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On incorrect representation of the shock process on shock polars in a viscous heat-conducting gas

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The shock gas-dynamic processes that have found wide application in rocket and space technology in the design and optimization of devices and power plants are considered. The analysis of known exact and asymptotic relations/conditions on a shock wave, in particular, generalized differential relations (GDRs) on a curvilinear oblique shock wave (SW) (COSW) for a model of a viscous heat-conducting gas at high Reynolds numbers is made. The advantages of using the discrete-analytical approach are shown, for example: 1) the ability to make maximal use of the smoothness of the shock gas-dynamic formation (shock wave) in the tangential direction; 2) to build efficient computational algorithms devoid of the negative effect of approximation/artificial viscosity on the schematized discontinuity. In parallel, a very common graphical method for mapping the results of gas-dynamic calculations on the plane of shock polars, proposed by Busemann, and a volumetric (3D) polaroid, proposed by V. N. Uskov, are considered. The mathematical apparatus of shock polars itself is based on exact relations of the Rankine-Hugoniot type and has proven itself well even in modeling the flows of a viscous heat-conducting gas. However, in numerous literature sources there are results (shock solutions) of both physical and computational experiments, which are not mapped strictly on shock polars. In this paper, we show that in rare cases this very common way of such mapping may be incorrect. It has been proven that the main causes of such a defect are the combined action of three main factors: the nonuniform flow in front of the shock formation, the edge/boudary effect behind it, the action of the external factor of viscosity and the heat conduction mechanism.

Keywords: shock gasdynamic process, gasdynamic discontinuity, generalized differential relations at the shock wave, shock polar and polaroid.

О некорректном представлении ударного процесса на ударных полярах в вязком теплопроводном газе

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Рассматриваются ударные газодинамические процессы, нашедшие широкое применение в ракетно-космической технике при конструировании и оптимизации устройств, энергетических установок. Производится анализ известных точных и асимптотических соотношений/условий на ударной волне, в частности – обобщенных дифференциальных соотношений (ОДС) на криволинейном косом скачке уплотнения (ККСУ) для модели вязкого теплопроводного газа при больших числах Рейнольдса. Показаны преимущества использования дискретно-аналитического подхода, например: 1) возможность максимально использовать гладкость ударного газодинамического образования (скачка) в касательном направлении; 2) строить эффективные вычислительные алгоритмы, лишенные негативного действия аппроксимационной/искусственной вязкости на схематизированном разрыве. Параллельно рассмотрен весьма распространенный графический способ отображения результатов газодинамических расчетов на плоскость ударных поляр, предложенный Буземаном, и объёмный (3D) поляроид, предложенный В. Н. Усковым. Сам математический аппарат ударных поляр построен на точных соотношениях типа Ренкина – Гюгонио и неплохо зарекомендовал себя даже при моделировании течений вязкого теплопроводного газа. Однако в многочисленных литературных источниках присутствуют результаты (ударные решения) как физического, так и вычислительного экспериментов, которые не отображаются строго на ударных полярах. В настоящей работе показано, что в редких случаях данный и весьма распространённый способ такого отображения может быть и некорректным. Доказано, что основными причинами такого дефекта является совместное действие трех основных факторов: неравномерность течения перед ударным образованием, краевой эффект за ним, действие внешних фактора вязкости и механизма теплопроводности.

Ключевые слова: ударный газодинамический процесс, газодинамический разрыв, обобщенные дифференциальные соотношения на скачке уплотнения, ударные поляра и поляроид.

