the invariant echo-jack is defined on the basis of laws of digital filtration [4].

$$H(z) = \frac{1 - \frac{1}{2}M_1(z) \cdot z^{-1} - \frac{1}{2}M_2(z) \cdot z^{-2}}{1 - \frac{1}{2}M_1(z) \cdot z^{-1} - \frac{1}{2}M_2(z) \cdot z^{-2}C},$$
 (4)

where  $M_1(z) = \frac{S_i(z)}{S_{i-1}(z)}$  is the 1st operating factor;

 $M_2(z) = \frac{S_i(z)}{S_{i-1}(z)} \cdot \frac{S_{i-1}(z)}{S_{i-2}(z)}$  is the 2nd operating factor;  $S_i(z) = S_i(z)$ 

z is the image of a transmission signal on *i*-block of processing; C < 1 - is an additional attenuator, included in a recursive chain.

For stable work of the offered structure it is necessary for the poles of the transmission characteristic to be inside the individual circle, then

$$\begin{array}{c}
M_1(z) \le 1, \\
M_2(z) \le 1.
\end{array}$$
(5)

When the chosen word length is equal to *m*, the maximum values  $M_1(z) \bowtie M_2(z)$  are  $2^m$ . If the operating factors have such values then the work of the invariant echo-jack will be unstable.

One of possible ways to solve the problem of stable work of the invariant echo-jack is the rationing of operating signals and echo-signals, together with reception signals.



Fig. 1. The structure of the invariant echo-jack of the second order

It will lead to the use of additional scaling two-port networks which should be included not only before an input of the invariant echo-jack itself (point a), but also on the inputs of the shapers of operating signals (points b and c).

The other way to provide the stability of the invariant echo-jack of the second order is the transformation of a recursive part into a non recursive digital filter. From the theory of digital filtration it is known, that any recursive digital filter (CF) can be transformed into a non recursive digital filter [4].

Then

$$\frac{1}{1 - \frac{1}{2}M_1(Z) \cdot Z^{-1} - \frac{1}{2}M_2(Z) \cdot C \cdot Z^{-2}} = 1 + \sum_{i=1}^k Q_i(Z) \cdot Z^{-i},$$
(6)  
where k is the quantity of taps.

For the first five taps of the transformed structure the values  $Q_i(Z)$  are accordingly equal to

$$Q_{1}(Z) = \frac{1}{2}M_{1}(Z);$$

$$Q_{2}(Z) = \frac{1}{4}M_{1}^{2}(Z) + \frac{1}{2}M_{2}(Z) \cdot C;$$

$$Q_{3}(Z) = \frac{1}{2}M_{1}(Z) \cdot M_{2}(Z) \cdot C + \frac{1}{8}M_{1}(Z);$$

$$Q_{4}(Z) = \frac{1}{4}M_{2}^{2}(Z) \cdot C^{2} + \frac{3}{8}M_{1}^{2}(Z) \cdot M_{2}(Z) \cdot C + \frac{3}{16}M_{1}^{3}(Z);$$

$$Q_{5}(Z) = \frac{3}{8}M_{1}(Z) \cdot M_{2}^{2}(Z) \cdot C^{2} + \frac{1}{16}M_{2}(Z) \cdot M_{1}^{2}(Z) \cdot C + \frac{3}{16}M_{1}^{3}(Z);$$

$$+ \frac{3}{16}M_{1}^{3}(Z) \cdot M_{2}(Z) \cdot C + \frac{1}{32}M_{1}^{4}(Z).$$

In figure 2 the structure of the invariant echo-jack consisting of two in cascade joined digital filters is presented. In the first digital filter the indemnification of echo-signals is made. In the second digital filter the restoration of reception signals is made. It should be noted that the phase-frequency characteristic of two in cascade joined digital filters will be linear [4].



Fig. 2. The transformed structure of the invariant echo-jack of the second order

For the definition of technical characteristics of the developed invariant echo-jack we assume  $M_1(z) = M_2(z) = 1$ . Then the transmission characteristic will be equal to

$$H(z) = \frac{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}C}.$$
 (7)

Let's decompose H(z) into two transmission functions of the first order

$$H(z) = H_0(z) \cdot H_1(z) = \frac{1 + \frac{1}{2}z^{-1}}{1 - B_0 z^{-1}} \cdot \frac{1 - z^{-1}}{1 - B_1 z^{-1}}, \qquad (8$$
$$B_0 = \frac{1}{4} - \frac{1}{4}\sqrt{1 + 8C}, \\B_1 = \frac{1}{4} + \frac{1}{4}\sqrt{1 + 8C},$$

where  $B_0$  and  $B_1$  are the poles of transmission function.

