UDC 539.3
Doi: 10.31772/2712-8970-2021-22-4-624-635

Для цитирования: Матвеев А. Д. Применение фиктивных дискретных моделей с переменными характерными размерами в расчетах на прочность композитных тел // Сибирский аэрокосмический журнал. 2021. Т. 22, № 4. С. 624–635. Doi: 10.31772/2712-8970-2021-22-4-624-635.

For citation: Matveev A. D. Application of fictitious discrete models with variable characteristic dimensions in calculations for the strength of composite bodies. *Siberian Aerospace Journal*. 2021, Vol. 22, No. 4, P. 624–635. Doi: 10.31772/2712-8970-2021-22-4-624-635.

Применение фиктивных дискретных моделей с переменными характерными размерами в расчетах на прочность композитных тел

А. Д. Матвеев

Институт вычислительного моделирования СО РАН
Российская Федерация, 660036, г. Красноярск, Академгородок, 50/44
E-mail: mtv241@mail.ru

Для анализа напряженно-деформированного состояния однородных и композитных тел (КТ) эффективно применяется метод многосеточных конечных элементов (ММКЭ), в котором используются многосеточные конечные элементы (МнКЭ). ММКЭ порождает многосеточные дискретные модели малой размерности, в которых неоднородная структура тел учитывается в рамках микроподхода с помощью МнКЭ. Базовые дискретные модели (БМ), учитывающие неоднородную структуру тел, имеют высокую размерность. Для понижения размерности дискретных моделей тел используется ММКЭ. Однако, существуют БМ КТ (например, БМ тел с микронеоднородной структурой), которые имеют такую высокую размерность, что реализация ММКЭ для таких БМ, в силу ограниченности ресурсов ЭВМ, затруднительна. Кроме того, для многосеточных дискретных моделей высокой размерности ММКЭ порождает численно неустойчивые решения, что связано с погрешностью вычислений ЭВМ. Для решения данных проблем здесь предлагается в расчетах использовать фиктивные дискретные модели, особенность которых состоит в том, что их размерности меньше размерностей БМ КТ.

В данной работе предлагается метод фиктивных дискретных моделей (МФДМ) для расчета на статическую прочность упругих композитных тел с неоднородной, микронеоднородной регулярной структурой. МФДМ реализуется с помощью ММКЭ с применением скорректированных условий прочности, которые учитывают погрешность приближенных решений. В основе МФДМ лежит положение, что решения, отвечающие БМ КТ, мало отличаются от точных, т. е. эти решения считаем точными.

Расчет КТ по МФДМ сводится к построению и расчету на прочность фиктивных дискретных моделей (ФМ), которые обладают следующими свойствами. ФМ отражают форму, характерные размеры, крепление, нагружение и вид неоднородной структуры КТ, распределение модулей упругости, отвечающее БМ КТ. Размерности ФМ меньше размерности БМ КТ. Последовательность, состоящая из ФМ, сходится к БМ, т. е. предельная ФМ совпадает с БМ. Как показывают расчеты, сходимость такой последовательности обеспечивает равномерную сходимость максимальных эквивалентных напряжений ФМ к максимальному эквивалентному напряжению БМ КТ, что позволяет применять такие ФМ в расчетах упругих тел на прочность.

Рассматриваются два типа ФМ. Первый тип – масштабированные ФМ, второй – ФМ с переменными характерными размерами. В данной работе подробно рассматриваются ФМ второго типа. Расчеты показывают, что реализация ММКЭ для ФМ с одним, двумя или тремя переменными характерными размерами приводит к большой экономии ресурсов ЭВМ, что позволяет использовать МФДМ для тел с микронеоднородной регулярной структурой. Расчеты на прочность КТ по МФДМ требуют в $10^3 \div 10^7$ *раз меньше объема памяти ЭВМ, чем аналогичный расчет с использованием БМ КТ, и не содержат процедуру измельчения БМ. Приведенный пример* *расчета на прочность трехмерной композитной балки по МФДМ с применением ФМ с тремя переменными характерными размерами показывает его высокую эффективность.*

Ключевые слова: упругость, композиты, скорректированные условия прочности, фиктивные дискретные модели, многосеточные конечные элементы.

Application of fictitious discrete models with variable characteristic dimensions in calculations for the strength of composite bodies

A. D. Matveev

Institute of Computational Modeling SB RAS
50/44, Akademgorodok, Krasnoyarsk, 660036, Russian Federation
E-mail: mtv241@mail.ru

To analyze the stress-strain state of homogeneous and composite bodies (CB), the method of multigrid finite elements (MMFE) is effectively applied, which uses multigrid finite elements (MgFE). MMFE generates multigrid discrete models of small dimension, in which the inhomogeneous structure of bodies is taken into account within the framework of a micro-approach using MgFE. Basic discrete models (BM), taking into account the heterogeneous structure of bodies, have a high dimension. To reduce the dimensionality of discrete models of bodies, MMFE is used. However, there are BM CB (for example, BM bodies with a micro-inhomogeneous structure), which have such a high dimension that the implementation of MMFE for such BM, due to limited computer resources, is difficult. In addition, for multigrid discrete models of high dimension, the MMFE generates numerically unstable solutions, which is associated with the error of computer calculations. To solve these problems, it is proposed here to use fictitious discrete models in calculations, the peculiarity of which is that their dimensions are smaller than the dimensions of BM CB.