Introduction

Impact gas-dynamic processes [1] are widely used in rocket and space technology in the design and optimization of devices, power plants, in modern technologies, and even in medicine. The “shock formation” itself (thin shock layer), with its correct idealization (separation of surface boundaries or schematization of a shock wave (SW), curvilinear oblique SW (COSW)) can be considered discontinuous, which made it possible to apply an analytical apparatus to relate gas-dynamic parameters on both sides of such a “discontinuity” [2–13]. Moreover, if there is an additional margin of smoothness in the direction tangent to the COSW, then it is possible to connect analytically not only the parameters themselves, but also the partial derivatives of them on both sides of the COSW. Such a connection for the model of an ideal gas in the form of differential conditions for dynamic compatibility on COSW (DCDC) is given in [8] by V. N. Uskov. In [10; 11] their generalized analog is given - generalized differential relations on COSW (GDR) for the model of viscous heat-conducting gas at high Reynolds numbers (Re_∞). The mathematical apparatus in the form of the GDR makes it possible to reduce from the Navier-Stokes equations of a viscous heat-conducting gas (ENSVHC) to the GDR system in terms of setting the viscous problem of COSW penetration into the shear layer. In the process of mathematical modeling based on the GDR, it is possible, within the framework of a single computational algorithm, to pass "through" from the gas-dynamic to the diffusion stages of COSW evolution in the shear layer, while specifying various boundary/edge effects (BE), and on the other hand, to significantly save computing resources: instead of many hours (ENSVHC), the calculation process on a PC takes seconds. However, the most important thing is that in such formulation of the problem (especially taking into account the viscosity and heat conducting external (on both sides of the discontinuity) factor (VHC factor)) the natural order of solution smoothness in the tangent to the COSW direction is preserved, which was the main motive for applying the discrete-analytical approach [11].

Busemann in his work laid the foundation of graphical methods for solving problems of gas-dynamic discontinuities interference using shock polars (SP) [13], which relate the intensity of oblique SWs to the angle of flow reversal on them. Such polars are called Busnemann polars in his honor, another name is heart-shaped curves because of their characteristic appearance, as well as isomachs, since each shock polar is constructed for a specific Mach number of the oncoming flow. In problems of gas-dynamic discontinuities interference, when one of the elements of a particular configuration can be a rarefaction wave, the term "shock-wave polar" (SWP) is more often used instead of SP [8–11].

The purpose of this paper is to study the correctness (rightness) of using the SWP apparatus for representing individual viscous shock solutions. Partially, such a study on the SWP plane was carried out in [11]. In the present paper such solutions are mapped, among other things, to the volumetric (3D) analog of the SWP - the shock-wave polaroid. It should be noted that the problems of such an incorrect representation/mapping of the obtained solutions can occur only in rare cases, since the SWP mathematical apparatus itself is built on exact Rankine-Hugoniot relations and has proven itself well even in modeling viscous heat-conducting gas flows. These rare but important cases can be observed under the combined action of the following three factors: 1) non-uniformity of the undisturbed flow in front of the COSW; 2) BE behind it, formed by overtaking disturbances; 3) VHC factor.

1. Classical and non-classical shock transition models

It is known that the shock formation itself, with its correct idealization (schematization) [1–13], allows one to approximately replace it with a mathematical discontinuity surface of the first kind, when the main gas-dynamic variables have a finite discontinuity. In stationary problems, SWs are considered instead of moving SWs [1; 7; 12]; although a SW moving at a constant speed can also be considered as a COSW in a moving coordinate system. The relations connecting the quantities on both sides of the SW are called the Rankine-Hugoniot relations, and similar relations on the COSW are called the relations on the oblique SW, including the normal COSW. In both cases, one can speak in general terms about relations of the Rankine-Hugoniot type. These relations form a one-parameter family of formulas, i.e., with known parameters before the discontinuity, it is enough to specify one parameter after it in order to determine (not always unambiguously!) all the relationships of quantities on both sides of the SW or COSW. In the latter case, the velocity of the SW itself often acts as such a parameter. These relations also have more complete implementations, namely:

1) their continuation in the form of differential conditions of dynamic compatibility (DCDC [8; 10; 11], V. N. Uskov);

2) GDR [10; 11] is a generalization of DCDC for the case of the action of an external VHC factor when using the viscous heat-conducting gas model (ENSVHC) at high Reynolds numbers;

3) differential relations (as well as DCDC in the inviscid approximation), which are satisfied at the front of a curvilinear SW moving with acceleration were obtained in [12] as applied to non-stationary gas-dynamic flows.

Without loss of generality, we consider the usual, i.e., without taking into account the external VHC factor, relations on an oblique SW (COSW), which follow from the integral conservation laws at the discontinuity

$$[\rho v_n] = \rho v_n - \hat{\rho}\hat{v}_n = 0, \quad \left[p + \rho v_n^2\right] = 0, \quad [\rho v_n v_\tau] = 0, \quad \left[h + \frac{v_n^2}{2}\right] = 0 \qquad (1)$$

and can be written in various known forms and are therefore not given. In (1) ρ is density; p is pressure; v_n and v_τ are the normal and tangential velocity components to the discontinuity surface; h is enthalpy (heat content); $h = \gamma/(\gamma-1)\, p/\rho$; $\gamma = c_p/c_v$ is the isentropic exponent.

