9. Signature verification using a "siamese" time delay neural network / J. Bromley, I Guyon., Y. LeCun et al. // NIPS. 1993. P. 737–744.

10. Learning the kernel matrix with semi-definite programming // G. Lanckriet, N. Cristianini, P. Bartlett, L. E. Ghaoui // J. of Machine Learning Research. 2004. Vol. 5. P. 27–72.

11. Weinberger K. Q., Saul L. K. Distance metric learning for large margin nearest neighbor classification // J. Machine Learning Research. 2009. Vol. 10 P. 207–244.

12. Yang L., Jin R. Distance metric learning: A comprehensive survey [Electronic resource] : Techn. Rep. Michigan State University. 2006. URL: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.91.4732 (data of visit: 30.07.2011).

13. Yang L. An overview of distance metric learning [Electronic resource] : Techn. Rep. Carnegie Mellon University. 2007. URL: http://www.cs.cmu.edu/~liuy/dist_overview.pdf (data of visit: 30.07.2011).

14. Learning to Learn / S. Thrun, L.Y. Pratt (ed.). Boston, MA : Kluwer Academic Publishers, 1998.

15. Regularized principal manifolds / A. J. Smola, S. Mika, B. Schlkopf, R. C. Williamson // JMLR. 2001. Vol. 1. P. 179–209.

16. Out-of-sample extensions for LLE, Isomap, MDS, Eigenmaps, and Spectral Clustering / Y. Bengio, J.-F. Paiement, P. Vincent et al. // NIPS. 2003. P. 177–184.

17. Globerson A., Tishby N. Sufficient dimensionality reduction // J. Machine Learning Research. 2003. Vol. 3. P. 1307–1331.

18. Ghahramani Z. Unsupervised Learning // Advanced Lectures in Machine Learning. Lecture Notes in Computer Sci. Berlin : Springer-Verlag, 2004. Vol. 3176. P. 72–112.

19. Luxburg U. A tutorial on spectral clustering // Statistics and Computing. 2007. Vol. 17. P. 395–416.

20. Jain A. K., Murty M. N., Flynn P. J. Data clustering : A review ACM Computing Surveys. 1999. P. 264–323.

21. Performance prediction challenge / I. Guyon, A. Saffari, G. Dror, J. Buhmann // IEEE/INNS conf. IJCNN 2006. Vancouver, Canada, July 16–21. 2006. P. 1649–1656.

22. Engel E. A. Modified artificial neural network for information processing with the selection of essential connections : Ph. D. thesis. Krasnoyarsk, 2004.

23. Engel E. A. Graphic information processing using intelligent algorithms // Vestnik. Sci. J. of Siberian State Aerospace Univ. № 4(25). 2009. P. 85–90.

24. Engel E. A. The hierarchical model of decision-making based on fuzzy neural networks for information processing, Vestnik. Sci. J. of Siberian State Aerospace Univ. № 1 (33). 2011. P. 83–86.

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ИСПОЛЬЗОВАНИЕ ИНТЕЛЛЕКТУАЛЬНЫХ МЕТОДОВ ДЛЯ ОБРАБОТКИ ИНФОРМАЦИИ НА ПРИМЕРЕ РЕШЕНИЯ ЗАДАЧ WCCI 2010

Рассмотрена актуальная проблема выбора стратегии решения слабоформализованных задач, предполагающих обработку как количественных, так и качественных данных, высокую размерность и пропуски в данных. Представлен детальный анализ моделей прогноза для обработки данных. Эксперименты подтверждают эффективность интеллектуальных алгоритмов, разработанных авторами.

Ключевые слова: обработка данных, слабоформализованные задачи, интеллектуальные алгоритмы.

УДК 517.95

M. B. Frost

DETERMINING THE SOURCE OF TRANSVERSE OSCILLATIONS OF AN ELASTIC ROD

Solvability of an inverse problem for the equations of transverse oscillations of an elastic rod (determining the source of oscillations on the basis of the rod deflection at the final time) is proved.

Keywords: elastic rod, inverse problem, source of oscillations, solvability.

Many issues of engineering, geophysics, and medicine involve problems whose sought quantities are elements of initial-boundary problems for differential equations. These unknown elements are determined on the basis of additional information. Such problems are called inverse problems for differential equations. The advanced theory of inverse problems and numerous publications can be found in [1].