In this paper, we propose a method of fictitious discrete models (MFDM) for calculating the static strength of elastic composite bodies with an inhomogeneous, micro-inhomogeneous regular structure. MFDM is implemented using MMFE with adjusted strength conditions application which takes into account the error of approximate solutions. The MFDM is based on the position that the solutions that meet the BM CB differ little from the exact ones, i. e. we consider these solutions to be accurate.

The calculation of CB by MFDM is reduced to the construction and calculation of the strength of fictitious discrete models (FM), which have the following properties. FM reflecst: the shape, characteristic dimensions, fastening, loading and type of inhomogeneous structure of the CB, and the distribution of elastic modulus corresponding to BM CB. The dimensions of FM are smaller than the dimensions of BM CB. The sequence consisting of FM converges to BM, i. e. the limiting FM coincides with BM. Calculations show that the convergence of such a sequence ensures uniform convergence of the maximum equivalent stresses of the FM to the maximum equivalent stress of the BM CB, which allows the application of such FM in the calculations of elastic bodies for strength.

Two types of FM are considered. The first type is scaled FM; the second type is FM with variable characteristic sizes. In this paper, the FM of the second type is considered in detail. Calculations show that the implementation of MMFE for FM with one, two or three variable characteristic sizes leads to a large saving of computer resources, which allows the use of MFDM for bodies with a micro-inhomogeneous regular structure. Calculations for the strength of CB according to MFDM require several times less computer memory than a similar calculation using BM CB, and does not contain a procedure for grinding BM. The given example of calculating the strength of a three-dimensional composite beam according to MFDM using FM with three variable characteristic dimensions shows its high efficiency.

Keywords: elasticity, composites, adjusted strength conditions, fictitious discrete models, multigrid finite elements.

Introduction

Static calculation on the strength of an elastic structure (body) V_0 is carried out, as a rule, according to the strength reserves [1-3] and comes down to determining the maximum equivalent stress of the structure. In this case, for the body V_0, the given strength conditions have the form $n_1 \le n_0 \le n_2$, where n_1, n_2 are set, n_0 - is the body safety factor V_0, $n_0 = \sigma_T / \sigma_0$, σ_T – the ultimate tension of the body V_0, σ_0 - the maximum equivalent stress of the body V_0, corresponding to the accurate solution of the elasticity problem, constructed for the body V_0. For maximum equivalent stresses, which are determined approximately, corrected strength conditions are used [4]. When analyzing the stress-strain state (SSS) of elastic bodies, the method of multigrid finite elements (MMFE) [5–11] is effectively applied, which uses multigrid finite elements (MgFE) [5–17]. The MMFE generates low-dimensional multigrid discrete models, in which the inhomogeneous structure of bodies is taken into account in the framework of the micro-approach [18] with the MgFE application. Basic discrete models (BM) of composite bodies (CB), which take into account their inhomogeneous, micro-inhomogeneous structure within the micro-approach, have a very high dimension. To reduce the dimensions of discrete models, the MMFE is very effectively used. However, for example, BM bodies with a micro-inhomogeneous regular structure have such a high dimension that the realisation of MMFE for such BM, due to limited computer resources, is difficult. In addition, for high-dimensional multigrid discrete models, the MMFE generates numerically unstable solutions, what is associated with the error of computer calculations. To solve these problems, in this article it is proposed to use fictitious discrete models in calculations, the peculiarity of which is that their dimensions are less than those of BM CB. The existing approximate approaches and methods for calculating CBs have complex formulations and are difficult for implementation for bodies with a complex inhomogeneous structure [19–26].

In this paper, a method of fictitious discrete models (MFDM) is proposed for calculating the strength of bodies with inhomogeneous, micro-inhomogeneous regular structure. MFDM is realised with MMFE application using corrected strength conditions which take into account the error of numerical solutions. Let us introduce a definition for fictitious discrete models.

Definition. Discrete models CB V will be called fictitious models (FM) if these FM have the following properties.

1. The inhomogeneous structures of the FM differ (do not differ) from the inhomogeneous structure of the BM CB V.

2. FM reflect: the form, characteristic dimensions, fastening, loading and type of inhomogeneous structure of CB V, and the distribution of elastic moduli corresponding to BM CB V.

3. A sequence consisting from FM converges to BM CB V, i.e., the limit FM of the sequence coincides with BM CB V.

4. The dimensions of the FM are less than the dimension of the BM CB V, except for the limiting FM, the dimension of which is equal to the dimension of the BM CB V.

Two main types of FM are considered here. The first type is scaled FM, the second is FM with variable characteristic sizes.

In [27], scaled composite discrete models are considered in detail as FM, the dimensions of which are less than the dimension of BM CB. The proposed FM, formed using a scaled regular CB cell, have the same characteristic dimensions, shape, fastenings and loadings as the BM, but the inhomogeneous structures of the FM differ from the inhomogeneous structure of the BM. The scaled FMs reflect the type of inhomogeneous structure of the BM CB and the distribution of elastic moduli corresponding to the BM. The convergence of the scaled FM sequence ensures uniform convergence of the maximum equivalent voltages of the FM to the maximum equivalent voltage of the BM. The high efficiency of using scaled FMs in MFDM is shown by the example of strength analysis of a beam with an inhomogeneous regular structure [27].

