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HEAT TRANSFER IN THE CENTRIFUGAL FORCE FIELD FOR GAS TURBINES ELEMENTS

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The study of heat transfer from combustion products (CP) to the impeller and the casing of gas turbines of liquid rocket engines (LRE) is an urgent task.

The solution of the flow problem, taking into account heat transfer, in rotational flows, in the flowing parts of the turbopump units (TPU) of the rocket engine, is carried out by the following methods: numerical methods; analytical approach, when solving the equations of dynamic and temperature boundary layers; as well as using empirical dependencies. The temperature parameter of the gaseous combustion products and, as a consequence, the heat exchange between the combustion products and the structural elements of the flow part, significantly affects the working and energy characteristics of the TPU LRE.

When designing gas turbines of LRE, it is necessary to take into account the presence of heat exchange processes, the working fluid temperature distribution and the structural element temperatures in the cavities of the TPU LRE (since energy losses and viscosity depend on the temperatures of the working fluid, and also determine the flow parameters). The temperature distribution in the structural elements determines the performance and reliability of the unit.

In the case of the use of cryogenic fuel components in the TPU LRE units the heating of the component leads to the implementation of cavitation modes and a drop in operating and energy characteristics. On the other hand, a lowered temperature of the working fluid leads to an increased viscosity of the components and, as a consequence, a decrease in the efficiency of the unit (especially when using gel-like components).

When studying heat transfer in the field of centrifugal forces for elements of rocket engine gas turbines it is necessary to obtain a joint solution of the equations of dynamic and temperature boundary layers in the boundary conditions of the flow parts.

This article offers a model of the distribution of dynamic and temperature boundary layers taking into account the convective component (for the case of a gaseous working fluid, i. e. Pr < 1), which is necessary for the analytical solution and determination of the heat transfer coefficient in the boundary conditions of the flow cavities of the LRE turbine. The energy equation has been analytically obtained for the boundary conditions of the temperature boundary layer, which allows integration over the surface of any shape, which is necessary in determining the thickness of the energy loss. Taking into account the integral relation, the heat transfer law of the turbulent boundary layer for the rotation cavities is written. The equations for determining the heat transfer coefficient in the form of the Stanton criterion for rectilinear uniform and rotational flows for cases of turbulent flow regimes were obtained analytically. The obtained equations for heat transfer coefficients are in good agreement with experimental data and dependences of other authors.

Keywords: heat transfer coefficient, temperature boundary layer, turbopump unit, power parameters, turbine.

ТЕПЛООТДАЧА В ПОЛЕ ЦЕНТРОБЕЖНЫХ СИЛ ДЛЯ ЭЛЕМЕНТОВ ГАЗОВЫХ ТУРБИН

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Исследование теплоотдачи от продуктов сгорания (ПС) к рабочему колесу и корпусу газовых турбин жидкостных ракетных двигателей (ЖРД) является актуальной задачей.

Решение задачи течения с учетом теплообмена при вращательных течениях в проточных частях турбонасосных агрегатов (ТНА) ЖРД осуществляется следующими способами: численными методами, аналитическим подходом при решении уравнений динамического и температурного пограничных слоев, а также с использованием эмпирических зависимостей. Параметр температуры газообразных ПС и, как следствие, теплообмен между ПС и конструктивными элементами проточной части значительно влияет на рабочие и энергетические характеристики ТНА ЖРД.

При проектировании газовых турбин ЖРД необходимо учитывать наличие теплообменных процессов, распределение температур рабочего тела и температур конструктивных элементов в полостях ТНА ЖРД (так как энергетические потери и вязкость зависят от температур рабочего тела, а также определяют параметры течения). Распределение температур в конструктивных элементах определяют работоспособность и надежность агрегата.

В случае применения криогенных компонентов топлива в агрегатах подачи ТНА ЖРД, подогрев компонента приводит к реализации кавитационных режимов и падению рабочих и энергетических характеристик. С другой стороны, пониженная температура рабочего тела приводит к повышенной вязкости компонентов и снижению КПД агрегата (особенно при использовании гелеобразных компонентов).

При исследовании теплоотдачи в поле центробежных сил для элементов газовых турбин ЖРД необходимо получить совместное решение уравнений динамического и температурного пограничных слоев в граничных условиях проточных частей.

