UDC 539.3
Doi: 10.31772/2712-8970-2021-22-3-432-451

Для цитирования: Матвеев А. Д. Обобщенные эквивалентные условия прочности в расчетах композитных тел // Сибирский аэрокосмический журнал. 2021. Т. 22, № 3. С. 432–451. Doi: 10.31772/2712-8970-2021-22-3-432-451.

For citation: Matveev A. D. Generalized equivalent strength conditions in the calculations of composite bodies. *Siberian Aerospace Journal.* 2021, Vol. 22, No. 3, P. 432–451. Doi: 10.31772/2712-8970-2021-22-3-432-451.

Обобщенные эквивалентные условия прочности в расчетах композитных тел

А. Д. Матвеев

Институт вычислительного моделирования СО РАН
Российская Федерация, 630036, г. Красноярск, Академгородок, стр. 50/44
E-mail: mtv241@mail.ru

Конструкции с неоднородной регулярной структурой (пластины, балки, оболочки) широко применяются в технике, особенно, в авиационной и ракетно-космической. В расчетах на прочность упругих композитных конструкций с помощью метода конечных элементов (МКЭ) важно знать погрешность решения. Для анализа погрешности решения необходимо использовать последовательность приближенных решений, построенных по МКЭ с применением процедуры измельчения для базовых дискретных моделей, которые учитывают в рамках микроподхода неоднородную, микронеоднородную структуру конструкций (тел). Реализация процедуры измельчения для базовых моделей требует больших ресурсов ЭВМ.

В данной работе кратко изложен метод эквивалентных условий прочности (МЭУП) для расчета на статическую прочность упругих тел с неоднородной регулярной структурой, для которых заданы множества различных нагружений. Согласно МЭУП, расчет на прочность композитного тела, для которого задано нагружение, сводится к расчету на прочность изотропного однородного тела (имеющего такое же нагружение, как композитное тело) с применением эквивалентных условий прочности. При численной реализации МЭУП используются скорректированные эквивалентные условия прочности, которые учитывают погрешность приближенных решений. Здесь МЭУП реализуется на основе МКЭ. Если для композитного тела задано множество различных нагружений, то в этом случае применяются обобщенные эквивалентные условия прочности. Показана процедура построения обобщенных эквивалентных условий прочности. Расчет на прочность композитных тел по МЭУП с использованием многосеточных конечных элементов требует в $10^3 \div 10^6$ *раз меньше объема памяти ЭВМ, чем аналогичный расчет с применением измельченных базовых моделей композитных тел. Приведенный пример расчета на прочность композитной балки, для которой задано множество нагружений, с помощью МЭУП с применением обобщенных эквивалентных условий прочности показывает его высокую эффективность.*

Ключевые слова: упругость, композиты, многосеточные конечные элементы, скорректированные и обобщенные эквивалентные условия прочности.

Generalized equivalent strength conditions in the calculations of composite bodies

A. D. Matveev

Institute of Computational Modeling SB RAS
50/44, Akademgorodok, Krasnoyarsk, 630036, Russian Federation
E-mail: mtv241@mail.ru

Structures with an inhomogeneous regular structure (plates, beams, shells) are widely used in engineering, especially in aviation and rocket and space. It is important to know the solution error in the strength elastic calculations for composite structures using the finite element method (FEM),. To analyze the error of the solution, it is necessary to use a sequence of approximate solutions constructed according to the FEM using the grinding procedure for basic discrete models that take into account the non-homogeneous, micro-homogeneous structure of structures (bodies) within the micro-approach. The implementation of the grinding procedure for basic models requires large computer resources.

This paper deals with the method of equivalent strength conditions (MESC) for testing the static strength of elastic bodies with an inhomogeneous regular structure, for which sets of different loads are given. According to the MESC, the calculation of the strength of a composite body for which the loading is set is reduced to the calculation of the strength of an isotropic homogeneous body (having the same loading as a composite body) using equivalent strength conditions. In the numerical implementation of the MESC, adjusted equivalent strength conditions are used, which take into account the error of approximate solutions. Here, the MESC is implemented on the basis of the FEM. If a set of different loads is specified for a composite body, then generalized equivalent strength conditions are applied in this case. The procedure for constructing generalized equivalent strength conditions is shown. The calculation of the strength of composite bodies according to the MESC using multigrid finite elements requires $10^3 \div 10^6$ *times less computer memory than a similar calculation using crushed basic models of composite bodies. The given example of calculating the strength of a composite beam, for which a number of loads is set with MESC using generalized equivalent strength conditions shows its high efficiency.*

Keywords: elasticity, composites, multigrid finite elements, corrected and generalized equivalent strength conditions.

Introduction

As a rule, the calculation of the strength of an elastic structure is carried out according to the safety factor and is reduced to determining the maximum equivalent stress of the structure (body) [1–3]. For an elastic body V_0, the given strength conditions are of the form $n_1 \le n_0 \le n_2$, where n_1, n_2, are given, the safety factor of the body n_0 corresponds to the exact solution of the problem of the theory of elasticity constructed for the body V_0. It is believed that the body does not collapse during operation if its safety factor satisfies the specified strength conditions. The determination of the safety factor n_0 for a composite body (CB), where is $n_0 = \sigma_T / \sigma_0$, σ_T the limit stress σ_0 [1], i.e. determination of the maximum equivalent stress [1] CT that meets the exact solution the task of elasticity is difficult. If the stresses in the bodies are determined approximately, then in this case we use the adjusted strength conditions [4], which take into account the error of solutions. In stress-strain analysis (VAT) the finite element method (FEM) is widely used [5; 6].

Finite element (discrete) basic models (BM), which take into account the heterogeneous structure of bodies within the framework of the micro approach [7], have a high dimension. In addition, to analyze the convergence and error of the solution, it is necessary to use the sequence of constructed solutions using the finite element grinding (FE) procedure of BM CB, which leads to a sharp increase in the dimensions of discrete models. For the analysis of CB VAT, the method of multigrid finite elements (MME) [8–14] is effectively used, in which multi-grid finite elements (MNKE) are used and which is a generalization of FEM, since if MNKE is used in FEM, then in this case, in fact, THE CMI is implemented. In the areas of MnKE [8–19], the heterogeneous structure is taken into account and the three-dimensional VAT is described.

It is important to note that MnKE generate discrete models whose dimensions are less than the dimensions of BM CB. For a number of CT (for example, for bodies with a microhedel structure), BMs have such a high dimension that the implementation of FEM using MnKE is also difficult. Existing methods for calculating CT [20–27] are based on hypotheses, have complex formulations and are difficult to implement.

The method of equivalent strength conditions (MESC) is proposed in this paper, to calculate the strength of elastic bodies with an inhomogeneous, microunitive regular structure, which is reduced to the calculation of the strength of elastic isotropic homogeneous bodies using equivalent strength conditions according to feM. Unlike the works [28; 29], the theorem that underlies the MESC is presented in detail here. In numerical implementation, MESC uses adjusted equivalent strength conditions that take into account the error of the solutions. For CB, for which many different loads are specified, generalized equivalent strength conditions are used in the calculations. Implementation of MEPM on the basis of FEM using MnKE requires $10^3 \div 10^6$ times less computer resources than FEM calculation on the basis of grinding BM CT. An example of CB calculation according to MESC shows its high efficiency.

1. Basic provisions of the method of equivalent strength conditions

MESC is applied to CB that satisfies the following provisions.

Regulation 1. CB consists of multi-module isotropic homogeneous bodies, the connections between which are ideal, i.e. at the general boundaries of isotropic homogeneous bodies, the functions of displacements and stresses are continuous.

Regulation 2. Displacements, deformations and stresses of multimodular isotropic homogeneous bodies correspond to the relations of the linear theory of elasticity [30].

Regulation 3. Approximate solutions of BM CB, built according to FEM, differ from accurate solutions. Such approximate decisions will be considered accurate.

2. Equivalent strength conditions

Let elastic bodies V_1, V_2 have the same characteristic dimensions, shape, fasteners and static loads, but differ in modulations of elasticity. Let for the safety factors n_1, n_2, respectively bodies V_1, V_2 , be given strength conditions

$$n_a^1 \le n_1 \le n_b^1, \qquad (1)$$

$$n_a^2 \le n_2 \le n_b^2, \qquad (2)$$

where $n_a^1, n_a^2 > 1$; $n_a^1, n_a^2, n_b^1, n_b^2$ are given; the reserve coefficient n_1 (n_2) corresponds to the exact solution of the problem of the theory of elasticity, built for the body V_1 (body V_2).