In this paper, the FM of the second type is considered in detail. Calculations show that the realisation of the MMFE for FM with one, two or three variable characteristic sizes leads to a large saving of computer resources, which makes it possible to use the MFDM for bodies with a micro-inhomogeneous regular structure.

Calculation of the strength of CB by MFDM requires $10^3 \div 10^7$ less computer memory than a similar calculation using BM CB, and does not contain the procedure for grinding of BM. The given example of strength calculation of a three-dimensional composite beam according to MFDM using FM with three variable characteristic dimensions shows its high efficiency.

1. Main provisions of the method of fictitious discrete models

MFDM is used for CBs with a regular structure which satisfy the following provisions.

Statement 1. CBs consist from multy-module isotropic homogeneous elastic bodies, the connections between which are ideal, i.e., on the common boundaries of multy-module isotropic homogeneous bodies, the functions of displacements and stresses are continuous.

Statement 2. Displacements, deformations and stresses of multy-module isotropic homogeneous bodies correspond to the relations of a three-dimensional linear problem of the theory of elasticity [28].

Statement 3. Approximate solutions, which correspond to BM CB, differ little from exact ones. Such approximate solutions will be considered as accurate. Note that, due to the convergence of the MMFE, such BMs for CBs always exist.

2. Fictitious discrete models with variable characteristic dimensions

In practice, composite beams and shells with a constant cross section are widely used, which are reinforced with continuous fibers of constant thickness. The fibers are parallel to the axis of the beam (shell).

Without losing the generality of judgments, for simplicity of presentation, we will consider the essence of constructing FM with one variable characteristic size using the example of a cantilever beam, the shape of a constant cross section of which is a symmetrical I-beam, consisting of 3 rectangles (fig. 1). CB V_0, located in a Cartesian rectangular coordinate system $Oxyz$, when $y = 0$ is rigidly fixed, i.e., when $y = 0$ we have: $u, v, w = 0$. The axis Oy in fig. 1 is parallel to the axis of the beam V_0. CB V_0 is reinforced with continuous fibers with a cross section $h \times h$ that are parallel to the axis Oy and have the same moduli of elasticity. Fig. 2 shows a section of a beam, consisting of 3 rectangles, the sections of the fibers are shaded.

At fig. 1, 3 a, b there are the characteristic dimensions of the cross section of the BM R_0 CB V_0 and FM R_n, L_0, (L_n) is the length of the BM R_0 (FM R_n), $L_n \le L_0$. FM R_n has the same fastening (i.e., when $y = 0$ FM R_n is rigidly fixed) and the same loading pattern as BM.

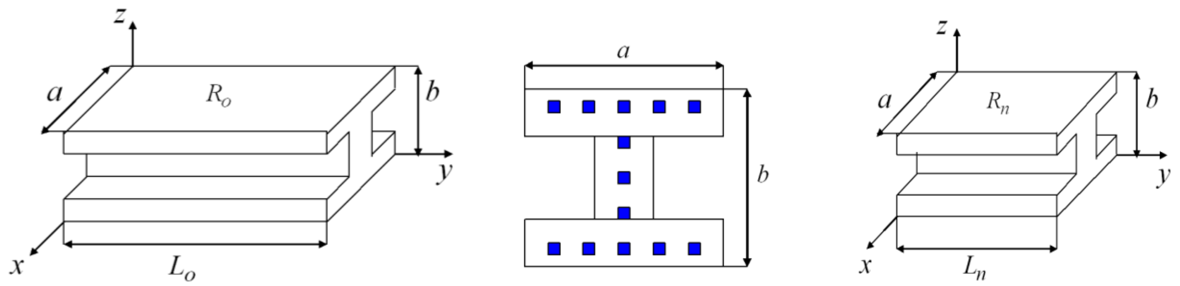

Рис. 1. Балка (КТ) V_0, БМ R_0 Рис. 2. Сечение КТ V_0 Рис. 3. ФМ R_n балки V_0

Fig. 1. Beam (CB) V_0, BM R_0 Fig. 2. Section of CB V_0 Fig. 3. FM R_n beams V_0

FM R_n has the same inhomogeneous structure as CB V_0, i.e. FM R_n is reinforced with continuous fibers parallel to the axis Oy with section $h \times h$ and has the same type of distribution of fibers in the cross section as CB V_0 (Fig. 2). The moduli of elasticity of the fibers and the binder material of CB V_0 and FM R_n are the same. For simplicity of presentation, let

$$h = L_0 / N, \quad (1)$$

where N is an integer; N – is set; $N >> 1$; h – few.

BM R_0 CB V_0 consists from the1st-order FE V_e of the shape of a cube with a side h (in which three-dimensional SSS is realised [28]), takes into account the inhomogeneous structure and complex shape of CB V_0. Let the BM R_0 generate a solution which differs little from the exact one and which we will assume to be exact (statement 3, item 1). The inhomogeneous structure in the FM R_n, as well as in the BM R_0, is taken into account with the help of the first-order FE V_e. Taking into account (1), the size L_n of the FM R_n is found by the formula

$$L_n = L_0 n / N = hn \quad (2)$$

where n is an integer, $n = n_0, ..., N$, n_0 is given, we have $L_n \le L_0$.