Предложена модель распределения динамического и температурного пограничных слоев с учетом конвективной составляющей (для случая газообразного рабочего тела, т. е. Pr < 1), необходимая для аналитического решения и определения коэффициента теплоотдачи в граничных условиях проточных полостей турбины ЖРД. Аналитически получено уравнение энергии для граничных условий температурного пограничного слоя, позволяющего вести интегрирование по поверхности любой формы, необходимое при определении толщин потери энергии. С учетом интегрального соотношения записан закон теплообмена турбулентного пограничного слоя для полостей вращения. Аналитическим путем получены уравнения для определения коэффициента теплоотдачи в виде критерия Стантона для прямолинейного равномерного и вращательных течений для случаев турбулентных режимов течения. Полученные аналитические уравнения для коэффициентов теплоотдачи хорошо согласуются с экспериментальными данными и зависимостями других авторов.

Ключевые слова: коэффициент теплоотдачи, температурный пограничный слой, турбонасосный агрегат, энергетические параметры, турбина.

Introduction. Considering the features of heat transfer in the flowing parts of turbopump units (TPU) of liquid rocket engines (LRE) is an urgent task. Currently, taking into account the features of the flow with heat transfer during the implementation of the potential and vortex rotational flow in the flow parts is mainly carried out by the following methods: using empirical equations, numerical and analytical methods for solving differential equations in partial derivatives [1].

The first method does not always provide the required accuracy in calculating the hydrodynamic and thermal characteristics of rotational flows taking heat transfer into account and requires additional specifying experimental studies. As a rule, this entails quite a bit more time and material costs for setting up and conducting research.

Numerical methods are quite difficult to use when carrying out engineering calculations and require their implementation in specialized software. Numerical research methods use direct numerical simulation (DNS method) and the averaged Navier-Stokes and Reynolds equations (RANS method). The choice of method depends on the complexity of the problem and the accuracy of the results. The RANS method is often used with the k-ε and k-ω turbulence models [2–7]. The issues of heat transfer during the flow around a curved surface with a longitudinal curvature of the working fluid gaseous flow were also considered in [8–10], where cases of flow around turbine blades were investigated. In [11], convective heat transfer in a channel with periodic protrusions was studied on the basis of multiblock computational technologies based on the solution of the Reynolds equations closed by the Mentor model of shear stress transfer using the factorized finite-volume method and the energy equation on multiscale intersecting structured grids.

The analytical method allows obtaining analytical dependences applicable for engineering calculations in a wide range of possible variations of design and operational parameters. Analytical methods, as a rule, were developed for a rectilinear uniform flow, and have a number of limitations. One of the earliest studies is the work of E. L. Knuth [12], in which the analysis is based on an extended Reynolds analogy with the transfer of heat, mass, and momentum in a developed turbulent flow in a pipe. The use of the velocity and temperature distribution profiles in the boundary layer was proposed by W. D. Rannie [13] and modified by D.L. Turcotte [14]. Turcotte sublayer analysis took into account the effect of heat transfer on turbulence. The analytical methods for determining the heat transfer coefficients proposed in [15; 16] take into account convective heat transfer in LRE chambers and are performed for a rectilinear turbulent flow. A one-dimensional analytical model for subcritical conditions was also proposed by S.R. Shine [17].

Design features of liquid rocket engine turbopump units (LRE TPU) turbines and the object of the study. During design, the diameter of the LRE TPU turbine is selected taking into account the layout and ensuring the minimum dimensions and is limited by the strength of the turbine rotor. From the analysis of adiabatic work it is known that with a selected working fluid the greatest value of adiabatic work is achieved at high temperatures and large pressure ratios.

The use of high temperatures is limited by the structure operability. LRE use uncooled turbines. Due to the design features and materials used the working fluid temperature in front of an uncooled turbine is limited, as a rule, for a reducing gas of 1000–1200 ^{0}C, for an oxidizing gas of 700–900 ^{0}C. The higher the temperature in front of the turbine can be assumed, the less pressure should be in front of the turbine. Due to the design features, LRE turbines are single-stage and rarely two-stage.

High temperatures of the working fluid lead to temperature deformations, including turbine disks [18]. When designing the flowing parts of the units and assemblies of the LRE TPU, it is necessary to take into account the change in the working fluid flow temperature along the length of the working channel, since the viscosity parameter is a function of temperature and determines the flow regime and, as a result, losses, in particular, disk friction and hydrodynamic losses in the flowing part.