The universality, and therefore the frequent applicability of nonlinear, but simple relations/conditions on the oblique SW (COSW) can be explained as follows:

1) even in the case of a complex spatial (3D) configuration of the SW (COSW), the gas-dynamic process is considered locally exclusively in two-dimensional space (2D) – the plane formed by the normal vector to the smooth surface of the shock and the velocity vector of the oncoming supersonic flow (Mach number - $\mathrm{M} > 1$);

2) the dissipative mechanism is not explicitly present in the relations themselves, but its operation inside the SW (COSW) is externally reflected, in particular, in the production/increase of entropy. There are no (jamps) in the total enthalpy and the velocity component tangent to the COSW: $\left[h + (v_n^2 + v_\tau^2)/2\right] = 0$, $\left[v_\tau\right] = 0$, but this is the case only with ordinary relations.

When using the ENSVHC model, it is possible to generalize the usual relations on a COSW by adding the action of a small external VHC factor [10; 11]. Additional terms with a small parameter (1/Re∞) appear at derivatives of gas-dynamic quantities in such generalized relations. However, even in this case, if there are no gradients of these values on both sides of the discontinuity, the usual relations on the oblique SW (COSW) are fully satisfiable regardless of the Reynolds number, which has a positive effect on the accuracy of the shock gas-dynamic process description and, accordingly, the frequent applicability of these relations.

Busemann in his work [13] laid the foundation of graphical methods for solving problems of gas-dynamic discontinuities interference with the help of shock polars, which relate the intensity of oblique SWs to the angle of flow reversal on them. Let us consider what the regular and irregular (Mach) reflection of an oblique SW from the wall looks like in the physical plane and the plane of shock polars.

Fig. 1 shows the types of SW reflection. The polars are plotted in coordinates $(\beta, \Lambda = \ln(J))$, where β is the angle of refraction/reversal of the velocity vector on the oblique SW, and $J = \hat{p}/p$ is its intensity (the ratio of the pressure behind the SW to the pressure in front of it). The polars are plotted from a specific Mach number and isentropic exponent $\gamma = c_p/c_v$. Here β_1 is the flow reversal angle on the incoming/incident shock, β_2 is a similar angle on the reflected shock, β_3 is the reversal angle on the main shock with irregular reflection, σ_1, σ_2, σ_3 are the incoming, reflected, and main shock (Mach stem), respectively. If the reflection is regular, then the secondary polar released from point 1 (constructed according to the Mach number behind the incoming shock) must cross the coordinate axis at point 2. Then the total reversal angle is $\beta_1 + \beta_2 = 0$, and the total degree of flow compression will be $\Lambda_1 + \Lambda_2$. If the reflection is irregular, then one or another triple configuration of shock waves is formed [8; 9], a similar calculation of which is based on the equality of pressures and slopes of the velocity vectors on the tangential discontinuity $\hat{\tau}$ emanating from the triple point (dash-dotted line in fig. 1, *b*).