From the foregoing, taking into account that according to (2) $L_n \to L_0$ when $n \to N$, it follows that

$$R_n \to R_0 \quad \text{when} \quad n \to N. \quad (3)$$

When $n = N$ due to (2), (3) we have $R_N = R_0$. Then the fulfillment of (3) implies

$$\sigma_n \to \sigma_0 \quad \text{when} \quad n \to N, \quad (4)$$

where σ_n (σ_0) is the maximum equivalent voltage of the FM R_n (BM R_0).

FM R_n and BM R_0 consist from the 1-st order FE V_e of the shape of a cube with a side h and the cross sections of these models are the same. This means that the sections FM R_n and BM R_0 have the same number of nodes, equal to N_0. Then the total number of nodes M_0 BM R_0 is equal to $M_0 = N_0(N+1)$. The total number of nodes M_n FM R_n is equal to $M_n = N_0(n+1)$. From this it follows that when $n_0 \le n < N$ we have

$$M_n < M_0. \quad (5)$$

When $n = N$ we have $M_N = M_0$, i.e. $R_N = R_0$. So, the application of FM R_n with a variable size L_n in strength calculations according to MFDM CB V_0, due to (5), leads to saving computer resources. Calculations show the greatest efficiency of MFDM when using FM with three variable characteristic sizes in calculations.

3. Results of numerical experiments

Let us consider a model problem of calculating the strength of a cantilever beam V_0 with a non-uniform regular fibrous structure with dimensions $48h \times 1152h \times 96h$ (Fig. 4). A regular cell G_0 of a beam with dimensions $6h \times 6h \times 6h$ in fig. 5 is located in the local Cartesian coordinate system $Oxyz$, fibers with a section $h \times h$ are directed along the axis Oy, fiber sections in the plane Oxz are shaded, $i, j, k = 1,...,7$.

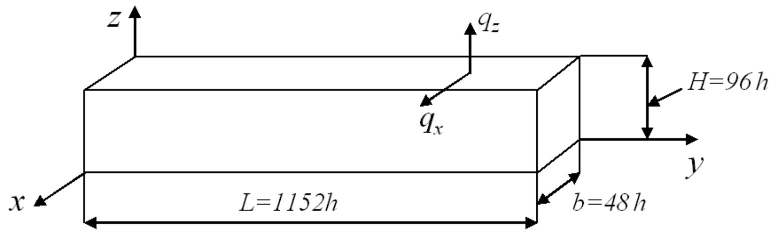

Рис. 4. Размеры балки V_0

Fig. 4. Dimensions of the beam V_0

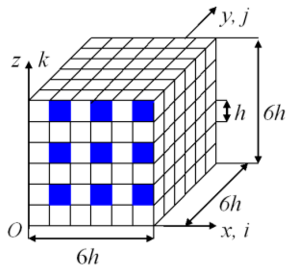

Рис. 5. Регулярная ячейка G_0

Fig. 5. Regular cell G_0

The beam is reinforced with longitudinal continuous fibers. When $y=0$ the beam is rigidly fixed, on the surface $z=H$ it has a load on q_x, q_z. For the safety factor n_0 of the beam, the strength conditions are given

$$1{,}8 \le n_0 \le 3{,}4\,. \tag{6}$$

For the model task, we have the following initial data:

$$h=0{,}2083;\quad \sigma_T=6;\quad E_c=1,\quad E_v=10,\quad \nu_c=\nu_v=0{,}3, \tag{7}$$

where E_c, E_v (ν_c, ν_v) – are Young's moduli (Poisson's ratios), matrix and fiber; σ_T – yield strength of the fiber, at the boundary $z=H$; $0{,}5L \le y \le L$ the load is set $q_z=q_x=0{,}000375$ (fig. 4).

The basic model R_0 CB V_0 is composed from single - grid finite elements (1gFE) V_j^h of the 1-st order cube shape with side h (in which three-dimensional SSS [28] is realised), takes into account the inhomogeneous structure of the CB V_0 and generates a uniform grid with step h of a dimension $49\times1153\times97$ with a total number of nodal unknowns of the finite element method (FEM) [29; 30] takes into account the heterogeneous structure of CB V_0 and generates a uniform grid with a step h of a dimension $49\times1153\times97$ with the total number of nodal unknowns of finite element method (FEM) [29; 30], equal to $N_0=16426368$, the tape width of the system of equations (SE) (FEM) is equal to $b_0=14556$. Let us consider, that BM R_0 CB V_0 satisfy Statement 3 item 1. In calculations we use FM R_n with three variable characteristic dimensions, $b_n \times L_n \times H_n$ (fig. 6), where

$$b_n=6hn,\quad L_n=24\times6hn,\quad H_n=2\times6hn, \tag{8}$$

i.e. FM R_n consists from regular cells G_0, fig. 5, $n=2,...,8$. At $n<8$ we have: $b_n<b$, $L_n<L$, $H_n<H$, at $n=8$: $b_8=b$, $L_8=L$, $H_8=H$, i.e. characteristic dimensions FM R_8 coinside with dimentions BM CB V_0. As FM and BM CB V_0 are presented with the same final elements (FE) (see point 2), then $R_8=\mathrm{R}_0$. At $y=0$ FM R_n is rigidly fixed, on the surface $z=H_n$, $0{,}5L_n \le y \le L_n$ has a load $q_z=q_x=0{,}000375$.