For bodies V_1, V_2, enter the following definition.

Definition. If from the fulfillment of conditions (2) for the coefficient n_2 follows the fulfillment of conditions (1) for the coefficient n_1 and vice versa, if from the fulfillment of conditions (1) for the coefficient n_1 follows the fulfillment of conditions (2) for the coefficient n_2, then the conditions of strength (1), (2) will be called equivalent conditions of strength co-responsible for bodies V_2, V_1.

3. Basic theorem of the method of equivalent strength conditions

Without losing the generality of judgments, we consider bodies with a fibrous structure, which are widely used in practice and in which the maximum equivalent stresses arise in the fibers. The MESC is based on the following theorem.

Theorem 1. Let the load and strength conditions of the form F be set for the safety factor n_0 of the elastic CT (fibrous structure)

$$n_1 \le n_0 \le n_2, \tag{3}$$

where the values n_1, n_2 are given, $n_1 > 1$, $n_0 = \sigma_T / \sigma_0$, σ_T, is the limit stress of CT (the yield strength of the fiber), σ_0 is the maximum equivalent stress of CB V_0, the stress σ_0 corresponds to the exact solution of the problem of the theory of elasticity, built for loading F CB V_0, the body fibers V_0 have the same modulus of elasticity.

Let the homogeneous isotropic body V^b and CB V_0 have the same shape, characteristic dimensions, fastenings and loading F. Let the elastic modules of the body V^b and the CB fibers are the same. Then there exists such a number $p > 0$ (equivalence coefficient) that if the safety factor of the body V^b satisfies the adjusted equivalent conditions of strength

$$\frac{pn_1}{1-\delta_\alpha} \le n_b \le \frac{pn_2}{1+\delta_\alpha}, \tag{4}$$

then the safety factor n_0 CB V_0 meets the specified conditions of strength (3), where , $n_b = \sigma_T / \sigma_b$ is σ_b the maximum equivalent stress of the body V^b, corresponding to the numbered solution constructed for loading the body with an error δ_b, $|\delta_b| \le \delta_\alpha$, where δ_α is the upper estimate of the error δ_b, satisfying the condition

$$\delta_\alpha < C_\alpha = (n_2 - n_1)/(n_2 + n_1) \tag{5}$$

<u>*Proof.*</u>

Safety factor n_0, n_b^0, respectively bodies V_0, V^b are found by formulas

$$n_0 = \sigma_T / \sigma_0, \tag{6}$$

$$n_b^0 = \sigma_T / \sigma_b^0, \tag{7}$$

where σ_b^0 is the maximum equivalent stress of the body V^b corresponding to the exact solution of the problem of the theory of elasticity constructed for loading F the body V^b.

Let the coefficient n_0 satisfy the conditions (3). Using (6) in (3), we have

$$n_1 \le \frac{\sigma_T}{\sigma_0} \le n_2 . \tag{8}$$

There is such a number $p > 0$ (equivalence coefficient) that

$$p = \frac{\sigma_0}{\sigma_b^0} . \tag{9}$$

Given (9) in (8), we get

$$pn_1 \le \frac{\sigma_T}{\sigma_b^0} \le pn_2 . \tag{10}$$

Using (7) in (10), we have

$$pn_1 \le n_b^0 \le pn_2 . \tag{11}$$

Let the body's V^b safe factor n_b^0 satisfies the conditions of strength (11).

Then, substituting (7) in (11) with (9), we have $pn_1 \le \frac{p\sigma_T}{\sigma_0} \le pn_2$. From where, taking into account (6), the strength conditions for the CB safety factor V_0 (3) follows. Let's consider the limit cases. Let $n_b^0 = pn_1$. Using the relations (7), (9) in the last equality, we obtain $p\frac{\sigma_T}{\sigma_0} = pn_1$. From where , taking into account (6) $n_b^0 = pn_1$ follows. Similarly, we show that if $n_0 = n_2$, the $n_b^0 = pn_2$ then . Let $n_0 = n_1$. Using (6), (9) in the last equality, we get $\frac{\sigma_T}{\sigma_b^0} = pn_1$. From where, taking into account (7) follows $n_b^0 = pn_1$. Similarly, we show that if $n_0 = n_2$, then $n_b^0 = pn_2$. So it is shown that (11) are equivalent strength conditions for CB V_0 (see definition of paragraph 2). Let the maximum equivalent stress σ_b e be found for the body V^b such that

$$|\delta_b| \le \delta_\alpha < C_\alpha = (n_2 - n_1)/(n_1 + n_2) , \tag{12}$$

where δ_b is the relative error for σ_b , i.e.

.

$$\delta_b = (\sigma_b - \sigma_b^0)/\sigma_b^0 . \tag{13}$$

From (13) follows $\sigma_b = (1+\delta_b)\,\sigma_b^0$. From here, considering (7) and that $n_b = \sigma_T / \sigma_b$, we get

$$n_b^0 = (1+\delta_b) n_b . \tag{14}$$

Note , that in (12) $C_\alpha < 1$. Let $\delta_0 = |\delta_b|$. Then due to (12)

$$0 \le \delta_0 = |\delta_b| \le \delta_\alpha < 1 . \tag{15}$$

Taking in (14) $\delta_b = -\delta_0$, $\delta_b = \delta_0$ sequentially , enter the coefficient

$$n_1^r = (1-\delta_0)n_b, \quad n_2^r = (1+\delta_0)n_b. \tag{16}$$

Then due to (14), (16) we get

$$n_b^0 = n_1^r \text{ or } n_b^0 = n_2^r \tag{17}$$

Let's enter the coefficients n_1^d, n_2^d according to the formulas

$$n_1^d = (1-\delta_\alpha)n_b, \quad n_2^d = (1+\delta_\alpha)n_b. \tag{18}$$

Because of $0 \le \delta_\alpha < 1$, $n_b > 0$, it follows from (18)

$$n_1^d \le n_2^d. \tag{19}$$

Adjusted equivalent strength conditions are of the form (4) or

$$pn_1(1+\delta_\alpha) \le n_b(1-\delta_\alpha^2) \le pn_2(1-\delta_\alpha), \tag{20}$$

where $n_b = \sigma_T / \sigma_b$, σ_T , is the limit voltage of CB (the yield strength of the fiber).

Let n_b the conditions of strength (20) be fulfilled, i.e. let $pn_1 \le (1-\delta_\alpha)n_b$ and $(1+\delta_\alpha)n_b \le pn_2$ Then it follows that for coefficients n_1^d, n_2^d, taking into account (18), (19) inequalities are fulfilled

$$pn_1 \le n_1^d \le n_2^d \le pn_2. \tag{21}$$

Comparing (16), (18) taking into accounts (15), the inequality $n_1^d \le n_1^r$, $n_2^r \le n_2^d$ follow.

Hence, given, that according to (16) $n_1^r \le n_2^r$, we get

$$n_1^d \le n_1^r \le n_2^r \le n_2^d. \tag{22}$$

Then by virtue of (21), (22) inequalities are fulfilled

$$pn_1 \le n_1^r \le n_2^r \le pn_2. \tag{23}$$

From the implementation (23) taking into account (17) follows the fulfillment of the conditions of strength (11) for the reserve factor n_b^0, therefore, the fulfillment of the specified conditions of strength (3). The constraints on the parameter δ_α are found from the condition of existence of strength conditions (4), i.e. let $pn_1(1+\delta_\alpha) \le pn_2(1-\delta_\alpha)$ .Where it comes from

$$\delta_\alpha < C_\alpha = (n_2 - n_1)/(n_1 + n_2). \tag{24}$$

Since $n_2 > n_1 > 1$, then from (24) it follows $0 < C_\alpha < 1$. If $\delta_\alpha = C_\alpha$, then from (4) it follows $n_b = p(n_1 + n_2)/2$ that it is difficult to perform in practice. Therefore, you should specify such δ_α that $\delta_\alpha < C_\alpha$.

In this case, the conditions (11) for the body V^b safety factor n_b^0 can be met using adjusted equivalent strength conditions (4) and numerical solutions that generate such errors δ_b for body V^b stresses that $|\delta_b| \le \delta_\alpha$. It has been shown that the fulfillment of conditions (11) follows the fulfillment of strength conditions (3). The theorem is proven.