The optimal level of the turbopump units (TPU) energy parameters stability is ensured in the course of development work by adjusting the geometric dimensions of parts and assembly the gas-dynamic path units, choosing technological schemes for dimensional processing, assembly and turbine testing with the involvement of static material significant amount. In this regard, the LRE TPU energy parameters' modelling represents an urgent scientific and technical problem. The parameter optimization of the workflow and the mathematical model of the propulsion system (PS) are considered by V.A. Grigoryev [19], where the analysis of models was carried out and the advantages and disadvantages for various design stages were revealed.

The requirements for the TPU design are formed on the basis of the tasks performed by the propulsion system (PS) into which the TPU is integrated as a single unit. And the requirements for the TPU design are formed on the basis of the tasks performed by the propulsion system (PS), into which the TPU is an integral part with which it is assembled as a single unit. Requirements for PS fully apply to the TPU: to ensure at all engine operating modes the supply of fuel components with the required flow rate and pressure with a high degree of reliability and efficiency; provide minimum dimensions and weight; simplicity of design and minimal cost.

The main object of study where the potential and vortex rotational flow occurs is the structural elements of the flowing parts of gas turbines of the rocket engine: inlet and outlet device, the cavity between the stator and the impeller [20].

The research problem. In the generalized problem formulation of fluid flow during heat exchange with the surface of aggregates, such as compressors, expanders, pumps of cryogenic components, etc., it is necessary to take into account the change in flow temperature along the length of the working channel, since viscosity, as a function of temperature, mainly determines the flow regime and, as a result, hydraulic losses [21].

For the case of incompressible fluid flow, it is necessary and sufficient to jointly solve the equations of motion and energy in the boundary conditions of the spatial boundary layer [22]; for a compressible fluid, the system must be supplemented with the equation of state.

The processes of heat transfer in power stations are largely similar, but there are certain differences in the analysis and derivation of heat transfer equations for the boundary conditions of the liquid propellant rocket engine. The main differences are as follows: extremely high values of heat fluxes, temperatures and pressures, the presence of high flow velocities, the initial turbulent state of flows in the core, working fluids can be in a gaseous and liquid state, surface curvature effects, the presence of density and compressibility gradients, non-stationarity heat fluxes and the presence of flow instability in the active zone of heat transfer [23].

The heat transfer law of a turbulent flow of a spatial boundary layer temperature. The integral relation of the energy equation. For Prandtl numbers less than unity ($Pr < 1$ is characteristic of gaseous working fluids, including combustion products), the thickness of the dynamic boundary layer is below the thickness of the temperature boundary layer, i. e., $\delta < \delta t$ (fig. 1). At very low Prandtl numbers (liquid metals), molecular thermal conductivity is the main mechanism of heat transfer and cannot be neglected even in a turbulent flow core. At low Prandtl numbers, thermal resistance is distributed over the entire flow section [24].

If we assume that at the boundary of the dynamic boundary layer, the temperature changes due to the transfer of velocity, and beyond its boundary – only due to molecular thermal conductivity. This assumption is in good agreement with Case's conclusions, since at very low Prandtl numbers the thickness of the dynamic boundary layer is much less than the thickness of the temperature boundary layer. Accordingly, thermal resistance is present throughout the thickness of the temperature boundary layer. Within the boundaries of the dynamic boundary layer, thermal resistance is due to turbulent heat transfer and outside the boundaries of the dynamic boundary layer thermal resistance is due to molecular thermal conductivity.

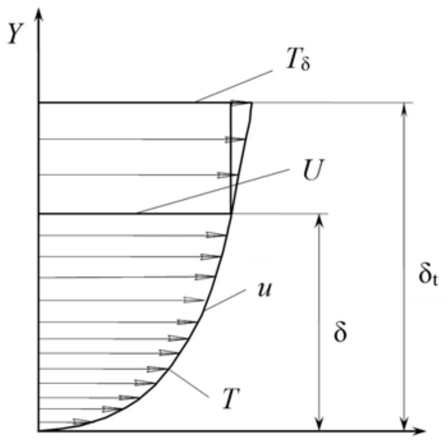

Fig. 1. Temperature distribution model and dynamic boundary layers at Pr < 1

Рис. 1. Модель распределения температурного и динамического пограничных слоев при Pr < 1

The equation of the thickness of the energy loss of the temperature boundary layer could be considered as [25]:

$$\delta_{t\varphi}^{**} = \int_0^{\delta_t} \frac{u}{U}\left(1 - \frac{T - T_0}{T_\delta - T_0}\right) dy . \quad (1)$$

The integration boundaries of the thickness of the energy loss (1) must be divided into two characteristic sections. The first integration section lies at the boundary of the thickness of the dynamic boundary layer δ, the second integration section lies at the boundary of the end of the thickness of the dynamic boundary layer δ to the end of the temperature boundary layer δ_t.