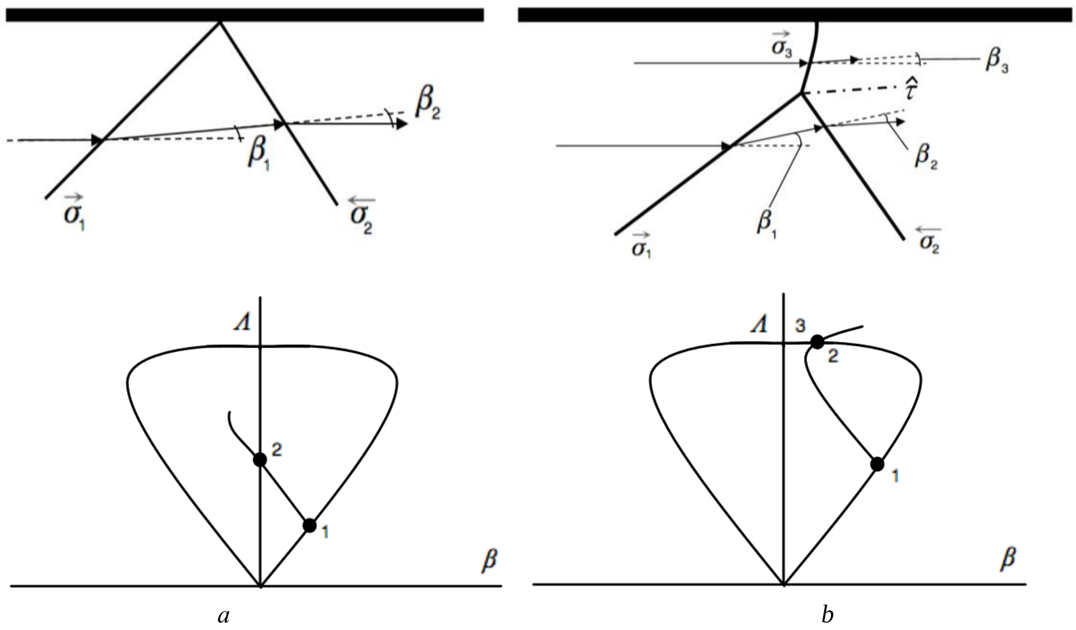

Fig. 1. Regular (*a*) and irregular (*b*) reflection of an oblique shock from the wall

Рис. 1. Регулярное (*а*) и нерегулярное (*б*) отражение косого скачка от стенки

The dependence $\beta(J)$ in the plane of polars, which follows from the usual relations on the oblique SW, is given as follows:

$$\operatorname{tg}\beta = \pm\sqrt{\frac{J_m - J}{J+\varepsilon}}\,\frac{(1-\varepsilon)(J-1)}{(J_m+\varepsilon)-(1-\varepsilon)(J-1)},$$

where $J_m = (1+\varepsilon)\mathrm{M}^2 - \varepsilon$ is the maximum SW intensity (normal SW), $\varepsilon = (\gamma-1)/(\gamma+1)$. Accordingly, for the secondary polar, it is required to take the Mach number beyond the primary oblique SW.

The most complete description of theoretical analysis of all possible interactions/interferences of stationary gas-dynamic discontinuities is given in [8; 9]; at the same time, the method of displaying the results of a physical or numerical experiment on the polar plane does not cause any particular complaints (see above). At the same time, in numerous modern works of a computational nature, there are often separate points (solutions) that for some reason (?) do not fall strictly on the shock polars. Let's look into this issue.

2. Shock penetration into the shear layer and mapping of the process on shock polars and polaroid

Let us consider the interaction of a COSW with a thin shear layer in an inviscid (vortex) and viscous setting (detailed in [10; 11]), which is schematically shown in fig. 2. The supersonic part of the boundary layer was used as a layer. We consider a thin layer to be formed at the moment of its interaction with the COSW, therefore, the influence of viscous forces can already be neglected locally in the interaction zone and the usual relations on the oblique SW, more precisely, their differential continuation - DCDC (see above) – can be used to calculate the interaction, since the COSW passing through the layer is smooth and curvilinear. This is what an inviscid or vortex formulation of the problem looks like. However, it is possible to take into account the effect of the external VHC factor and make a calculation based on the universal GDRs (see above), from which DCDC follow automatically when the VCH factor is excluded. Note that the GDRs are implemented within the framework of the discrete-analytical approach [11], with the help of which these calculations were performed. It is essential that in this approach there are fundamentally no (!) such negative effects as circuit or artificial viscosity, as well as effects caused by the operation of the so-called limiters (monotonic limiters) [14].