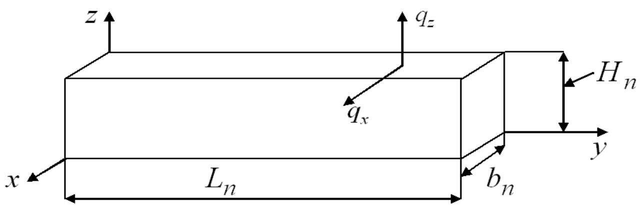

Рис. 6. Переменные характерные размеры ФМ R_n балки V_0

Fig. 6. Variable characteristic dimensions of the FM R_n beam V_0

In calculations, we use a two-grid FE (2gFE) $V_d^{(2)}$, having dimensions $6h \times 6h \times 6h$ (fig. 7), i. e. consisting of one regular cell G_0 (fig. 5). On the basis of model R_n we build a two-grid model R_n^o, which consists from 2gFE $V_d^{(2)}$.

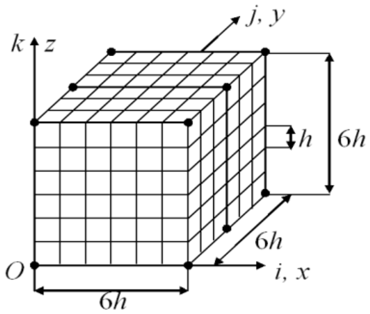

Рис. 7. Мелкая и крупная сетки 2сКЭ $V_d^{(2)}$

Fig. 7. Small and large grids 2gFE $V_d^{(2)}$

In fig. 7 2gFE $V_d^{(2)}$ is located in the local Cartesian coordinate system $Oxyz$. When constructing a 2gFE $V_d^{(2)}$ we use two nested grids: a small grid h_d with a step h dimension $7 \times 7 \times 7$ and a large grid – H_d with dimension $2 \times 3 \times 2$. Along the axes Ox, Oz the grid H_d has a step $6h$, along the axis Oy - a step $3h$. At fig. 7 the grids h_d and H_d are shown and nodes of a large grid are marked with dots, 12 nodes. Fine grid h_d is generated by the base partition R_d 2gFE $V_d^{(2)}$, which is composed from 1gFE V_j^h of the 1-st order cube shape with side h (in which three-dimensional SSS is realised) and takes into account the heterogeneous structure 2gFE $V_d^{(2)}$. The procedure for constructing the stiffness matrix and the vector of nodal forces 2gFE $V_d^{(2)}$ is described in detail in the work [27].

The calculation results are given in the table 1, where σ_n^o – is model maximum equivalent voltage R_n^o, found according to the 4th theory of strength, N_n^o и b_n^o – dimension and width of the tape SE MMFE of the model R_n^o, $n = 2,...,7$, relative error $\delta_n(\%)$ is determined by the formula

$$\delta_n(\%) = 100\% \times |\sigma_n^o - \sigma_{n-1}^o| / \sigma_n^o, \quad n = 2,...,7. \tag{9}$$

Analysis of the results shows a uniform monotonic convergence of stresses σ_n^0 and errors $\delta_n(\%)$, $n = 2,...,7$.

Table 1

Calculation results for models $R_2^o - R_7^o$

n	R_n^o	σ_n^o	$\delta_n(\%)$	N_n^o	b_n^o	n	R_n^o	σ_n^o	$\delta_n(\%)$	N_n^o	b_n^o
2	R_2^o	1.801	–	4320	105	5	R_5^o	2.373	7.33	47520	420
3	R_3^o	1.993	9.61	12096	186	6	R_6^o	2.525	6.03	78624	573
4	R_4^o	2.199	9.36	25920	291	7	R_7^o	2.661	5.12	120960	751

As $R_8 = \mathrm{R}_0$, then the voltage σ_8^o FM R_8^o, is equal to $\sigma_8^o = 2,785$, we consider the exact solution, i.e. $\sigma_0 = 2,785$ (see item 2, Statement 3 item 1). In calculations for the strength of elastic bodies according to MFDM, the corrected strength conditions are used (taking into account the error of approximate solutions), which are presented in the following theorem.

Theorem. Let for the safety factor n_0 of an elastic body V_0 the strength conditions are given

$$n_1 \le n_0 \le n_2, \tag{10}$$

where n_1, n_2 – are set, $n_1 > 1$; $n_0 = \sigma_T / \sigma_0$, σ_T – ultimate stress of the body V_0; σ_0 – the maximum equivalent stress of the body V_0, which corresponds to the exact solution of the problem of the theory of elasticity, built for the body V_0 .

Let the safety factor n_b of the body V_0, corresponding to the approximate solution of the problem of the theory of elasticity, satisfie the corrected strength conditions

$$\frac{n_1}{1-\delta_\alpha} \le n_b \le \frac{n_2}{1+\delta_\alpha}. \tag{11}$$

Then the safety factor n_0 of the body V_0, corresponding to the exact solution of the problem of the theory of elasticity, satisfies the specified strength conditions (10), where $n_b = \sigma_T / \sigma_b$, σ_b – maximum equivalent body stress V_0, corresponding to an approximate solution of the problem of elasticity theory, constructed for the body V_0, and found with such an error δ_b, that

$$|\delta_b| \le \delta_\alpha < C_\alpha = \frac{n_2 - n_1}{n_1 + n_2}, \tag{12}$$

where δ_α – upper estimate of the relative error, δ_b, δ_α – is given, error δ_b for tension σ_b is determined by the formula $\delta_b = (\sigma_0 - \sigma_b)/\sigma_0$.