The thickness of the energy loss equation (1) for the distribution of the temperature model and dynamic boundary layers under consideration, with the accepted model of splitting into two characteristic integration sections, can be written as:

$$\delta_{t\varphi}^{**} = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{T - T_0}{T_\delta - T_0}\right) dy + \int_\delta^{\delta_t} \frac{u}{U}\left(1 - \frac{T - T_0}{T_\delta - T_0}\right) dy . \quad (2)$$

Using equation (2), it becomes possible to determine the form of the heat transfer law for the case Pr < 1. For further use, equation (2) needs to be integrated taking into account the accepted laws of distribution of the velocity profile in the boundary layer. Next, we first consider the power-law profile, then the gradient profile.

The distribution of the dynamic boundary layer is approximated by a power function of the form

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{m}} .$$

The distribution of the temperature boundary layer is approximated by a function of the form

$$\frac{T - T_0}{T_\delta - T_0} = \left(\frac{y}{\delta_t}\right)^{\frac{1}{m}} .$$

Given the accepted model of the distribution of dynamic and temperature boundary layers and the distribution function of the parameters of the dynamic and temperature layers, we write the equation for the thickness of the energy loss:

$$\delta_{t\varphi}^{**} = \int_0^{\delta} \left(\frac{y}{\delta}\right)^{\frac{1}{m}} \left(1 - \left(\frac{y}{\delta_t}\right)^{\frac{1}{m}}\right) dy + .$$

$$+ \int_\delta^{\delta_t} \left(\frac{y}{\delta}\right)^{\frac{1}{m}} \left(1 - \left(\frac{y}{\delta_t}\right)^{\frac{1}{m}}\right) dy. \quad (3)$$

If we take into account that in the first term of equation (3), the distribution profiles of the dynamic and temperature boundary layers coincide. Then, within the boundaries of integration of the first term of equation (3), the thicknesses of the temperature and dynamic boundary layers coincide, i. e. $\delta_t = \delta$. Considering the second term of equation (3), we note that there is no change in the dynamic boundary layer along the Y axis at the integration boundaries and the flow velocity is equal to the flow velocity in the flow core. Then the distribution of the dynamic boundary layer in the second term of equation (3) can be written as

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{m}} = 1 .$$

Also, in the second term of equation (3.3), a change in the diagram of the temperature boundary layer occurs only due to molecular thermal conductivity. It should be noted that for this section of the profile diagram is as follows:

$$\frac{T - T_\delta}{T_{\delta_t} - T_\delta} = \lambda \frac{y}{\delta_t} .$$

Taking into account the condition of conjugation of the temperature and dynamic profiles for the physical model of thermal conductivity at Pr < 1 (fig. 2) with the convective component, we write:

$$\frac{T - T_0}{T_\delta - T_0} = \lambda\left(\frac{y}{\delta} - 1\right) + 1 .$$

In this case, the equation for the thickness of the energy loss, taking into account the accepted two-layer model of the distribution of the temperature boundary layer with the presence of heat transfer under the condition Pr < 1, can be rewritten as:

$$\delta_{t\varphi}^{**} = \int_0^{\delta} \left(\frac{y}{\delta}\right)^{\frac{1}{m}} \left(1 - \left(\frac{y}{\delta}\right)^{\frac{1}{m}}\right) dy + \int_\delta^{\delta_t} \left(1 - \left(\lambda\left(\frac{y}{\delta} - 1\right) + 1\right)\right) dy =$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{\frac{1}{m}} \left(1 - \left(\frac{y}{\delta}\right)^{\frac{1}{m}}\right) dy + \int_\delta^{\delta_t} \left(-\lambda\left(\frac{y}{\delta} - 1\right)\right) dy. \quad (4)$$