Pulse reaction CFVH-2 is defined by convolution  $h_0(nT) \bowtie h_1(nT)$ .

The general level of self-noise CFVH-2 is defined by a known parity [4]:

$$\sigma^{2} = \frac{\Delta_{0^{2}}}{12} \sum_{n=0}^{N-1} h^{2}(nT) + \frac{\Delta^{2}}{12} \sum_{j=1}^{M} \sum_{n=0}^{N-1} h_{j}^{2}(nT) , \qquad (9)$$

where  $\Delta$  – is a step of a quantum entrance word;  $\Delta_0$  is a step of quantum in CFVH-2; h(nT) – is pulse reaction CFVH-2;  $h_j(nT)$  – is pulse reaction from *j*-th source of noise in CFVH-2 to an exit.

Let's find the value of separate components included in equality (9), using Cauchy–Bunyakowsky inequality

$$\sum_{n=0}^{N-1} h^{2}(nT) = \sum_{n=0}^{N-1} \left[ h_{0}(nT) * h_{1}(nT) \right]^{2} \leq \\ \leq \sum_{n=0}^{N-1} \left[ \sum_{n=0}^{N-1} h_{0}(nT) \right]^{2} \cdot \left[ \sum_{n=0}^{N-1} h_{1}(nT) \right]^{2}, \quad (10)$$

where symbol \* designates the convolution operation.

If we omit intermediate calculations, the definitive result will be the following:

$$\sum_{n=0}^{N-1} h^{2}(nT) = \left[\frac{(B_{0} + \frac{1}{2})(B_{0}^{N-2} - 1)}{(B_{0}^{2} - 1)}\right]^{2} \times \left[\frac{(B_{1} - 1)(B_{1}^{N-2} - 1)}{(B_{1}^{2} - 1)}\right]^{2}.$$
(11)

For the simplicity of computations in the process of calculation of the second composed expression (9) we will suppose that all pulse reactions from *j*-th source of noise to an exit are equal among themselves and are defined by value h(nT). Then the second composed expression (9) will be equal to (taking into account Cauchy–Bunyakowsky inequality)

$$\sum_{j=1}^{M} \sum_{n=0}^{N-1} \left[ h_j(nT) \right]^2 = 5 \sum_{n=0}^{N-1} h^2(nT).$$

If  $\Delta = \Delta_0$  then expression (9) becomes simpler and it will be equal to

$$\sigma_{own}^{2} \leq \frac{\Delta^{2}}{2} \left[ \frac{(B_{0} + \frac{1}{2})(B_{0}^{N-2} - 1)}{(B_{0}^{2} - 1)} \right] \times \left[ \frac{(B_{1} - 1)(B_{1}^{N-2} - 1)}{(B_{1}^{2} - 1)} \right]^{2}.$$
 (12)

Substituting the values of poles  $B_0$  and  $B_1$  into equation (12) we will definitively receive (at  $N \ge 1000$ )

$$\sigma_{\rm own}^2 \le \frac{\Delta^2}{2} \cdot \frac{(5 + 4C + \sqrt{1 + 8C})^2}{(6 + 5C)^2}.$$
 (13)

The noise level of undercompensation in the invariant echo-jack will be equal to

$$\sigma_{\text{inv.und}}^2 = \sigma_{\text{cc}}^2 \cdot \sum_{n=0}^{\infty} h^2(nT), \qquad (14)$$

where h(nT) – is pulse reaction of the invariant echo-jack of the second order;  $\sigma_{cc}^2$  – capacity of noise of a communication channel. At

$$M_1(Z) = M_2(Z) = 1$$
 size  $\sum_{n=0}^{\infty} h^2(nT) \approx 1.003.$ 

The overall performance of any method of signal processing can be compared with an overall performance of known methods. As an analogue we will use the characteristics of Widrow classical algorithm which is realized in the equipment of xDSL. Comparison will be made taking into account the level of self-noise and the value of undercompensation of an echo-signal.

In Widrow algorithm the adaptive filter joins in parallel with a sending device and models the parameters of an unknown system, i. e. of an echo-path. Pulse reaction of the adaptive filter depends on the connected channel (line) and can be arbitrary. For the simplicity of computations we will suppose that the pulse reaction of the adaptive filter is described by the readings of the homogeneous digital filter and is equal

$$h_{\rm AF}(nT) = \{1; c; c^2; K \dots c^n\},$$
(15)

where *c* is the factor of transmission of the second tap of the adaptive filter (AF); c < 1.