According to theorem 1, the implementation of the MESC is reduced to the determination of the coefficient p and the safety factor n_b of the body V^b, i.e. to the determination of the maximum equivalent stress σ_b of the body V^b with an error $|\delta_b| < \delta_\alpha$, $n_b = \sigma_T / \sigma_b$.

4. Implementation of the method of equivalent strength conditions

Without losing the commonality of judgments, for simplicity of presentation, the procedure for implementing the MESC will be considered on the example of a body V_0 with an inhomogeneous regular structure of sizes $H \times H \times H$, where $H = 6Nh$, N is the whole, $N >> 1$, h little. CB V_0, located in the Cartesian coordinate system $Oxyz$, with $y = 0$ rigidly fixed, i.e. at $y = 0$: $u, v, w = 0$. The regular cell G_0 CB V_0, having the shape of a cube with a side $6h$, is located in the local Cartesian coordinate system $Oxyz$, $i, j, k = 1,...,7$ (Fig. 1), the fibers are directed along the axis Oy by cross-section $h \times h$, the fiber sections are painted over. So, the body V_0 is reinforced with pa-rally axes Oy of continuous fibers. Strength conditions (3) are set for CB V_0. BM R_0 CB V_0, consisting of finite elements (CE) V_j^h of the 1st order of the shape of the cube with a side h (in which the three-dimensional VAT is realized), takes into account the heterogeneous structure of the body V_0 and generates a uniform grid with a step h. We think that 3 MESC for CB V_0 is performed.

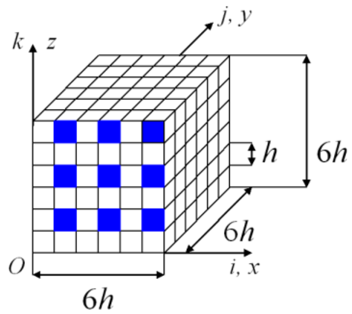

Рис. 1. Регулярная ячейка G_0

Fig. 1. Regular cell G_0

Note that the implementation of the MESC is reduced to the determination of the equivalence coefficient p, the body V^b reserve coefficient n_b and the construction of adjusted equivalent strength conditions (4).

Finding the equivalence coefficient *p*

According to the MESC, we will introduce an isotropic homogeneous body V^b and CB as R^0 are follows that the bodies V^b, R^0 and V_0 have the same shape, characteristic dimensions, given fastenings and loads, but differ in modulations of elasticity. The moduluses of elasticity of the body V^b are equal to the modules of elasticity of the CB V_0 fiber. For the body V^b (for CB R^0) we define discrete models V_n^b (models R_n^0) that form sequences $\{V_n^b\}_{n=1}^N$, $\{R_n^0\}_{n=1}^N$. The model V_N^b is BM body V^b. The model V_n^b (model R_n^0) consists of CE $V_j^{(n)}$ 1st order of cube-shaped FE with a side h_n in which a three-dimensional stress state is realized and that generates a uniform grid with a dimension $n_1^{(n)} \times n_2^{(n)} \times n_3^{(n)}$, step h_n, where

$$n_1^{(n)} = 6n + 1, \quad n_2^{(n)} = 6n + 1, \quad n_3^{(n)} = 6n + 1, \quad n = 1,...,N. \tag{25}$$

According to (25), the model V_n^b (model R_n^0) consists of a finite number of isotropic homogeneous bodies G_n^b s (CT G_n^0) of the same shape and size , with dimensions $6h_n \times 6h_n \times 6h_n$, where

$$h_n = H / (6n) = \beta_n h \, , \tag{26}$$

where $H = 6Nh$, $\beta_n = N / n$, $n = \overline{1, N}$ at $n < N$: $\beta_n > 1$, $h_n > h$, when $n \to N$ we have $h_n \to h$, $h_N = h$.

CB G_n^0 has the same number of grid nodes (343 nodes), the number of fibers (cross-section $h_n \times h_n$) and the same mutual arrangement as a regular cell G_0 (Fig. 1). Fibers and matrix CB G_n^0 and G_0 have the same modules of elasticity $n = \overline{1, N}$, (Fig. 2), where $h_n > h$ at $n < N$, $i, j, k = 1, ..., 7$.
C B G_n^0, G_0, (their heterogeneous structures) geometrically differ only in scale. For the convenience of reasoning, formally for CT G_n^0, G_0, let's write down the ratio

$$G_n^0 = \beta_n G_0 \, , \tag{27}$$

where β_n is the scale coefficient, $\beta_n = N / n$, $n = \overline{1, N}$, at $n \to N$: $\beta_n \to 1$, $\beta_N = 1$, $G_N^0 = G_0$.

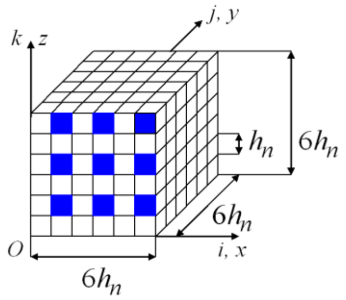

Рис. 2. КТ G_n^0 (регулярная ячейка модели R_n^0)

Fig. 2. CB G_n^0 (regular cell body R_n^0)

Note that since in the regular cell G_0 the heterogeneous structure is taken into account, then due to (27) and in CB G_n^0 ($n = \overline{1, N}$)the heterogeneous structure using the CE $V_j^{(n)}$ of the 1st order of the shape of the cube with a side h_n is also taken into account, i.e. the model R_n^0 takes into account the heterogeneous structure. Note that CB G_n^0, in fact, is a regular cell of the model. So, the models V_n^b, R_n^0 have the same shape, dimension, the same characteristic dimensions, uniform grids with a step h_n, fastening and loading, like CB V_0, i.e. models V_n^b, R_n^0, differ from each other only in modulations of elasticity. Note the following advantages of the models V_n^b, R_n^0 .

1. Dimensions of models V_n^b, R_n^0 with $n < N$ the force of (25), (26) less than the dimension of BM R_0.

2. When building models $\{R_n^0\}_{n=1}^N$, BM R_0 grinding is not used.

To reduce the dimensions of the models V_n^b, R_n^0, multi-grid FE are used.

Due to (26), (27) at $n = N$ ($h_N = h$, $\beta_N = 1$ i.e. $G_N^0 = G_0$) models V_N^b, R_N^0, and BM R_0 CB V_0 have the same dimension, and models R_N^0 and R_0 in force (27) coincide, i.e. $R_N^0 = \mathrm{R}_0$. Since, according to (27), at $n \to N$ we have $G_n^0 \to G_0$, then we get

$$R_n^0 \to R_N^0 = \mathrm{R}_0 \quad \text{at } n \to N . \tag{28}$$

Since the models R_N^0, V_N^b, have the same dimension as the BM R_0, for which the positions 3 MESC are executed, then we consider that the maximum equivalent stress σ_N^0 (stress σ_N^b) of the model R_N^0 (model V_N^b) differs little from the exact σ_0 (σ_b^0). Therefore, we believe that

$$\sigma_0 = \sigma_N^0 , \sigma_b^0 = \sigma_N^b , \tag{29}$$

where σ_b^0 is the maximum equivalent stress of the body V^b corresponds to the exact solution of the three-dimensional problem of the theory of elasticity constructed for the body V^b.

The equivalence coefficient p is found by the formula (9), i.e. $p = \sigma_0 / \sigma_b^0$ or including (29)

$$p = \sigma_N^0 / \sigma_N^b . \tag{30}$$

The approximate value of the equivalence coefficient p_n is found by the formula

$$p_n = \sigma_n^0 / \sigma_n^b , \tag{31}$$

where σ_n^0 (σ_n^b) is the maximum equivalent stress of the model R_n^0 (model V_n^b).

Due to (26) at $V_n^b \to V_N^b$, $n \to N$ should follow . Hence, given (28), we have

$$\sigma_n^0 \to \sigma_N^0 , \quad \sigma_n^b \to \sigma_N^b \quad \text{at } n \to N . \tag{32}$$

Taking into account (32), (29), (30) in (31), we get

$$p_n \to p \quad \text{at } n \to N . \tag{33}$$

Let it be $\delta_n = | p_n - p_{n-1} | / p_n$ few where $n = 2,3,...$. Then we accept

$$p = p_n . \tag{34}$$

Calculations show uniform (monotonous) convergence of stress σ_n^0, σ_n^b, and parameter p_n respectively to stress σ_N^0, σ_N^b, and parameter p.