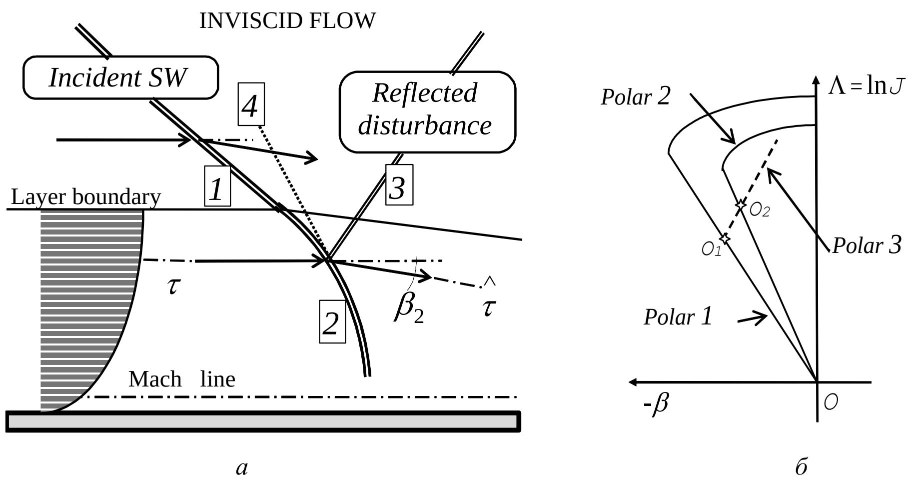

Fig. 2. The scheme of interaction of a shock wave (SW) with a shear layer:
a – physical plane; *b* – polar plane

Рис. 2. Схема взаимодействия скачка уплотнения со сдвиговым слоем:
а – физическая плоскость; *б* – плоскость поляр

On fig. 2, *a 1–3* are the main elements of the shock refraction, *4* are overtaking disturbances that carry the BE, and τ and $\hat{\tau}$ are the same streamline in front of and behind the SW, respectively, which is a degenerate tangential discontinuity: its intensity is inversely proportional to the number of divisions of the continuous velocity profile. The same can be said about the reflected disturbance (fig. 2, *a 3*). The layer was preliminarily calculated on the basis of ENSVHC, and then tabulated [11] with the help of smooth interpolants, so that the main parameters inside it were smooth up to the second derivatives.

With the known current intensity J of the COSW at any point of the layer the process of penetration of the COSW into it with a greater degree of schematization can be mapped on the SWP plane (fig. 2, *b*). For this, it is convenient to represent this continuous process as a discrete one. Polars 1 and 2 correspond to two adjacent streamlines in the undisturbed flow (fig. 2, *a*) with a small difference (due to the continuity of the given profile) of the Mach numbers, and the linear analogue of the secondary polar *3* corresponds to the streamline behind the incident COSW (corresponds to the Mach number $\hat{M}$).

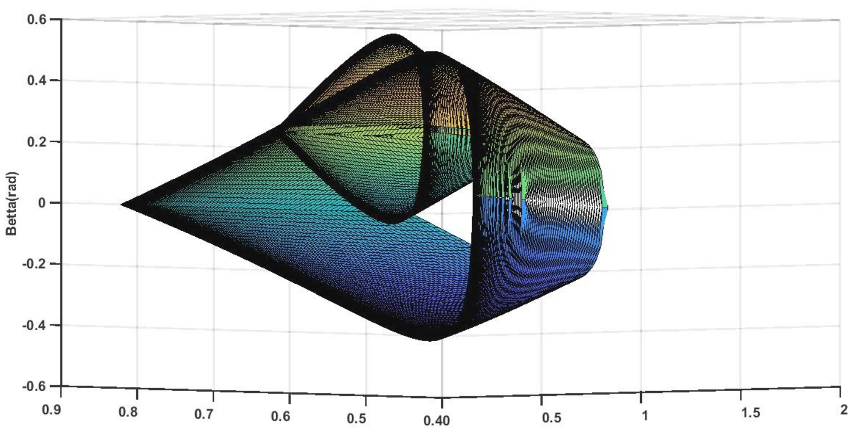

Fig. 3. Shock-wave polaroid

Рис. 3. Ударно-волновой поляроид

The process of COSW penetration into the layer can be mapped on a volumetric shock-wave polaroid (V. N. Uskov's term, fixed in [11]), which is a 3D analog of SWP (see fig. 3). Note that there is no need to represent secondary polars in the problem under consideration: it is only desirable to indicate the direction of the corresponding branch, as in the schematic fig. 2, *b*.

In the calculation [11] on the external streamline in the undisturbed flow (see fig. 2, *a*): $\rho_\infty = 1{,}18$; $W_\infty = 0{,}95$ (full dimensionless speed); $\mathrm{M}_\infty = 2{,}275$, and behind the SW $\hat{\mathrm{M}} = 1{,}759$ and with each line the values decrease; $\mathrm{Re}_x = 1{,}6 \cdot 10^3$, $\mathrm{Pr} = 1$ (important only for viscous setting). BE behind the COSW was debilitating.