Let us replace the variables in the equation of the thickness of the energy loss of the temperature boundary layer (4):

$$A1=\int_0^{\delta}\left(\frac{y}{\delta}\right)^{\frac{1}{m}}\left(1-\left(\frac{y}{\delta}\right)^{\frac{1}{m}}\right)dy,$$

$$A2=\int_{\delta}^{\delta_t}\left(-\lambda\left(\frac{y}{\delta}-1\right)\right)dy.$$

Consider the first term of the equation:

$$A1=\int_0^{\delta}\left(\frac{y}{\delta}\right)^{\frac{1}{m}}\left(1-\left(\frac{y}{\delta}\right)^{\frac{1}{m}}\right)dy=\frac{m\delta}{(m+1)(m+2)}. \quad (5)$$

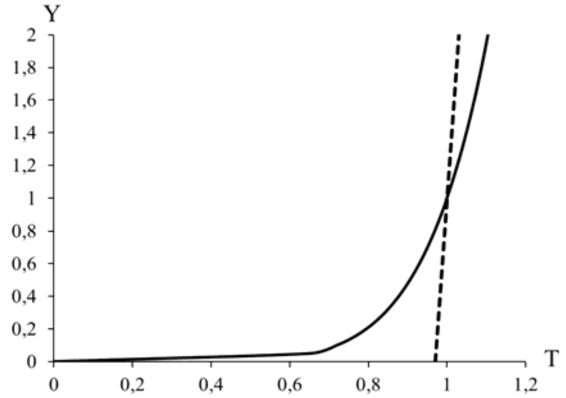

Fig. 2. The pairing of temperature and dynamic profiles for the physical model at Pr < 1: ▬ – dynamic boundary layer with a power-law velocity distribution; ‐‐‐ – temperature boundary layer

Рис. 2. Сопряжение температурного и динамического профилей для физической модели при Pr < 1: ▬ – динамический пограничный слой со степенным распределением скорости; ‐‐‐– температурный пограничный слой

We define the second term of the equation to determine the thickness of the energy loss of the temperature boundary layer:

$$A2=\int_{\delta}^{\delta_t}-\lambda\left(\frac{y}{\delta}-1\right)dy=-\left.\frac{\delta\lambda\left(\frac{y}{\delta}-1\right)^2}{2}\right|_{\delta}^{\delta_t}=-\frac{\lambda(\delta-\delta_t)^2}{2\delta}. \quad (6)$$

Then the equation for the thickness of the energy loss (4) taking into account (5) and (6) can be written as

$$\delta_{t\varphi}^{**}=\frac{m\delta}{(m+1)(m+2)}-\frac{\lambda(\delta-\delta_t)^2}{2\delta}. \quad (7)$$

Taking into account that

$$\frac{\delta}{\delta_t}=x,$$

after transformations we get

$$\delta_{t\varphi}^{**}=\frac{m\delta}{(m+1)(m+2)}-\frac{\lambda\left(\delta-\frac{\delta}{x}\right)^2}{2\delta}, \quad (8)$$

or

$$\delta_{t\varphi}^{**}=\frac{m\delta}{(m+1)(m+2)}-\frac{\delta\lambda(x-1)^2}{2x^2}=$$

$$=\delta\left(\frac{m}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x^2}\right). \quad (9)$$

For the heat transfer law in the form of the Stanton criterion, it is necessary to determine the derivative of the temperature boundary layer on the wall of the heat transfer surface. Since the derivative does not exist formally for $m < 1$, we determine it from the accepted two-layer model of turbulence with a laminar sublayer. Given that in the laminar sublayer, the profiles of the dynamic and temperature boundary layers coincide, based on the accepted model

$$\frac{\partial}{\partial y}\left(\frac{T-T_0}{T_\delta-T_0}\right)_{y=0}=\frac{U}{\alpha_л^2\nu}\left(\frac{\alpha_л^2\nu}{U\delta_t}\right)^{\frac{2}{m+1}}, \quad (10)$$

we express the thickness of the dynamic boundary layer from (9):

$$\delta=\frac{\delta_{t\varphi}^{**}}{\frac{m}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x^2}}. \quad (11)$$

Then the temperature boundary layer thickness is defined as

$$\delta_t=\frac{\delta_{t\varphi}^{**}}{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}. \quad (12)$$