Construction of adjusted equivalent strength conditions

Substituting the found equivalence coefficient p and given values of δ_α, n_1, n_2 in (4), we determine the adjusted equivalent strength conditions for CB V_0.

Finding a safety factor n_b for a homogeneous isotropic body V^b

Let it be $\delta_n^\sigma = | \sigma_n^b - \sigma_{n-1}^b | / \sigma_n^b$ few and $| \delta_n^\sigma | \le \delta_\alpha$, where $\delta_\alpha < C_\alpha$, $n = 2,3,...$. Then we assume

$$\sigma_b = \sigma_n^b . \tag{35}$$

Using (35) in the formula $n_b = \sigma_T / \sigma_b$, we determine the safety factor n_b for the body V^b

$$n_b = \sigma_T / \sigma_n^b . \qquad (36)$$

Checking the specified strength conditions

Let the safety factor of the isotropic homogeneous body V^b found by the formula (36), i.e., corresponding to the numerical solution of the elasticity problem, satisfies the adjusted equivalent strength conditions (4) constructed for CBV_0. Then, according to theorem 1 (see paragraph 3), the safety factor n_0 of CBV_0, corresponding to the exact solution of the elasticity problem satisfies the specified conditions of strength (3).

The procedure for constructing generalized equivalent strength conditions for CBV_0, for which many different loads are specified, without losing the commonality of judgments, will be considered with the CBV_0 example. Let on the surface S of the CB the loading of the form q_x, q_y, q_z acts, where q_x, q_y, q_z are the surface loads acting respectively in the direction of the coordinate axes Ox, Oy, Oz; $q_x, q_y, q_z \in Q_{xyz}$, Q_{xyz} is a set of different loads given for CB V_0,

$$Q_{xyz} = \{q_x, q_y, q_z : \quad q_x, q_y, q_z - \text{гладкие функции, заданные на } S\} \qquad (37)$$

smooth functions given on *S*. ВСТАВИТЬ ТЕКСТ

To find the (upper, lower) boundaries for the set **P** of equivalence coefficients corresponding to the load set (37), we calculate for a number of characteristic loads of CBV_0: $q_x = q_x^{(n)}$, $q_y = q_y^{(n)}$, $q_z = q_z^{(n)}$ ($q_x^{(n)}$, $q_x^{(n)}$, $q_z^{(n)}$ smooth functions), $n = \overline{1, N_0}$, N_0 - given. Enter the coefficients

$$p_1 = \min(p^{(n)}),\ p_2 = \max(p^{(n)}),\ n = \overline{1, N_0}, \text{ т. е. } \forall p \in \mathbf{P}:\ p_1 \le p \le p_2 . \qquad (38)$$

Let the condition for CTV_0 be met

$$p_2 C_1 \le p_1 C_2 , \qquad (39)$$

where is $C_1 = n_1 / (1 - \delta_\alpha)$, $C_2 = n_2 / (1 + \delta_\alpha)$.

For the equivalence coefficient $p_q \in [p_1, p_2]$, which is found by FEM for body loading $q_x, q_y, q_z \in Q_{xyz}$, the strength conditions (4) take the form

$$p_q C_1 \le n_b \le p_q C_2 , \qquad (40)$$

where n_b is the safety factor of the isotropic homogeneous body V^b.

According to (38) we have $p_q C_1 \le p_2 C_1$, $p_1 C_2 \le p_q C_2$. Using these inequalities and (39), we get $p_q C_1 \le p_2 C_1 \le p_1 C_2 \le p_q C_2$. Let the loading of the body $q_x, q_y, q_z \in Q_{xyz}$ be such that

$$p_q C_1 \le p_2 C_1 \le n_b \le p_1 C_2 \le p_q C_2 , \qquad (41)$$

i.e. the following conditions of strength are met for the body V^b safety factor n_b

$$p_2 C_1 \le n_b \le p_1 C_2 . \qquad (42)$$

Let the coefficient n_b for the body V^b satisfies the conditions of strength (42) for loading $q_x, q_y, q_z \in Q_{xyz}$. Then the conditions (41) are met for the coefficient n_b, i.e. the conditions of strength (40). According to theorem 1 (see paragraph 3), from the fulfillment of the conditions of strength (41) follows the fulfillment of the specified strength conditions (3) for loading $q_x, q_y, q_z \in Q_{xyz}$ CBV_0. Note that according to the MESC, the body V^b and the CBV_0 have the same loads (see paragraph 4). Thus, it is shown that from the fulfillment of the conditions of strength (42) for a body having a load, the fulfillment of the strength conditions (3) follows for loading $q_x, q_y, q_z \in Q_{xyz}$ CTV_0. Conditions (42) will be called generalized equivalent conditions of strength. In fact, the following statement is proved above.

Theorem 2. Let for the set Q of different loads given for CBV_0, according to the MESC, generalized equivalent strength conditions (42) are constructed. Let for the reserve factor n_b of an isotropic homogeneous body V^b having a load $F \in Q$, the strength conditions (42) are met. Then the specified strength conditions (3) for loading F CBV_0 are met.

6. Results of numerical experiments

Consider the model problem of calculating the strength of a cantilever composite beam V_0, the dimensions $H \times L \times H$, where $H = 96h$, $L = 1152h$, h is given (Fig. 3). The regular cell G_0 of CB V_0 has the shape of a cube with a side $6h$, the fibers $h \times h$ are parallel to the axis Oy (Fig. 4), the fiber sections in the plane Oxz are painted over. So, the body V_0 is reinforced with parallel axes Oy with continuous fibers, the distance between the fibers is $2h$. When $y = 0$ CT V_0 is rigidly fixed and at $z = H$ has a load of the form q_x, q_z, where q_x (q_z) is the force acting on the beam in the direction of the axis Ox (axis Oz).

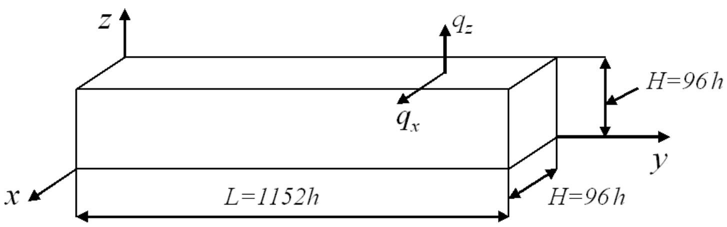

Рис. 3. Размеры тела V_0 (тела V^b, моделей V_n^b, R_n^0)

Fig. 3. Dimensions of the body V_0 (body V^b, models V_n^b, R_n^0)

The basic discrete model $\mathbf{R}_0$ of CBV_0, consisting of single-grid finite elements (1 cCE) V_j^h of the 1st order of the cube shape with a side h [5; 6] (in which the three-dimensional VAT is implemented [30]), takes into account the heterogeneous structure of the body V_0 and generates an equal-dimensional (basic) grid with dimension $97 \times 1153 \times 97$ step h. Fig. 4 shows the base grid of the regular cell G_0. Since the BM $\mathbf{R}_0$ has 32517504 (over 32 million) unknown FEMs and since $h / H << 1$ ($h / H = h / (96h) = 0,0104$), we will assume that the maximum equivalent voltage of the BM $\mathbf{R}_0$ differs little from the exact solution, i.e. put. 3 MESC for CBV_0 is performed (see paragraph 1).

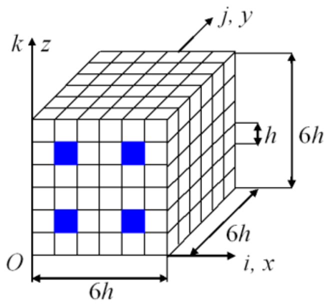

Рис. 4. Регулярная ячейка G_0

Fig. 4. Regular cell G_0

For the CB V_0 safety factor n_0, the strength conditions of the type are set

. $$1{,}3 \le n_0 \le 3{,}5\,.$$ 43)

Initial data for CB V_0: $h = 0{,}2083$; $\sigma_T = 5$; $\nu_c = \nu_v = 0{,}3$ $E_c = 1$, $E_v = 10$, where E_c, E_v (ν_c, ν_v)are the Jung modules (Poisson coefficients) respectively of the binder material and fiber, on the surface $S = \{0{,}5L \le y \le L,\ z = H\}$ of the CB V_0 there is a uniform loading $q_z = q_x = 0{,}000285$, σ_T is the yield strength of the fiber.