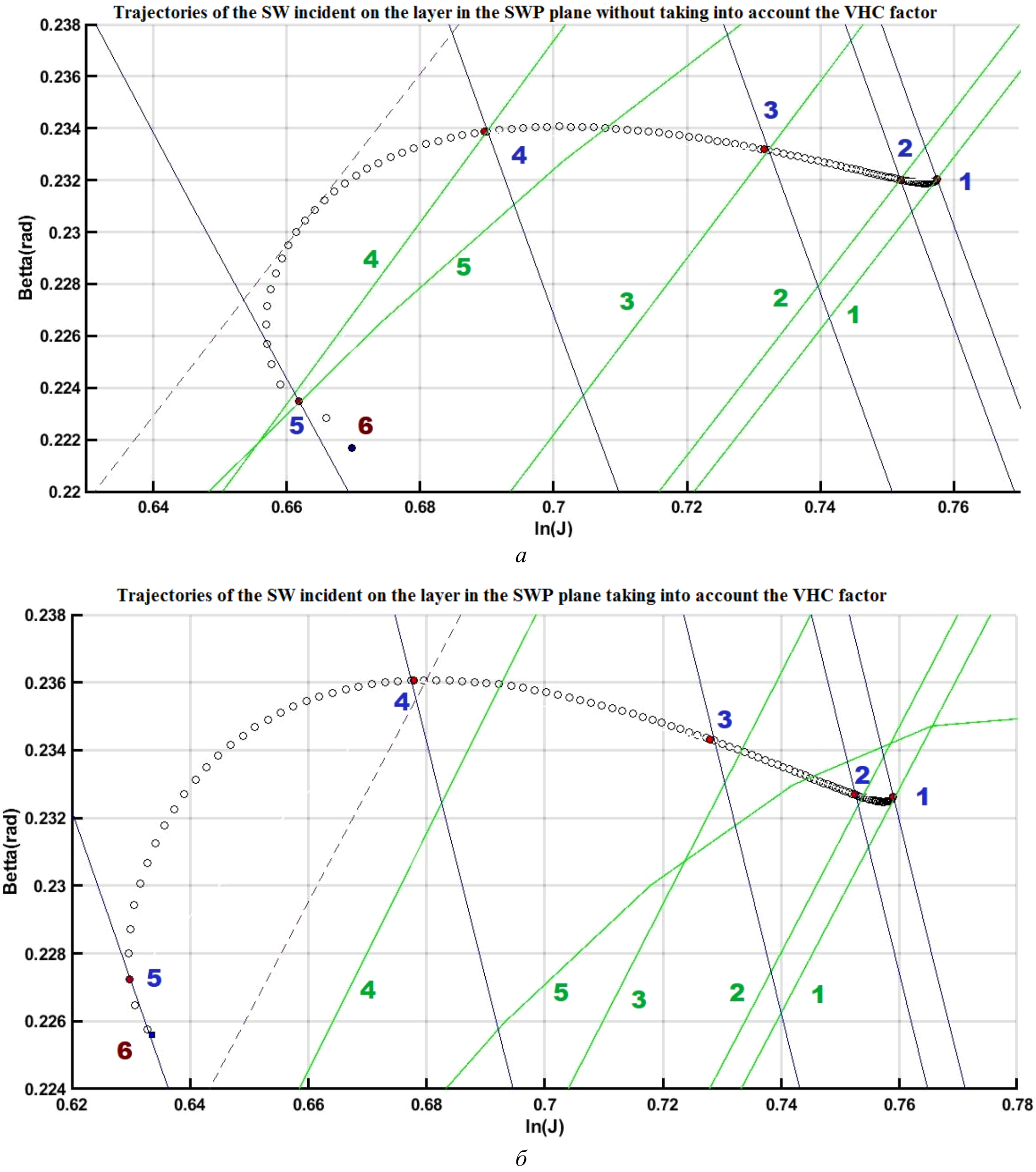

Fig. 4. SW track:
a – excluding VHC (Viscosity and Heat Conducting) factor; *б* – taking into account VHC factor. *1–5* – main SPs (Shock Polars) and trajectory points corresponding to streamlines intersected in physical space; *6* – the last point of the trajectory of the SW (Shock Wave); dotted line – envelope of all SPs. The main SPs are shown with green lines running across the linearized SWP

Рис. 4. Трек СУ:
а – без учета фактора ВТ; *б* – с учетом фактора ВТ. *1–5* – основные УП и точки траектории, соответствующие пересекаемым в физическом пространстве линиям тока; *6* – последняя точка траектории СУ; пунктир – огибающая всех УП. Основные УП показаны зелеными линиями, поперек идущими линеаризованным УВП

Let us analyze the difference between inviscid and viscous (taking into account the VHC factor) solutions in the problem of COSW penetration into the shear layer on the SWP plane and the polaroid (fig. 4).