Using the equation for the thickness of the energy loss (12) and performing the transformations for the derivative on the wall of the heat exchange surface of the temperature boundary layer distribution profile (10), and also taking into account the heat transfer law, we obtain

$$\mathrm{St}=\frac{\lambda}{\rho C_p U}\cdot\frac{U}{\alpha_л^2\nu}\left(\frac{\alpha_л^2\nu}{U\frac{\delta_{t\varphi}^{**}}{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}}\right)^{\frac{2}{m+1}} \quad (13)$$

or

$$\mathrm{St}=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\cdot\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\times$$

$$\times\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}. \quad (14)$$

Considering the distribution profile turbulization degree of the thermal and dynamic boundary layers at $m=7$, we write

$$\mathrm{St}=\frac{\lambda}{\rho C_p U^{0,25}\alpha_л^{1,5}\nu^{0,75}}\cdot\frac{1}{(\delta_{t\varphi}^{**})^{0,25}}\times$$

$$\times\left(\frac{7}{72}x-\frac{\lambda\left(x-1\right)^2}{2x}\right)^{0,25}. \qquad (15)$$

For the practical implementation of the heat transfer law, it is necessary to determine the value of $\alpha_л$. It is found from the condition that the laminar sublayer closes with the turbulent profile [25] at $y=\delta_л$ and profile degree at 7:

$$\alpha_л=\sqrt[1,5]{\frac{\left(\frac{xm}{(m+1)(m+2)}\right)^{0,25}\times\left(\left(\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}\right)x\frac{(m+1)}{(m+3)}\right)^{\frac{2}{m+1}}}{\left(\left(\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}\right)\frac{(m+1)}{(m+3)}\right)^{\frac{2}{m+1}}0,012\,56}}.$$

Now we finally get the equation for determining the laminar sublayer coefficient of the turbulent temperature boundary layer distribution profile for the admitted model:

$$\alpha_л=12,5496\,\mathrm{Pr}^{\frac{1}{18}}.$$

We write the integral relation of the energy equation of the boundary layers taking into account the heat transfer law (14):

$$\frac{1}{H_\varphi}\cdot\frac{\partial}{\partial\varphi}\left(\delta_{t\varphi}^{**}\right)+\frac{1}{H_\psi}\cdot\frac{\partial}{\partial\psi}\left(\delta_{t\psi}^{**}\right)+$$

$$+\frac{1}{H_\varphi H_\psi}\cdot\frac{\partial H_\psi}{\partial\varphi}\delta_{t\varphi}^{**}+\frac{1}{H_\varphi H_\psi}\cdot\frac{\partial H_\varphi}{\partial\psi}\delta_{t\psi}^{**}=$$

$$=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\times$$

$$\times\frac{1}{\left(\delta_t^{**}\right)^{\frac{2}{m+1}}}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)}. \qquad (16)$$

Assuming relative characteristic thickness

$$J=\frac{1}{\varepsilon}\cdot\frac{\delta_{t\psi}^{**}}{\delta_{t\varphi}^{**}}, \qquad (17)$$

we write

$$\frac{1}{H_\varphi}\cdot\frac{\partial}{\partial\varphi}\left(\delta_{t\varphi}^{**}\right)+\frac{J}{H_\psi}\cdot\frac{\partial}{\partial\psi}\left(\varepsilon\delta_{t\varphi}^{**}\right)+$$

$$+\frac{1}{H_\varphi H_\psi}\cdot\frac{\partial H_\psi}{\partial\varphi}\delta_{t\varphi}^{**}+\frac{J}{H_\varphi H_\psi}\cdot\frac{\partial H_\varphi}{\partial\psi}\delta_{t\varphi}^{**}=$$

$$=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\times$$

$$\times\frac{1}{\left(\delta_t^{**}\right)^{\frac{2}{m+1}}}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)}. \qquad (18)$$

We consider a rectilinear uniform flow under the following boundary conditions: $\frac{\partial}{\partial\psi}=0$, $H_\psi=H_\varphi=1$, $\frac{\partial H_\psi}{\partial\varphi}=\frac{\partial H_\varphi}{\partial\psi}=0$ [26].