According to the MESC, we will introduce an isotropic homogeneous body V^b and CB R^0 such that the bodies V^b, R^0 and V_0 have the same shape, characteristic dimensions, given fastenings and loads, but differ in modulations of elasticity. Body V^b elasticity modules are equal to fiber elasticity moduluses CB V_0. For the body V^b (for CB R^0) define discrete models V_n^b (models R_n^0) that form sequences $\{V_n^b\}_{n=1}^{16}$, $\{R_n^0\}_{n=1}^{16}$. The model V_n^b (model R_n^0) consists of a 1st order cube shaped 1cCE $V_j^{(n)}$ with a side h_n in which the three-dimensional VAT is realized and which generate a uniform grid with a dimension $n_1^{(n)} \times n_2^{(n)} \times n_3^{(n)}$ step h_n, where

$$n_1^{(n)} = 6n + 1,\ n_2^{(n)} = 12 \times 6n + 1,\ n_3^{(n)} = 6n + 1,\ n = 1, 2, 3, \ldots. \qquad (44)$$

The steps $h_x^{(n)}$, $h_y^{(n)}$, $h_y^{(n)}$ of the grids of the model V_n^b (model R_n^0) respectively along the axis Ox, Oy, Oz, are equal to $h_x^{(n)} = H/(6n)$, $h_y^{(n)} = L/(72n)$, $h_z^{(n)} = H/(6n)$. Since $L = 12H$, then $h_n = h_x^{(n)} = h_y^{(n)} = h_z^{(n)}$. From here, given that $H = 96h$, we get

$$h_n = \beta_n h\,, \qquad (45)$$

where $\beta_n = 16/n$, $n = 1, 2, 3, \ldots$, at, $n \le 15$ $\beta_n > 1$, $h_n > h$.

According to (44), the model V_n^b (model R_n^0) (Fig. 3) consists of a finite number of isotropic homogeneous bodies G_n^b (CB G_n^0) of the same shape and size $6h_n \times 6h_n \times 6h_n$ (Fig. 5).

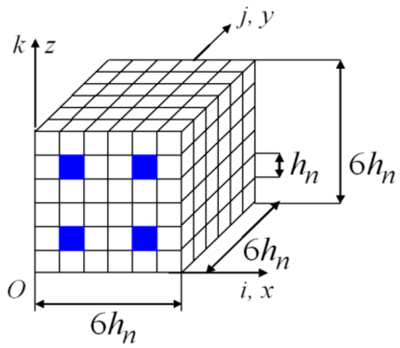

Рис. 5. Регулярная ячейка G_n^0

Fig. 5. Regular cell G_n^0

CB G_n^0 has the same number of fibers (cross-section $h_n \times h_n$) and their same mutual arrangement as a regular cell G_0 (Fig. 4), fibers and binder material CB G_n^0 and G_0 have the same modules of elasticity $n=\overline{1,16}$. So, CB G_n^0, G_0, (their heterogeneous structures) geometrically differ only in scale. Then, for the convenience of reasoning, taking into account (45), for CB G_n^0, G_0 we formally write down the ratio

$$G_n^0 = \beta_n G_0, \qquad (46)$$

where β_n is the scale coefficient, with $n \to 16$, we have $\beta_n \to 1$, $\beta_{16} = 1$, i.e. $G_{16}^0 = G_0$.

Note that since the regular cell G_0 takes into account the heterogeneous structure, then due to (46) and in CB G_n^0 ($n = 1, 2, 3, ...$) the heterogeneous structure is also taken into account with the help of 1cCE $V_j^{(n)}$ of the 1st order of the shape of the cube with a side h_n, i.e. the model R_n^0 takes into account the heterogeneous structure. Note that CB G_n^0 is essentially a regular cell of the model R_n^0, $n = 1, 2, 3, ...$. So, the models V_n^b, R_n^0 have the same shape and dimension, the same characteristic dimensions, uniform grids with step h_n, fastenings and loads like CB V_0.

In the calculations we use two-grid FE (2sKE). When constructing a 2sCE $V_d^{(2)}$ with dimensions $6h \times 6h \times 6h$ [15–19], we use two nested grids: a small uniform grid h_d with dimension $7 \times 7 \times 7$ step h and a large grid H_d of dimension $2 \times 3 \times 2$, $H_d \subset h_d$. Along the axes Oy, Oz the grid H_d has a step $6h$, along the axis - a step. $3h$ On Fig. 6 grid H_d nodes marked with dot 12 knots. The grid h_d is generated by the basic partition of R_d 2cKE, which consists of 1cCE V_j^h of the 1st order of the cube shape with a side h (in which the three-dimensional VAT, $j = 1, ..., M$ is realized, M is the total number of 1sKE V_j^h , $M = 216$) and takes into account the heterogeneous structure of the 2sKE $V_d^{(2)}$.

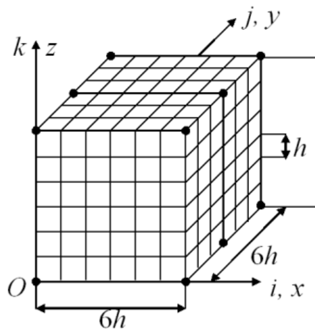

Рис. 6. Мелкая и крупная сетки 2сКЭ $V_d^{(2)}$

Fig. 6. Small and large grids 2gFE $V_d^{(2)}$

On the partition R_d we build a superelement V_S using the condensation method [5]. The total potential energy $\varPi_S$ of the superelement V_S is represented in the form of

$$\varPi_S = \frac{1}{2}\mathbf{q}_S^T[K_S]\mathbf{q}_S - \mathbf{q}_S^T\mathbf{F}_S, \tag{47}$$

where T is transposion; $[K_S]$ is the matrix of stiffness (dimension 654×654); $\mathbf{F}_S$, $\mathbf{q}_S$ are the vectors of nodal forces and displacements (dimensions 654) of the superelemen V_S t. The basic function $N_{ijk}(x,y,z)$ for the node i,j,k of a large grid H_d using Lagrange polynomials is written in the form $N_{ijk} = L_i(x)L_j(y)L_k(z)$, where

$$L_i(x) = \prod_{\alpha=1,\alpha\neq i}^{2} \frac{x - x_\alpha}{x_i - x_\alpha}, L_j(y) = \prod_{\alpha=1,\alpha\neq j}^{3} \frac{y - y_\alpha}{y_j - y_\alpha}, L_k(z) = \prod_{\alpha=1,\alpha\neq k}^{2} \frac{z - z_\alpha}{z_k - z_\alpha},$$

where x_i, y_j, z_k are the coordinates of the grid H_d node i,j,k in the coordinate system $Oxyz$; ijk is an integer coordinate system introduced for grid nodes H_d, $i,k = 1,2$; $j = 1,2,3$ (see Figure 6).

Denote: $N_e = N_{ijk}$, $u_e = u_{ijk}$, $v_e = v_{ijk}$, $w_e = w_{ijk}$, where u_{ijk}, v_{ijk}, w_{ijk} are the values of the movements *u*, *v*, *w* in the grid H_d, $i,k = 1,2$, $j = 1,2,3$, $e = 1,...,12$ node i,j,k, Then the approximation functions of displacements $u^{(2)}$, $v^{(2)}$, $w^{(2)}$, 2sKE $V_d^{(2)}$ represent

$$u^{(2)} = \sum_{e=1}^{12} N_e u_e, \quad v^{(2)} = \sum_{e=1}^{12} N_e v_e, w^{(2)} = \sum_{e=1}^{12} N_e w_e. \tag{48}$$

Denote: $\mathbf{q}_d$ – vector of nodal displacements of a large grid H_d (dimension 36), i.e. vector of nodal unknown 2sKE $V_d^{(2)}$. Using (48), the vector $\mathbf{q}_S$ of nodal displacements of the superelement V_S is expressed through the vecto $\mathbf{q}_d$ r , i.e.

$$\mathbf{q}_S = [A_S^d]\,\mathbf{q}_d, \tag{49}$$

where $[A_S^d]$ is the rectangular matrix (dimensions 654×36).