Let us consider an inverse problem of determining the source of transverse oscillations of an elastic rod. The initial-boundary problem of transverse oscillations of a simply supported rod with a constant cross section and a length l under the action of a distributed load $q(x,t)$ is posed [2] as follows:

$$u_{tt} + a^2 u_{xxxx} = q(x,t), \quad (1)$$

$$u(x,0) = m(x),\ u_t(x,0) = n(x), \quad (2)$$

$$u(0,t) = u(l,t) = u_{xx}(0,t) = u_{xx}(l,t) = 0, \quad (3)$$

Here, $u = u(x,t)$, $0 \le x \le l$, $0 \le t \le T$ is the deflection, a is a constant, $m(x)$ and $n(x)$, $0 \le x \le l$ are given functions satisfying the conditions

$$m(0) = n(0) = m(l) = n(l) = 0. \quad (4)$$

Let us consider an inverse problem of determining the source of oscillations. We assume that

$$q(x,t) = \omega(t)\lambda(x), \quad (5)$$

where $\omega(t)$, $0 \le t \le T$, is a known function of time, and $\lambda(x)$, $0 \le x \le l$ is an unknown function determined with the use of an additional condition (deflection at the final time $t = T$):

$$u(x,T) = p(x),\ 0 \le x \le l,\ p(0) = p(l) = 0,\ T > 0. \quad (6)$$

We integrate Eq. (1) with respect to t from 0 to t, and then from 0 to T. Taking into account equalities (2–5), we obtain

$$p(x) - m(x) - Tn(x) + a^2 \int_0^T \left(\int_0^t u_{xxxx}(x,\tau)\,d\tau \right) dt = \lambda(x) \int_0^T \left(\int_0^t \omega(\tau)\,d\tau \right) dt. \quad (7)$$

Let us assume that

$$h \equiv \int_0^T (T-\tau)\omega(\tau)\,d\tau \ne 0. \quad (8)$$

Then, equality (7) yields

$$\lambda(x) = h^{-1}\Big(p(x) - m(x) - Tn(x) + a^2 \int_0^T (T-\tau) u_{xxxx}(x,\tau)\,d\tau \Big). \quad (9)$$

Substituting Eq. (9) into Eq. (1), we obtain

$$u_{tt} + a^2 u_{xxxx} - h^{-1} a^2 \omega(t) \int_0^T (T-\tau) u_{xxxx}(x,\tau)\,d\tau = h^{-1}\big(p(x) - m(x) - Tn(x)\big)\omega(t). \quad (10)$$

Thus, the problem of determining the function $u(x,t)$ is reduced to the initial-boundary problem for the loaded equation of oscillations (10) under conditions (2–4). In this case, condition (6) is satisfied automatically. The function $\lambda(x)$, $0 \le x \le l$ is then determined from equality (9).

Let us study solvability of the initial-boundary problem (10), (2–4) under the following assumptions:

– the function is $\omega(t) > 0$ (or $\omega(t) < 0$) for $0 < t < T$, $\omega(0) = 0$, $\omega(T) \ne 0$; the function $\omega(t)$ is continuously differentiable on $[0,T]$;

– the following expansions are valid:

$$m(x) = \sum_{k=1}^{\infty} m_k \sin\left(\frac{k\pi x}{l}\right),\ |m_k| < \frac{m^*}{k^{7+\varepsilon}};$$

$$n(x) = \sum_{k=1}^{\infty} n_k \sin\left(\frac{k\pi x}{l}\right),\ |n_k| < \frac{n^*}{k^{5+\varepsilon}}; \quad (11)$$

$$p(x) = \sum_{k=1}^{\infty} p_k \sin\left(\frac{k\pi x}{l}\right),\ |p_k| < \frac{p^*}{k^{7+\varepsilon}}$$

(ε, m^*, n^* and p^* are positive constants).

We seek for the solution of problem (10), (2–4) by the Fourier method of separation of variables in the form

$$u(x,t) = \sum_{k=1}^{\infty} U_k(t) \sin\left(\frac{k\pi x}{l}\right). \quad (12)$$

Substituting Eq. (12) into Eq. (10) and taking into account Eq. (2), (11), we obtain an infinite system of ordinary differential equations with initial conditions for the functions $U_k(t)$, $k = 1,2,\dots$:

$$\frac{d^2 U_k}{dt^2} + \mu_k^2 U_k(t) = e_k \omega(t).\ k = 1,2,\dots, \quad (13)$$

$$U_k(0) = m_k,\ \left.\frac{dU_k}{dt}\right|_{t=0} = n_k,\ \mu_k = \frac{ak^2\pi^2}{l^2}, \quad (14)$$

$$e_k = h^{-1}(p_k - m_k - n_k T) + h^{-1}\mu_k \int_0^T U_k(\tau)(T-\tau)\,d\tau. \quad (15)$$

Solving Eq. (13) with the initial condition (14) for $k = 1,2,\dots$, we obtain

$$U_k(t) = \frac{e_k}{\mu_k} \int_0^t \omega(\tau) \sin\big(\mu_k(t-\tau)\big)\,d\tau + m_k \cos(\mu_k t) + \frac{n_k}{\mu_k}\sin(\mu_k t). \quad (16)$$

Let us use $t = T$, $U_k(t) = p_k$ in Eq. (16) and assume that

$$\omega_k \equiv \int_0^T \omega(\tau) \sin\big(\mu_k(T-\tau)\big)\,d\tau \ne 0,\ k = 1,2,\dots. \quad (17)$$

The following equality is valid:

$$e_k = \frac{\mu_k}{\omega_k}\left(p_k - m_k \cos(\mu_k T) - \frac{n_k}{\mu_k}\sin(\mu_k T) \right). \quad (18)$$

As $\mu_k \to \infty$ with $k \to \infty$, then, by virtue of the assumptions made for the function $\omega(t)$, we obtain [3] for large values of k

$$|\omega_k| \sim \frac{l^2 \omega(T)}{a\pi^2 k^2}. \quad (19)$$

Therefore, the series determining the function $u(x,t)$ by equality (12) with allowance for Eqs. (16, 18, 19), is absolutely converging, and it can be differentiated four times with respect to x and two times with respect to t. It satisfies Eq. (10) and conditions (1–4).