Then the integral relation of the energy equation for the rectilinear uniform flow (18) is transformed to the equation

$$\frac{\partial}{\partial\varphi}\delta_{t\varphi}^{**}=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\times$$

$$\times\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)}. \qquad (19)$$

For a rotational flow realized in the cavities of energy and power-generating units, the stream line is an annular line. Let us write the integral relation of the energy equation (18) in cylindrical coordinates, taking into account that for an axisymmetric flow at ε = const the relations are $\varphi=\alpha,\ \psi=R,\ H_\varphi=R,\ \frac{\partial H_\varphi}{\partial\psi}=\frac{\partial R}{\partial R}=1,\ H_\psi=1,\ \frac{\partial H_\psi}{\partial\varphi}=0,$ $\frac{\partial}{\partial\varphi}=0$ [27; 28]:

$$J\varepsilon\frac{\partial}{\partial R}\delta_{t\varphi}^{**}+\frac{J\varepsilon}{R}\delta_{t\varphi}^{**}=$$

$$=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\times$$

$$\times\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)}. \qquad (20)$$

The integral relation of the energy equation (20) is necessary for recording and determining the energy loss thickness of the temperature spatial boundary layer, which is included in the equation for determining the local heat transfer coefficient in the Stanton criterion form.

Local heat transfer under various laws of the external flow. For a rectilinear uniform flow, we neglect the dissipative term and obtain the integral relation of the energy equation (19) in the following form:

$$\frac{\partial}{\partial\varphi}\delta_{t\varphi}^{**}=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\times$$

$$\times\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}. \qquad (21)$$

Next, we separate the variables and integrate from zero to the current value of the variables:

$$\int_0^{\delta_{t\varphi}^{**}}\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}d\delta_{t\varphi}^{**}=$$

$$=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\mu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\int_0^{\varphi}d\varphi, \quad (22)$$

$$\delta_{t\varphi}^{**}=\left(\frac{\lambda\varphi}{\rho C_p U^{\frac{2}{m+1}}}\right)^{\frac{m+1}{3+m}}\times$$

$$\times\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2m+2}{(m+1)(m+3)}}\left(\frac{3+m}{m+1}\right)^{\frac{m+1}{3+m}}. \quad (23)$$

Taking into account the heat transfer law equation (14) and the resulting equation for the thickness of the energy loss (23), we write:

$$\mathrm{St}=\frac{\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}}{\left[\left(\frac{\lambda\varphi}{\rho C_p U^{\frac{2}{m+1}}}\right)^{\frac{m+1}{3+m}}\times\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2m+2}{(m+1)(m+3)}}\cdot\left(\frac{3+m}{m+1}\right)^{\frac{m+1}{3+m}}\right]^{\frac{2}{m+1}}}. \quad (24)$$

After transformations and considering equations for the Prandtl criteria $\left(\mathrm{Pr}=\frac{\mu C_p}{\lambda}\right)$ and Reynolds $\left(\mathrm{Re}=\frac{\rho U\varphi}{\mu}\right)$, we obtain an equation for determining the local heat transfer coefficient in the form of the Stanton criterion of a rectilinear uniform flow with a power-law distribution profile of the velocity and temperature diagrams at Pr < 1:

$$\mathrm{St}=\frac{1}{\mathrm{Pr}^{\frac{m+1}{m+3}}}\left(\frac{\left(\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}\right)(m+1)}{\alpha_{\text{л}}^{m-1}(m+3)\,\mathrm{Re}_U}\right)^{\frac{2}{m+3}}. \quad (25)$$

Then we will consider the rotational flow. Neglecting the dissipative term in the integral relation of the energy equation (20), we obtain

$$J\varepsilon\frac{\partial}{\partial R}\left(\delta_{t\varphi}^{**}\right)+\frac{J\varepsilon}{R}\delta_{t\varphi}^{**}-\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\times$$

$$\times\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\cdot\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}=0. \quad (26)$$

We take into account that the rotational flow is realized according to the law of a solid body $\left(\frac{U}{R}=\omega=\text{const}\right)$ and then equation (26) is transformed to

$$\frac{\partial}{\partial R}\delta_{t\varphi}^{**}+\frac{\delta_{t\varphi}^{**}}{R}-\frac{\lambda}{J\varepsilon\rho C_p\omega^{\frac{2}{m+1}}}\times$$

$$\times\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\cdot\frac{1}{R^{\frac{2}{m+1}}\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}=0. \quad (27)$$

We introduce the intermediate notation:

$$\delta_{t\varphi}^{**}=y,$$

$$B=\frac{\lambda}{J\varepsilon\rho C_p\omega^{\frac{2}{m+1}}}\left(\frac{\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}}{\alpha_{\text{л}}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}.$$