Fig. 4 shows the trajectories of the COSW incident on the layer in the SWP plane in semilogarithmic coordinates without taking into account the VHC factor (fig. 4, *a*) and taking into account this factor (fig. 4, *b*). Separate selected points of the trajectory corresponding to five selected streamlines intersected by the COSW, as well as fragments of the main SPs for the corresponding Mach numbers, are numbered. The dotted line shows a fragment of the envelope of all SPs, for which the analytical dependence was obtained by V. N. Uskov [8]. In the viscous case, the selected points on the trajectories (tracks) of the COSW no longer coincide with the corresponding SPs (their numbers), and such a mismatch accumulates as the SW penetrates into the gradient part of the layer; the SW track goes through the SP envelope (!). The usual relations on the oblique SW in this case are not strictly fulfilled (!) and, as a result, the SWP apparatus turns out to be less suitable. The action of the VHC factor leads to a sharper decrease in the COSW intensity than that which occurs only under the action of the BE that weakens it.

Fig. 5 shows both inviscid and viscous solutions as trajectories (tracks) on the surface of the main (primary) polaroid. We see that the viscous solution (red track) flakes off from this polaroid, which indicates the poor feasibility of the usual conditions on the oblique SW and, accordingly, the incorrect use of polars or polaroid when displaying the results of calculations.

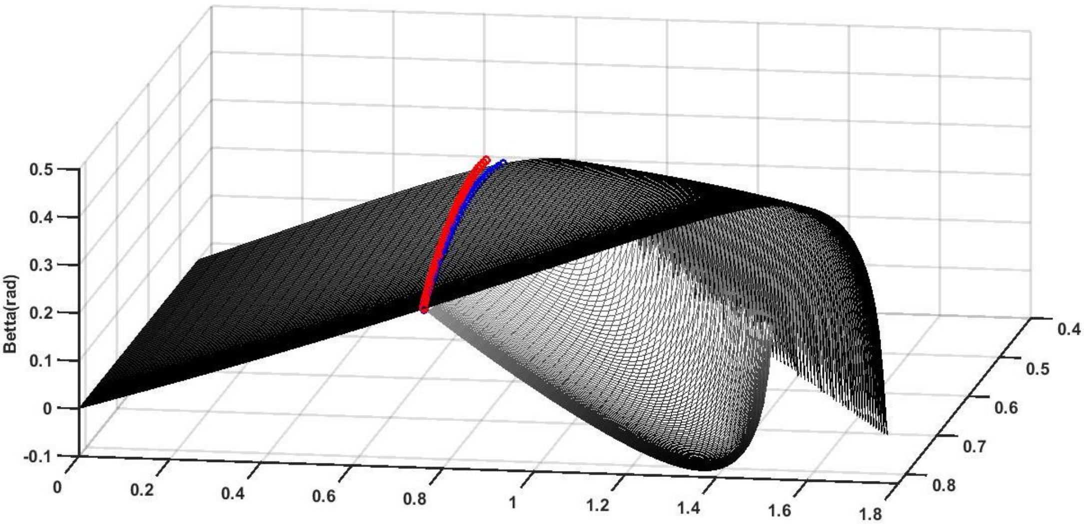

Fig. 5. Mapping the trajectory of the SW incident on a layer on the polaroid surface.
Blue color – excluding VHC factor. Red color - taking into account the VHC factor

Рис. 5. Отображение траектории падающего на слой СУ на поверхность поляроида.
Синий цвет – без учета фактора ВТ. Красный цвет – с учетом фактора ВТ

Conclusion

The paper shows that in rare cases, with the simultaneous action of several factors: nonuniform flow in front of the curvilinear oblique SW, the edge/boudary effect, and the effective external VHC factor - a very common way to map the solution to the plane of shock-wave polars or a 3D polaroid may be incorrect. Thus, it has been proven that even when using "ideal" (without scheme/artificial viscosity, limiters) computational methods, this phenomenon can occur.

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