Note, that if the body V_0 consists of plastic materials, then σ_T – is the yield strength. From (12) follows, that if $n_2 - n_1$ few, then σ_b must be determined with a small error δ_b.

The proof of the theorem is given in the article [4].

For the given $n_1 = 1{,}8$ и $n_2 = 3{,}4$ according to (12) we have $C_\alpha = 0{,}31$. Calculations show that if $\delta_n(\%) \le 10\ \%$, then the tension error σ_n^o of model R_n^o is not more than $15\ \%$. Tensions $\sigma_6^o = 2{,}525$ and $\sigma_5^o = 2{,}373$ differ at $\delta_6(\%) = 6{,}028\ \%$ (look table 1), then the tension error σ_6^o is not more than $15\ \%$, i. e. we have $\delta_\alpha \le 0{,}15$. Note, that $\sigma_6^o = 2{,}525$ differes from the exact voltage $\sigma_0 = 2{,}785$ on $9{,}33\ \%$. Let's accept that $\delta_\alpha = 0{,}15$, $\sigma_b = \sigma_6^o$. Condition (12) for δ_α is satisfied, i.e. we have $\delta_\alpha = 0{,}15 < C_\alpha = 0{,}31$. Using $\delta_\alpha = 0{,}15$, $n_1 = 1{,}8$ and $n_2 = 3{,}4$ in (11) we get

$$2{,}12 \le n_b \le 2{,}96. \tag{13}$$

Using $\sigma_b = 2{,}525$, $\sigma_T = 6$, we find the safety factor n_b for CB V_0 according to the formula

$$n_b = \sigma_T / \sigma_b = 6 / 2{,}525 = 2{,}38. \tag{14}$$

As the found coefficient n_b satisfies the corrected strength conditions (13), then, according to the above formulated theorem, the safety factor n_0 CB V_0, corresponding to the exact solution of the elasticity problem found for CB V_0, satisfies the specified strength conditions (6), i.e. $1{,}8 \le n_0 \le 3{,}4$. Indeed, $n_0 = \sigma_T / \sigma_0 = 6/2{,}785 = 2{,}15$, safety factor $n_0 = 2{,}15$ CB V_0 satisfies the specified strength conditions (6), i.e. we have $1{,}8 < 2{,}15 < 3{,}4$.

In strength calculations CB V_0 according to MFDM, we use the discrete model R_6^o, which requires $\mathrm{k} = \frac{N_0 \times b_0}{N_6^o \times b_6^o} = \frac{16426368 \times 14556}{78624 \times 573} = 5307{,}30$ times less than the amount of computer memory, i.e., almost in $5{,}3 \times 10^3$ times less, than BM R_0 CB V_0, what shows the high efficiency of the realization of the MFDM using the FM with three variable characteristic sizes.

Conclusion

The method of fictitious discrete models (MFDM) is proposed for calculating the static strength of elastic bodies with an inhomogeneous, micro-inhomogeneous regular structure. The proposed method is reduced to the construction and calculation of the strength of fictitious discrete models (FM), the dimensions of which are less than the dimensions of the basic discrete models (BM) of composite bodies (CB), and is realised using the multigrid finite element method (MMFE) and corrected conditions strength, which take into account the error of approximate solutions. Here, FMs are represented by two main types. The first type is scaled FM; the second type is FM with variable characteristic dimensions. In this paper, FM of the second type is considered in detail. Calculations show that the implementation of MMFE for FM with variable characteristic dimensions leads to a large saving of computer resources, what makes it possible to use MFDM for bodies with a microinhomogeneous regular structure. Calculations for the strength of CB using MFDM require less computer memory than a similar calculation using BM CB, and do not contain the procedure for grinding CB. The given example of calculating the strength of a composite beam according to MFDM using FM with three variable characteristic dimensions shows its high efficiency.

References

1. Pisarenko G. S., Yakovlev A. P., Matveev V. V. *Spravochnik po soprotivleniyu materialov* [Handbook of resistance materials']. Kiev, Nauk. Dumka Publ., 1975, 704 p.

2. Birger I. A., Shorr B. F., Iosilevich G. B. Raschet na prochnost' detalej mashin [Calculation of the strength of machine parts]. Moscow, Mashinostroenie Publ., 1993, 640 p.

3. Moskvichev V. V. *Osnovy konstrukcionnoy prochnosti tekhnicheskih sistem i inzhenernyh sooruzheniy* [Fundamentals of structural strength of technical systems and engineering structures]. Novosibirsk, Nauka Publ., 2002, 106 p.

4. Matveev A. D. [Calculation of elastic structures using the adjusted terms of strength]. *Izvestiya AltGU.* 2017, No. 4, P. 116–119. Doi: 10.14258/izvasu (2017)4-21.

5. Matveev A. D. [The method of multigrid finite elements in the calculations of three-dimensional homogeneous and composite bodies]. *Uchen. zap. Kazan. un-ta. Seriia: Fiz. -matem. Nauki.* 2016, Vol. 158, Is. 4, P. 530–543 (In Russ.).

6. Matveev A. D. [Multigrid method for finite elements in the analysis of composite plates and beams]. *Vestnik KrasGAU.* 2016, No. 12, P. 93–100 (In Russ.).

7. Matveev A. D. Multigrid finite element method in stress of three-dimensional elastic bodies of heterogeneous structure. *IOP Conf, Ser.: Mater. Sci. Eng.* 2016, Vol. 158, No. 1, Art. 012067, P. 1–9.