Substituting (49) in (47) we get $\Pi_S = \Pi_S(\mathbf{q}_d)$. From the execution $\partial\Pi_S / \partial\mathbf{q}_d = 0$ we obtain the equality $[K_d]\,\mathbf{q}_d = \mathbf{F}_d$, where $[K_d] = [A_S^d]^T[K_S][A_S^d]$, where $[K_d]$, $\mathbf{F}_d$, is the matrix of stiffness (dimension 36×36) and the vector of nodal forces (dimension 36) 2cCE $V_d^{(2)}$.

The solution built for the 2cCE $V_d^{(2)}$ grid H_d is projected using the formula (49) on the grid of the superelement V_S, then using the ratios of the condensation method - on a fine grid h_d of 2cKE $V_d^{(2)}$ which allows you to find voltages in 1sKE V_j^h basic partitioning R_d 2cKE $V_d^{(2)}$.

On the model V_n^b (R_n^0) we build a two-grid discrete model, which consists of a 2cCE type $V_d^{(2)}$ of size $6h_n \times 6h_n \times 6h_n$, where $h_n = 16h/n$, $n = \overline{1, 11}$ and which we denote V_n^0 (R_n). Note that the models V_n^0, R_n, have the same dimension. For models V_n^0, R_n, we find (according to the 4th theory of strength [1]) respectively the maximum equivalent stresses σ_n^b, σ_n^0, $n = 3,5,...,11$. . The results of the calculations are presented in Table. 1, where

$$\delta_n^p(\%) = 100\% \times | p_n - p_{n-2} | / p_n, \tag{50}$$

where $n = 5,7,\ 9,\ 11$; $p_n = \sigma_n^0 / \sigma_n^b$; N_n^o, b_n^o are the dimension and width of the tape SU FEM model V_n^0, $n = 3,5,...11$.

The analysis of the results of the calculations shows a uniform monotonous convergence of the stress σ_n^b, σ_n^0, parameter p_n and error δ_n^p. Let's consider the calculation of CB V_0 on the basis of BM. Note that in CB calculations, as a rule , three (or more) discrete models are used to analyze the convergence and error of numerical solutions. In this case, we use three models: $\mathrm{R}_1 = \mathbf{R}_0$, models R_2 and R_3 obtained by grinding BM $\mathbf{R}_0$. On a discrete model R_n, using a 2cCE type $V_d^{(2)}$ of size $6\mathrm{h}_n \times 6\mathrm{h}_n \times 6\mathrm{h}_n$, define a two-grid discrete model $\mathrm{R}_n^{\mathrm{o}}$, where is the step of the uniform grid of the model R_n, $\mathrm{h}_n = h/n$, $n = 1,2,3$.

The results of calculations for models R_n, $\mathrm{R}_n^{\mathrm{o}}$ are given in Table. 2, where N_n, b_n, is the dimension and width of the tape SU FEM model R_n; $\mathrm{N}_n^{\mathrm{o}}$ and $\mathrm{b}_n^{\mathrm{o}}$ are the dimension and width of the model tap $\mathrm{R}_n^{\mathrm{o}}$; $n = 1,2,3$. e;. The coefficient k_n is found by the formula $\mathrm{k}_n = (\mathrm{N}_n^{\mathrm{o}} \times \mathrm{b}_n^{\mathrm{o}}) / (N_{11}^o \times b_{11}^o)$, where $\mathrm{N}_n^{\mathrm{o}} \times \mathrm{b}_n^{\mathrm{o}}$ is the amount of computer memory required for the model $\mathrm{R}_n^{\mathrm{o}}$; $n = 1,2,3$; $N_{11}^o \times b_{11}^o$ – the amount of computer memory required for the model V_{11}^0, where $N_{11}^o = 114048$, $b_{11}^o = 906$, which is used in the calculations of CB V_0 according to MESC (see Table 1). So, the implementation of MESC in the calculation of CB V_0 V_0 requires $1{,}169 \times 10^3$ several times less computer memory than the implementation of the calculation of CB V_0 based on the grinding of the BM $\mathbf{R}_0$ (see Table 2). Find the voltage σ_b for the body V^b and the equivalence coefficient p. Since the stresses $\sigma_9^b = 0{,}477$ and $\sigma_{11}^b = 0{,}515$ differ by a small amount $\delta = (0{,}515 - 0{,}477) / 0{,}515 = 0{,}07379$ (see Table 1), let $\sigma_b = \sigma_{11}^b$, i.e. $\sigma_b = 0{,}515$. Test calculations show that the stress σ_{11}^b is found with an error of no more than 15 % . Then we assume $\delta_\alpha = 0{,}15$. Note that the condition (24) is fulfilled, i.e. we have $\delta_\alpha = 0{,}15 < C_\alpha = 0{,}458$. Since $\delta_{11}^p = 0{,}221$ (%) is the small value (see Table 1), we take $p = p_{11} = 4{,}54183$.

Table 1

CB V_0 calculation results

n	V_n^0	N_n^o	b_n^o	σ_n^b	R_n	σ_n^0	p_n	δ_n^p (%)
3	V_3^0	3456	114	0.319	R_3	0.169	0.52907	–
5	V_5^0	12960	240	0.383	R_5	1.741	4.54020	88.35
7	V_7^0	32256	414	0.434	R_7	1.979	4.55590	0.345
9	V_9^0	94800	636	0.477	R_9	2.173	4.55185	0.089
11	V_{11}^0	114048	906	0.515	R_{11}	2.339	4.54183	0.221

Table 2

Calculation results for models $\mathbf{R}_n$, $\mathbf{R}_n^o$

n	h_n	R_n	N_n	b_n	R_n^o	N_n^o	b_n^o	k_n
1	h	R_1	32517504	28524	R_1^o	332928	1791	5.77
2	$h/2$	R_2	257465088	112332	R_2^o	2509056	6639	161.21
3	$h/3$	R_3	865945728	251436	R_3^o	8297856	14559	1169.18

Substituting in the representation (4) $p = 4{,}54183$, $n_1 = 1{,}3$, $n_2 = 3{,}5$, $\delta_\alpha = 0{,}15$, for CB V_0 we obtain the adjusted equivalent strength conditions

$$6{,}95 \le n_b \le 13{,}82 \,. \tag{51}$$

For a body V^b, the reserve factor n_b is determined by the formula $n_b = \sigma_T / \sigma_b$, taking into account that $\sigma_T = 5$, $\sigma_b = 0{,}515$, we obtain $n_b = 5/0{,}515 = 9{,}71$. The reserve factor $n_b = 9{,}71$ of the body V^b satisfies the conditions of strength (51). Then the safety factor n_0 of CB V_0 satisfies the conditions of strength (43) (see theorem 1 of paragraph 3).

7. Application of generalized equivalent strength conditions

Let us consider the construction of generalized equivalent strength conditions for CB V_0 (Fig. 3), for which a set of different loads Q_{xz} of the form

$$Q_{xz} = \{q_x, q_z{:}\ q_x = \alpha, \quad q_z = \beta, \ \ 0 \le \alpha, \beta < \infty\} \,. \tag{52}$$

is given at the CB V_0 boundary $S = \{0{,}5L \le y \le L, \ \ z = H\}$.

To find the (upper, lower) boundaries for the set $\mathbf{P}$ of equivalence coefficients corresponding to the load set (52), calculations are made for a number of characteristic loads of CB V_0: $q_x = q_x^{(n)}$, $q_z = q_z^{(n)}$ ($q_x^{(n)}$, $q_z^{(n)} = \mathrm{const}$). The results of the calculations are given in Table. 3, where the equivalence coefficient $p^{(n)}$ is found for loading $q_x^{(n)}$, $q_z^{(n)}$, using the models V_{11}^0, R_{11}, see p. 6 $n = \overline{1,4}$, .