This solution is unique in the class of functions that admit twofold differentiation with respect to t and fourfold differentiation with respect to x. The function $\lambda(x)$ for $0 \le x \le l$ is uniquely determined by equality (9) in the form of an absolutely converging series

$$\lambda(x) = \sum_{k=1}^{\infty} e_k \sin\frac{k\pi x}{l}.$$

Equality (6) is also satisfied. The constructed functions $u(x,t)$ and $\lambda(x)$ uniquely determine the solution of the inverse problem (1–6).

Note that condition (17) is always satisfied for the linear function $\omega(t) = \alpha t$ for $0 \le t \le T$, where α is a constant, because

$$1 - \frac{\sin(\mu_k T)}{\mu_k T} > 0 \text{ for all } \mu_k,\ k = 1,2,...$$

The algorithm of the numerical solution of problem (1–6) can be constructed as follows. Let $n = 0,1,2,...$ be the number of the iteration. We seek for the function $u^n(x,t)$ numerically, using the grid technique or the finite-element method [4] to solve the equation of rod oscillations with a known right side determined via the function $u^{n-1}(x,t)$

$$\begin{aligned} u_{tt}^n + a^2 u_{xxxx}^n = \frac{a^2}{h}\omega(t)\int_0^T u_{xxxx}^{n-1}(x,\tau)(T-\tau)\,d\tau + \\ + h^{-1}\left(p(x) - m(x) - Tn(x)\right)\omega(t), \end{aligned} \tag{20}$$

and satisfying conditions (2–4). An arbitrary function satisfying conditions (2–4) can be used as the initial approximation. The calculations are performed until

$$\int_0^l \left(\int_0^T \left(u_{xxxx}^n(x,\tau) - u_{xxxx}^{n-1}(x,\tau) \right)(T-\tau)\,d\tau \right)^2 dx < \delta,$$

where δ is an arbitrary, rather small number.

This condition means a small value of the integral

$$\int_0^l \left(\lambda^n(x) - \lambda^{n-1}(x) \right)^2 dx,$$

where $\lambda^n(x)$ is calculated via $u^n(x,\tau)$ by Eq. (9).

In conclusion, we can mention that the Fourier method can be used in a similar manner to prove unique solvability of the inverse problem of determining the source of oscillations of an elastic rectangular constant-thickness plate with sides a and b, which is simply supported on its edges. In this case, $u = u(x,y,t)$, $0 \le x \le a$, $0 \le y \le b$, $0 \le t \le T$; u_{xxxx} in Eq. (1) should be replaced by a biharmonic operator $\Delta^2 u$ for the variables x and y [5]; the transverse load should be taken in the form $q(x,y,t) = \omega(t)\lambda(x,y)$, where $\omega(t)$ is a known function of time and $\lambda(x,y)$ is an unknown function of coordinates, which is determined from the known value of the deflection at the final time $t = T$.

An inverse problem for the equation of transverse oscillations of an elastic rod with a constant cross section (determining the source of oscillations on the basis of the deflection at the final time) is considered in the paper. The problem is reduced to solving an initial-boundary problem for the loaded equation of oscillations containing a functional of the solution. Unique solvability of the initial-boundary problem for the loaded equation and then of the inverse problem is proved by the Fourier method. An algorithm of the numerical solution of the inverse problem is proposed.

References

1. Prilepko A. J., Orlovsky D. G., Vasin J. A. Methods for solving inverse problems in mathematical physics. Monographs and textbooks in pure and applied mathematics. N.-Y. : M. Dekker, 2000.
2. Krylov A. N. On some differential equations of mathematical physics, which have engineering applications. M.–L. : Gos. Izdat. Tekhn. – Teor. Lit., 1950.
3. Kopson E. T. Asymptotic Expansions. Cambridge : Cambridge Univ. Press, 1965.
4. Basov K. A. Ansys in Examples and Problems. M. : Computer Press, 2002.
5. Panovko Ya. G. Fundamentals of the Applied Theory of Oscillations and Impacts. L. : Politekhnika, 1990.

М. Б. Фрост

ОПРЕДЕЛЕНИЕ ИСТОЧНИКА ПОПЕРЕЧНЫХ КОЛЕБАНИЙ УПРУГОГО СТЕРЖНЯ

Рассмотрена задача определения источника поперечных колебаний упругого стержня по дополнительной информации – значению прогиба в финальный момент времени.

Ключевые слова: упругий стержень, обратная задача, источник поперечных колебаний, разрешимость.