8. Matveev A. D. [Multigrid finite element Method in the calculations of composite plates and beams of irregular shape]. *The Bulletin of KrasGAU*. 2017, No. 11, P. 131–140.

9. Matveev A. D. [Multigrid finite element Method]. *The Bulletin of KrasGAU*. 2018, No. 2, P. 90–103 (In Russ.).

10. Matveev A. D. [The method of. multigrid finite elements of the composite rotational and bi-curved shell calculations]. *The Bulletin of KrasGAU*. 2018, No. 3, P. 126–137 (In Russ.).

11. Matveev A. D. [Method of. multigrid finite elements to solve physical boundary value problems]. *Information technologies and mathematical modeling*. Krasnoyarsk, 2017, P. 27–60.

12. Matveev A. D. [Some approaches of designing elastic multigrid finite elements]. *VINITI Proceedings*. 2000, No. 2990-B00, P. 30 (In Russ.).

13. Matveev A. D. [Mixed discrete models in the analysis of elastic three-dimensional inhomogeneous bodies of complex shape]. *Vestnik PNIPU. Mekhanika*. 2013, No. 1, P. 182–195 (In Russ.).

14. Matveev A. D. [Multigrid modeling of composites of irregular structure with a small filling ratio]. *J. Appl. Mech. Tech. Phys*. 2004, No. 3, P. 161–171 (In Russ.).

15. Matveev A. D. [The construction of complex multigrid finite element heterogeneous and micro-inhomogeneities in structure]. *Izvestiya AltGU. Seriya: Matematika i mekhanika*. 2014, No. 1/1, P. 80–83 (In Russ.). Doi: 10.14258/izvasu (2014)1.1-18.

16. Matveev A. D. [Method of generating finite elements]. *The Bulletin of KrasGAU*. 2018, No. 6, P. 141–154 (In Russ.).

17. Matveev A. D. [Construction of multigrid finite elements to calculate shells, plates and beams based on generating finite elements]. *PNRPU Mechanics Bulletin*. 2019, No. 3, P. 48–57 (In Russ.). Doi: 10/15593/perm.mech/2019.3.05.

18. Fudzii T., Dzako M. *Mekhanika razrusheniya kompozicionnyh materialov* [Fracture mechanics of composite materials]. Moscow, Mir Publ., 1982, 232 p.

19. Golushko S. K., Nemirovskij Y. V. *Pryamye i obratnye zadachi mekhaniki uprugih kompozitnyh plastin i obolochek vrashcheniya* [Direct and inverse problems of mechanics of elastic composite plates and shells of rotation]. Moscow, FIZMATLIT Publ., 2008, 432 p.

20. Nemirovskij Y. V., Reznikov B. S. *Prochnost' elementov konstrukciy iz kompozitnyh materiallov* [Strength of structural elements made of composite materials]. Novosibirsk, Nauka Publ., 1984, 164 p.

21. Kravchuk A. S., Majboroda V. P., Urzhumcev Y. S. Mekhanika polimernyh i kompozicionnyh materialov [Mechanics of polymer and composite materials]. Moscow, Nauka Publ., 1985, 201 p.

22. Alfutov N. A., Zinov’ev A. A., Popov B. G. *Raschet mnogoslojnyh plastin i obolochek iz kompozicionnyh materialov* [Calculation of multilayer plates and shells made of composite materials]. Moscow, Mashinostroenie Publ., 1984, 264 p.

23. Pobedrya B. E. *Mekhanika kompozicionnyh materialov* [Mechanics of composite materials]. Moscow, MGU Publ., 1984, 336 p.

24. Andreev A. N., Nemirovskij Y. V. *Mnogosloynye anizotropnye obolochki i plastiny. Izgib, ustoychivost’, kolebaniya* [Multilayer anisotropic shells and plates. Bending, stability, vibration]. Novosibirsk, Nauka Publ., 2001, 288 p.

25. Vanin G. A. *Mikromekhanika kompozicionnyh materialov* [Micromechanics of composite materials]. Kiev, Naukova dumka Publ., 1985, 302 p.

26. Vasil’ev V. V. *Mekhanika konstrukciy iz kompozicionnyh materialov* [Mechanics of structures made of composite materials]. Moscow, Mashinostroenie Publ., 1988, 269 p.

27. Matveev A. D. [The method of fictitious discrete models in calculations bodies with in inhomogeneous regular structure]. *Siberion Aerospace Journal*. 2021, Vol. 22, No. 2. P. 244–260.
(In Russ.) Doi: 10.31772/2712-8970-2021-22-2-244-260.

28. Samul’ V. I. Osnovy teorii uprugosti i plastichnosti [Fundamentals of the theory of elasticity and plasticity]. Moscow, Vysshaia shkola Publ., 1982, 264 p.

29. Norri D., de Friz Zh. Vvedenie v metod konechnykh elementov [Introduction to the finite element method]. Moscow, Mir Publ., 1981, 304 p.

30. Zenkevich O. *Metod konechnykh elementov v tekhnike* [Finite element method in ingineering]. Moscow, Mir Publ., 1975, 544 p.

Библиографические ссылки

1. Писаренко Г. С., Яковлев А. П., Матвеев В. В. Справочник по сопротивлению материалов. Киев: Наук. думка, 1975. 704 с.