For the calculation of self-noise level of the adaptive filter we will use expression (9). We will consider, that the word length of an entrance word  $\Delta_0$  and the word length of processing  $\Delta$  in the adaptive filter are equal. Besides we will suppose, that pulse reactions  $h_j(nT)$  from *j*-th source of noise in the adaptive filter are equal among themselves and are defined by pulse reaction of all adaptive filter  $h_{AF}(nT)$ . Omitting intermediate transformations, we definitively receive analytical expression of the calculation of self-noise level in Widrow classical algorithm:

$$\sigma_{\rm AF\,own}^2 = (N+1) \cdot \frac{\Delta^2}{12} \cdot \frac{1-c^{2n}}{1-c^2},$$
 (16)

where N – is the quantity of taps of the adaptive filter;  $n = \{0; 1; ...\infty\}$  – is a supervision step; c – is a factor of transmission of the second tap of the adaptive filter;  $\Delta$  – is a quantization step;  $\Delta = \frac{1}{2^m}$ , m – is the word length of processing.

Let's estimate the noise level of undercompensation in Widrow classical algorithm which is caused by the influence of noise of a communication channel. The noise level of undercompensation at an exit of the adaptive filter in classical algorithm is equal to [5]

$$\sigma_{\rm AF\,undercomp}^2 = \frac{\gamma \cdot N \cdot R}{\left|2 - \gamma \cdot N\right|},\tag{17}$$

where  $\gamma$  - is a fine tuning step (in relative sizes),  $\gamma \in \{0, 1\}$ ; N - is the quantity of taps in the adaptive filter;  $R = \sum_{n=N+1}^{\infty} h_{AF}^2(nT) + \sigma_{cc}^2$ ;  $\sum_{n=N+1}^{\infty} h_{AF}^2(nT)$  - considers noise of work of the adaptive filter because of truncation of its pulse reaction to size *N*;  $\sigma_{cc}^2$  – is the capacity of noise of a communication channel.

At an intake input (a subtracter exit) in classical algorithm the level of noise of a communication channel doubles. It is caused by the fact that undercompensation noise  $\sigma^2_{AFundercomp}$ and communication channel noise  $\sigma^2_{cc}$  are not correlated.

The gain value in relative sizes will be equal to

$$\Delta A = 10 \, \text{lg} \cdot \frac{\sigma_{A\Phi}^2}{\sigma^2}, \qquad (18)$$

where  $\sigma^2$  – defines either the self-noise level or noise level of undercompensation in the invariant echo-jack;  $\sigma^2_{A\Phi}$  – defines either the self-noise level or noise level of undercompensation in Widrow algorithm.

For c = 0.9; N = 100; n = 1,000; m = 12 the size of  $\Delta A_{own}$  will be equal to 21.79db.

Similarly, for c = 0.9; N = 100;  $\gamma = 0.05$ ;  $P_{\text{ur.cc}} = 40$ db; the size  $\Delta A_{\text{undercomp}}$  will be equal to 5.3 db.

The received gain can be explained by several reasons:

Firstly, the invariant echo-jack is controlled by a transmission signal. The classical echo-jack is controlled by a signal of an error from a subtracter exit.

Secondly, the invariant echo-jack uses the readings of hindrance taken directly from a communication channel. The classical echo-jack calculates the echo-signal estimation artificially. Thirdly, the work of the invariant echo-jack does not depend on correlation communications of signals of two directions.

The structure of the invariant echo-jack of the second order is synthesized. The overall performance of the invariant echo-jack is proved. The invariant echo-jack of the second order can find application in systems of telecommunications and objects control.

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## THE NOISE IMMUNITY OF THE INVARIANT SYSTEM OF INFORMATION TRANSMISSION BASED ON COHERENT RECEPTION UNDER WEAK CORRELATION COMMUNICATIONS

The invariant system of information processing based on obtaining of the rectangular envelope by using a synchronous detector is considered. The indexes of the noise immunity of such system are calculated. It is supposed that the closest readings of the rectangular envelope are interfered with the additive noise whose readings are weakly correlated with each other. The quantitative estimation of the operation of such system is compared with the quantitative indexes of the known invariant system under non-correlativity of the noise readings.

*Keywords: noise immunity, invariant, invariant relative amplitude modulation, probability of pairwise transition, signal/noise relation, coefficient of correlation.* 

The method of analysis of the qualitative parameters of the invariant system using synchronous detector under the weak correlativity of the noise readings is developed. The analytical expression of calculation of the probability density of invariants transition is worked out on the basis of the expression of invariant estimation.

The results obtained under non-correlativity of the noise readings are presented. All this helps to use the offered structure for qualitative transmission of information. The invariant systems of information transmission can be based on different methods of information processing. The aim is to reduce the influence of the multiplicative noise using the algorithm of the particular information parameter to the training one [1].

The authors considered the four ways of signals processing with the help of invariant relative amplitude modulation and noise readings of different correlativity.