Table 3

Calculation results for loadings $q_x^{(n)}$, $q_z^{(n)}$,

n	$q_x^{(n)} \times 10^{-3}$	$q_z^{(n)} \times 10^{-3}$	$p^{(n)}$
1	$q_x^{(1)} = 0$	$q_z^{(1)} = 0,225$	4.53868
2	$q_x^{(2)} = 0,180$	$q_z^{(2)} = 0,325$	4.54185
3	$q_x^{(3)} = 0,275$	$q_z^{(3)} = 0$	4.55305
4	$q_x^{(4)} = 0,750$	$q_z^{(4)} = 0,750$	4.54129

Because of the linearity of the problem of the theory of elasticity and the relation (30), the equivalence coefficient p, which is defined for loading $q_x = \alpha_0 q_x^{(n)}$, $q_z = \alpha_0 q_z^{(n)}$ does not depend on α_0, where $\alpha_0 = \text{const}$, $0 < \alpha_0 < \infty$, $n = \overline{1,4}$. Then, for any $\alpha_0 > 0$ for loads $q_x = \alpha_0 q_x^{(3)}$, $q_z = 0$ $q_x = 0$, $q_z = \alpha_0 q_z^{(1)}$ ($q_x = \alpha_0 q_x^{(2)}$, $q_z = \alpha_0 q_z^{(2)}$ and $q_x = \alpha_0 q_x^{(4)}$, $q_z = \alpha_0 q_z^{(4)}$, where $q_x^{(4)} = q_z^{(4)}$) respectively we get $p = p^{(3)}$ и $p = p^{(1)}$ ($p = p^{(2)}$ и $p = p^{(4)}$) (see Table 3). It follows that if $q_x \to q_z$, then $p \to p^{(4)}$; if $q_z = \alpha_0$, $q_x \to 0$ then $p \to p^{(1)}$; if $q_x = \alpha_0$, $q_z \to 0$, then $p \to p^{(3)}$, if $q_x \neq q_z$, $q_x, q_z \neq 0$, then $p^{(1)} < p < p^{(3)}$, that is confirmed by the calculations. So, for any loads q_x, q_z, in (52) we have $\forall p \in \mathbf{P}$: $p^{(1)} \le p \le p^{(3)}$.

Enter the coefficients

$$p_1 = \min(p^{(n)}),\ p_2 = \max(p^{(n)}),\ n = \overline{1,4}, \text{ т. е. } \forall p \in \mathbf{P}:\ p_1 \le p \le p_2. \tag{53}$$

For CBV_0, the condition (39) is met, i.e. we have

$$p_2 C_1 \le p_1 C_2, \tag{54}$$

where is $C_1 = n_1 / (1 - \delta_\alpha)$, $C_2 = n_2 / (1 + \delta_\alpha)$.

In fact, following the initial data for CBV_0 and the results of Table. 3, we have $C_1 = 1,5294$, $C_2 = 3,0435$, $p_1 = p^{(1)} = 4,53868$, $p_2 = p^{(3)} = 4,55305$. We get $p_2 C_1 = 6,963$, $p_1 C_2 = 13,81$, i.e. condition (54) for CBV_0 is met. For the body V^b reserve factor n_b, the generalized equivalent strength conditions are of the form (42), i.e.,

$$p_2 C_1 \le n_b \le p_1 C_2. \tag{55}$$

Thus, the calculation of strength according to MESC CBV_0, for which many different loads are given (52), is reduced to the construction of generalized equivalent strength conditions (55). According to theorem 2, if the reserve factor n_b of a body V^b having a load $q_x, q_z \in Q_{xz}$ satisfies the generalized equivalent strength conditions (55), then the coefficient reserve n_0 CBV_0 meets the specified conditions of strength (43) for loading $q_x, q_z \in Q_{xz}$.

For CBV_0, the generalized equivalent strength conditions (55) are of the form

$$6,96 \le n_b \le 13,81. \tag{56}$$

In this example $p_1 = 4{,}53868$, $p_2 = 4{,}55305$, and since $\Delta p = p_2 - p_1 = 0{,}01437$ is little, the strength conditions (51) and (56) are almost the same (see paragraph 6).

The advantage of the generalized equivalent strength conditions (55) is that they are applied to all different loads of the CB V_0 set Q_{xz}. Consequently, there is no need to define equivalent strength conditions (40), i.e. the equivalence coefficient p_q for each given load $q_x, q_z \in Q_{xz}$, which leads to a reduction in the time spent on the implementation of the MESC when using different loads $q_x, q_z \in Q_{xz}$ in the calculations for the strength of the CB V_0.

Conclusion

We briefly described the method of equivalent strength conditions for calculating the strength of a body with an inhomogeneous, microunhomogeneous regular structure, for which many different static loads are specified. The proposed method is implemented on the basis of FEM using multi-grid finite elements and is reduced to the calculation of the strength of isotropic homogeneous bodies using generalized equivalent strength conditions. The implementation of the method requires low time and computer resources.

Библиографические ссылки

1. Писаренко Г. С., Яковлев А. П., Матвеев В. В. Справочник по сопротивлению материалов. Киев : Наук. думка, 1975. 704 с.

2. Биргер И. А., Шорр Б. Ф., Иосилевич Г. Б. Расчет на прочность деталей машин. М. : Машиностроение, 1993. 640 с.

3. Москвичев В. В. Основы конструкционной прочности технических систем и инженерных сооружений. Новосибирск : Наука, 2002. 106 с.

4. Матвеев А. Д. Расчет упругих конструкций с применением скорректированных условий прочности // Известия АлтГУ. Математика и механика. 2017. № 4. С. 116–119. Doi: 10.14258/izvasu(2017)4-21.

5. Норри Д., Ж. де Фриз. Введение в метод конечных элементов. М. : Мир, 1981. 304 с.

6. Зенкевич О. Метод конечных элементов в технике. М. : Мир, 1975. 542 с.

7. Фудзии Т., Дзако М. Механика разрушения композиционных материалов. М. : Мир, 1982. 232 с.

8. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах трехмерных однородных и композитных тел // Учен. зап. Казан. ун-та. Серия: Физ.-матем. науки. 2016. Т. 158, кн. 4. С. 530–543.

9. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах композитных пластин и балок // Вестник КрасГАУ. 2016. № 12. С. 93–100.

10. Matveev A. D. Multigrid finite element method in stress of three-dimensional elastic bodies of heterogeneous structure // IOP Conf. Ser.: Mater. Sci. Eng. 2016. Vol. 158, No. 1. Art. 012067, P. 1–9.

11. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах композитных пластин и балок сложной формы // Вестник КрасГАУ. 2017. № 11. С. 131–140.

12. Матвеев А. Д. Метод многосеточных конечных элементов // Вестник КрасГАУ. 2018. № 2. С. 90–103.

13. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах композитных оболочек вращения и двоякой кривизны // Вестник КрасГАУ. 2018. № 3. С. 126–137.

14. Матвеев А. Д. Метод многосеточных конечных элементов в решении физических краевых задач // Информационные технологии и математическое моделирование. Красноярск, 2017. С. 27–60.

15. Матвеев А. Д. Некоторые подходы проектирования упругих многосеточных конечных элементов // Деп. в ВИНИТИ. 2000. № 2990–В00. 30 с.

16. Матвеев А. Д. Многосеточное моделирование композитов нерегулярной структуры с малым коэффициентом наполнения // Прикладная механика и техническая физика. 2004. № 3. С. 161–171.

17. Матвеев А. Д. Построение сложных многосеточных конечных элементов с неоднородной и микронеоднородной структурой // Известия АлтГУ. Серия: Математика и механика. 2014. № 1/1. С. 80–83. Doi: 10.14258/izvasu(2014)1.1-18.

18. Матвеев А. Д. Метод образующих конечных элементов // Вестник КрасГАУ. 2018. № 6. С. 141–154.

19. Матвеев А. Д. Построение многосеточных конечных элементов для расчета оболочек, пластин и балок на основе образующих конечных элементов. // Вестник Пермского нац. исслед. политех. ун-та. Механика. 2019. № 3. С. 48–57. Doi: 10/15593/perm.mech/2019.3.05.

20. Голушко С. К., Немировский Ю. В. Прямые и обратные задачи механики упругих композитных пластин и оболочек вращения. М. : Физматлит, 2008. 432 с.

21. Немировский Ю. В., Резников Б. С. Прочность элементов конструкций из композитных материалов. Новосибирск : Наука, Сибирское отделение. 1984. 164 с.

22. Кравчук А. С., Майборода В. П., Уржумцев Ю. С. Механика полимерных и композиционных материалов. М. : Наука. 1985. 201 с.