2. Биргер И. А., Шорр Б. Ф., Иосилевич Г. Б. Расчет на прочность деталей машин. М.: Машиностроение, 1993. 640 с.

3. Москвичев В. В. Основы конструкционной прочности технических систем и инженерных сооружений. Новосибирск: Наука, 2002. 106 с.

4. Матвеев А. Д. Расчет упругих конструкций с применением скорректированных условий прочности // Известия АлтГУ. Математика и механика. 2017. № 4. С. 116–119. Doi: 10.14258/izvasu (2017)4-21.

5. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах трехмерных однородных и композитных тел // Учен. зап. Казан. ун-та. Серия: Физ.-матем. науки. 2016. Т. 158, кн. 4. С. 530–543.

6. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах композитных пластин и балок // Вестник КрасГАУ. 2016. № 12. С. 93–100.

7. Matveev A. D. Multigrid finite element method in stress of three-dimensional elastic bodies of heterogeneous structure // IOP Conf. Ser.: Mater. Sci. Eng. 2016. Vol. 158, № 1. P. 1–9.

8. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах композитных пластин и балок сложной формы // Вестник КрасГАУ. 2017. № 11. С. 131–140.

9. Матвеев А. Д. Метод многосеточных конечных элементов // Вестник КрасГАУ. 2018. № 2. С. 90–103.

10. Матвеев А. Д. Метод многосеточных конечных элементов в расчетах композитных обо-лочек вращения и двоякой кривизны // Вестник КрасГАУ. 2018. № 3. С. 126–137.

11. Матвеев А. Д. Метод многосеточных конечных элементов в решении физических краевых задач. // Информационные технологии и математическое моделирование. 2017. С. 27–60.

12. Матвеев А.Д. Некоторые подходы проектирования упругих многосеточных конечных элементов. // Деп. в ВИНИТИ № 2990–В00. 2000. 30 с.

13. Матвеев А. Д. Смешанные дискретные модели в анализе упругих трехмерных неоднородных тел сложной формы // Вестник ПНИПУ. Механика. Пермь: Изд-во Перм. нац. исслед. Политехн. ун-та. 2013. № 1. С. 182–195.

14. Матвеев А. Д. Многосеточное моделирование композитов нерегулярной структуры с малым коэффициентом наполнения // Прикладная механика и техническая физика. 2004. № 3. С. 161–171.

15. Матвеев А. Д. Построение сложных многосеточных конечных элементов с неоднородной и микронеоднородной структурой // Известия АлтГУ. Серия: Математика и механика. 2014. № 1/1. С. 80–83. Doi: 10.14258/izvasu(2014)1.1-18.

16. Матвеев А. Д. Метод образующих конечных элементов // Вестник КрасГАУ. 2018. № 6. С. 141–154.

17. Матвеев А. Д. Построение многосеточных конечных элементов для расчета оболочек, пластин и балок на основе образующих конечных элементов // Вестник ПНИПУ. Механика. 2019. № 3. С. 48–57. Doi: 10/15593/perm.mech/2019.3.05.

18. Фудзии Т., Дзако М. Механика разрушения композиционных материалов. М. : Мир, 1982. 232 с.

19. Голушко С. К., Немировский Ю. В. Прямые и обратные задачи механики упругих композитных пластин и оболочек вращения. М.: ФИЗМАТЛИТ. 2008. 432 с.

20. Немировский Ю. В., Резников Б. С. Прочность элементов конструкций из композитных материалов. Новосибирск: Наука, Сибирское отделение. 1984. 164 с.

21. Кравчук А. С., Майборода В. П., Уржумцев Ю. С. Механика полимерных и композиционных материалов. М.: Наука. 1985. 201 с.

22. Алфутов Н. А., Зиновьев А. А., Попов Б. Г. Расчет многослойных пластин и оболочек из композиционных материалов. М.: Машиностроение, 1984. 264 с.

23. Победря Б. Е. Механика композиционных материалов. М.: МГУ, 1984. 336 с.

24. Андреев А. Н., Немировский Ю. В. Многослойные анизотропные оболочки и пластины. Изгиб, устойчивость, колебания. Новосибирск: Наука, 2001. 288 с.

25. Ванин Г. А. Микромеханика композиционных материалов. Киев: Наукова думка. 1985. 302 с.

26. Васильев В. В. Механика конструкций из композиционных материалов. М.: Машиностроение, 1988. 269 с.

27. Матвеев А. Д. Метод фиктивных дискретных моделей в расчетах тел с неоднородной регулярной структурой // Сибирский аэрокосмический журнал. 2021. Т. 22, № 2. С. 244–260. Doi: 10.31772/2712-8970-2021-22-2-244-260.

28. Самуль В. И. Основы теории упругости и пластичности. М.: Высшая школа, 1982. 264 с.

29. Норри Д., Ж. де Фриз. Введение в метод конечных элементов: М.: Мир, 1981. 304 с.

30. Зенкевич О. Метод конечных элементов в технике. М.: Мир, 1975. 542 с.

Матвеев Александр Данилович – кандидат физико-математических наук, доцент, старший научный сотрудник; Институт вычислительного моделирования СО РАН. E-mail: mtv241@mail.ru.

Matveev Alexander Danilovich – Cand. Sc., associate Professor, senior researcher; Institute of computational modeling SB RAS. E-mail: mtv241@mail.ru.