23. Алфутов Н. А., Зиновьев А. А., Попов Б. Г. Расчет многослойных пластин и оболочек из композиционных материалов. М. : Машиностроение, 1984. 264 с.

24. Победря Б. Е. Механика композиционных материалов. М. : МГУ. 1984. 336 с.

25. Андреев А. Н., Немировский Ю. В. Многослойные анизотропные оболочки и пластины. Изгиб, устойчивость, колебания. Новосибирск : Наука, 2001. 288 с.

26. Ванин Г. А. Микромеханика композиционных материалов. Киев : Наукова думка. 1985. 302 с.

27. Васильев В. В. Механика конструкций из композиционных материалов. М. : Машиностроение, 1988. 269 с.

28 Матвеев А. Д. Расчет на прочность композитных конструкций с применением эквивалентных условий прочности // Вестник КрасГАУ. 2014. № 11. С. 68–79.

29. Матвеев А. Д. Метод эквивалентных условий прочности в расчетах композитных конструкций регулярной структуры с применением многосеточных конечных элементов // Сибирский журнал науки и технологий. 2019. Т. 20, № 4. С. 423–435. Doi: 10.31772/2587-6066-2019-20-4-423-435.

30. Самуль В. И. Основы теории упругости и пластичности. М. : Высшая школа, 1982. 264 с.

References

1. Pisarenko G. S., Yakovlev A. P., Matveev V. V. *Spravochnik po soprotivleniyu materialov* [Handbook of resistance materials']. Kiev, Nauk. Dumka Publ., 1975, 704 p.

2. Birger I. A., Shorr B. F., Iosilevich G. B. *Raschet na prochnost' detalej mashin* [Calculation of the strength of machine parts]. Moscow, Mashinostroenie Publ., 1993, 640 p.

3. Moskvichev V. V. *Osnovy konstrukcionnoj prochnosti tekhnicheskih sistem i inzhenernyh sooruzhenij* [Fundamentals of structural strength of technical systems and engineering structures]. Novosibirsk, Nauka Publ., 2002, 106 p.

4. Matveev A. D. [Calculation of elastic structures using the adjusted terms of strength]. *Izvestiya AltGU*. 2017, No. 4, P. 116–119. Doi: 10.14258/izvasu(2017)4-21.

5. Norri D., de Friz Zh. *Vvedenie v metod konechnykh elementov* [Introduction to the finite element method]. Moscow, Mir Publ., 1981, 304 p.

6. Zenkevich O. *Metod konechnykh elementov v tekhnike* [Finite element method in engineering]. Moscow, Mir Publ., 1975, 544 p.

7. Fudzii T., Dzako M. *Mekhanika razrusheniya kompozicionnyh materialov* [Fracture mechanics of composite materials]. Moscow, Mir Publ., 1982, 232 p.

8. Matveev A. D. [The method of multigrid finite elements in the calculations of three-dimensional homogeneous and composite bodies]. *Uchen. zap. Kazan. un-ta. Seriia: Fiz.-matem. Nauki*. 2016, Vol. 158, No. 4, P. 530–543 (In Russ.).

9. Matveev A. D. [Multigrid method for finite elements in the analysis of composite plates and beams]. *Vestnik KrasGAU*. 2016, No. 12, P. 93–100 (In Russ.).

10. Matveev A. D. Multigrid finite element method in stress of three-dimensional elastic bodies of heterogeneous structure. *IOP Conf, Ser.: Mater. Sci. Eng*. 2016, Vol. 158, No. 1, Art. 012067, P. 1–9.

11. Matveev A.D. *Metod mnogosetochnyh konechnyh elementov v raschetah kompozitnyh plastin i balok slozhnoj formy* [Multigrid finite element Method in the calculations of composite plates and beams of irregular shape]. // *The Bulletin of KrasGAU*, 2017, No. 11, P. 131–140.

12. Matveev A. D. [Multigrid finite element Method]. *The Bulletin of KrasGAU*. 2018, No. 2, P. 90–103 (In Russ.).

13. Matveev A. D. [The method of. multigrid finite elements of the composite rotational and bicurved shell calculations]. *The Bulletin of KrasGAU*. 2018, No. 3, P. 126–137 (In Russ.).

14. Matveev A. D. [Method of. multigrid finite elements to solve physical boundary value problems]. *Ministry of information technologies and mathematical modeling*. Krasnoyarsk, 2017, P. 27–60.

15. Matveev A. D. [Some approaches of designing elastic multigrid finite elements]. *VINITI Proceedings*. 2000, No. 2990-B00, P. 30.

16. Matveev A. D. [Multigrid modeling of composites of irregular structure with a small filling ratio]. *J. Appl. Mech. Tech. Phys.* 2004, No. 3, P. 161–171 (In Russ.).

17. Matveev A. D. [The construction of complex multigrid finite element heterogeneous and microinhomogeneities in structure]. *Izvestiya AltGU*. 2014, No. 1/1, P. 80–83 (In Russ.). Doi: 10.14258/izvasu(2014)1.1-18.

18. Matveev A. D. [Method of generating finite elements]. *The Bulletin of KrasGAU*. 2018, No. 6, P. 141–154 (In Russ.).

19. Matveev A. D. [Construction of multigrid finite elements to calculate shells, plates and beams based on generating finite elements]. *PNRPU Mechanics Bulletin*. 2019, No. 3, P. 48–57 (In Russ.). Doi: 10/15593/perm.mech/2019.3.05.

20. Golushko S. K., Nemirovskij Y. V. *Pryamye i obratnye zadachi mekhaniki uprugih kompozitnyh plastin i obolochek vrashcheniya* [Direct and inverse problems of mechanics of elastic composite plates and shells of rotation]. Moscow, Fizmatlit Publ., 2008, 432 p.

21. Nemirovskij Y. V., Reznikov B. S. *Prochnost' elementov konstrukcij iz kompozitnyh materiallov* [Strength of structural elements made of composite materials]. Novosibirsk, Nauka Publ., Sibirskoe ot-delenie. 1984, 164 p.

22. Kravchuk A. S., Majboroda V. P., Urzhumcev Y. S. *Mekhanika polimernyh i kompozicionnyh materialov* [Mechanics of polymer and composite materials]. Moscow, Nauka Publ., 1985, 201 p.

23. Alfutov N. A., Zinov'ev A. A., Popov B. G. *Raschet mnogoslojnyh plastin i obolochek iz kompozicionnyh materialov* [Calculation of multilayer plates and shells made of composite materials]. Moscow, Mashinostroenie Publ., 1984, 264 p.

24. Pobedrya B. E. *Mekhanika kompozicionnyh materialov* [Mechanics of composite materials]. Moscow, MGU Publ., 1984, 336 p.

25. Andreev A. N., Nemirovskij Y. V. *Mnogoslojnye anizotropnye obolochki i plastiny. Izgib, ustojchivost', kolebaniya* [Multilayer anisotropic shells and plates. Bending, stability, vibration]. Novosibirsk : Nauka Publ., 2001, 288 p.

26. Vanin G.A. *Mikromekhanika kompozicionnyh materialov* [Micromechanics of composite materials]. Kiev, Naukova dumka Publ., 1985, 302 p.

27. Vasil'ev V. V. *Mekhanika konstrukcij iz kompozicionnyh materialov* [Mechanics of structures made of composite materials]. Moscow, Mashinostroenie Publ., 1988, 269 p.

28. Matveev A. D. [Calculation of the strength of composite structures using equivalent strength conditions]. *The Bulletin of KrasGAU*. 2014, No. 11, P. 68–79 (In Russ.).

29. Matveev A. D. [The method of equivalent strength conditions in calculating composite structures regular structure using multigrid finite elements]. *Siberian Journal of Science and Technology.* 2019. Vol. 20, No. 4, P. 423-435. Doi: 10.31772/2587-6066-2019-20-4-423-435.

30. Samul' V. I. *Osnovy teorii uprugosti i plastichnosti* [Fundamentals of the theory of elasticity and plasticity]. Moscow, Vysshaia shkola Publ., 1982, 264 p.

Матвеев Александр Данилович – кандидат физико-математических наук, доцент, старший научный сотрудник; Институт вычислительного моделирования СО РАН. E-mail: mtv241@mail.ru.

Matveev Alexander Danilovich – Cand. Sc., associate Professor, senior researcher; Institute of computational modeling SB RAS. E-mail: mtv241@mail.ru.