Then

$$\frac{dy}{dR}+\frac{y}{R}-\frac{B}{R^{\frac{2}{m+1}}y^{\frac{2}{m+1}}}=0.$$

This equation is solved by the substitution method, where $y = u\vartheta$:

$$\frac{du}{dR}\vartheta + \frac{d\vartheta}{dR}u + u\frac{\vartheta}{R} = \frac{B}{u^{\frac{2}{m+1}}\vartheta^{\frac{2}{m+1}}R^{\frac{2}{m+1}}},$$

$$u\left(\frac{d\vartheta}{dR} + \frac{\vartheta}{R}\right) + \vartheta\frac{du}{dR} = \frac{B}{u^{\frac{2}{m+1}}\vartheta^{\frac{2}{m+1}}R^{\frac{2}{m+1}}}.$$

The function ϑ must satisfy the condition $\frac{d\vartheta}{dR} + \frac{\vartheta}{R} = 0$, then $\vartheta = \frac{1}{R}$, where

$$u = \sqrt[\frac{m+3}{m+1}]{\frac{BR^2(m+3)}{2(m+1)}},$$

$$y = \delta_{t\varphi}^{**} = \frac{1}{R}\sqrt[\frac{m+3}{m+1}]{\frac{BR^2(m+3)}{2(m+1)}} =$$

$$= \sqrt[\frac{m+3}{m+1}]{\frac{BR^2(m+3)}{2(m+1)}R^{\left(\frac{m-1}{m+1}\right)}}, \qquad (28)$$

$$\delta_{t\varphi}^{**} = \sqrt[\frac{m+3}{m+1}]{\frac{\dfrac{\lambda}{J\varepsilon\rho C_p\omega^{\frac{2}{m+1}}} \times \left(\dfrac{\dfrac{xm}{(m+1)(m+2)} - \dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}} \cdot (m+3)}{2(m+1)} \cdot R^{\frac{m-1}{m+1}}}.$$

With the provision for the heat transfer law equations (14) and the resulting equation for the energy loss thickness (28), we determine the Stanton criterion for rotational flow according to the law of a solid body of the power-law distribution profile of the temperature boundary layer and Pr < 1:

$$\mathrm{St} = \frac{\dfrac{\lambda}{J\varepsilon\rho C_p U^{\frac{2}{m+1}}}\left(\dfrac{\dfrac{xm}{(m+1)(m+2)} - \dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}}{\left[\sqrt[\frac{m+3}{m+1}]{\dfrac{\lambda}{J\varepsilon\rho C_p\omega^{\frac{2}{m+1}}} \times \left(\dfrac{\dfrac{xm}{(m+1)(m+2)} - \dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}} \dfrac{(m+3)}{2(m+1)} R^{\frac{m-1}{m+1}}}\right]^{\frac{2}{m+1}}}. \qquad (29)$$

After the transformation and considering Prandtl's criteria $\left(\mathrm{Pr} = \frac{\mu C_p}{\lambda}\right)$ and Reynold's $\left(\mathrm{Re} = \frac{\rho U\varphi}{\mu}\right)$, the local heat transfer coefficient in the form of the Stanton criterion for rotational flow according to the law of a solid body for the case Pr < 1 can be defined as:

$$\mathrm{St} = \frac{1}{\mathrm{Pr}^{\frac{m+1}{m+3}}}\left(\frac{2J\varepsilon}{\alpha_{л}^{m-1}\,\mathrm{Re}}\frac{(m+1)}{(m+3)}\right)^{\frac{2}{m+3}} \times$$

$$\times \left(\frac{xm}{(m+1)(m+2)} - \frac{\lambda(x-1)^2}{2x}\right)^{\frac{2(m+3)-4}{(m+1)(m+3)}}. \qquad (30)$$

Next, the rotational flow will be considered, which is carried out according to the free vortex law $(UR = C = \mathrm{const})$, in this case, the energy equation integral relation (26) takes the form

$$\frac{d\delta_{t\varphi}^{**}}{dR} + \frac{\delta_{t\varphi}^{**}}{R} - \frac{\lambda}{J\varepsilon\rho C_p C^{\frac{2}{m+1}}} \times$$

$$\times \left(\frac{\dfrac{xm}{(m+1)(m+2)} - \dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}} \cdot \frac{R^{\frac{2}{m+1}}}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}} = 0. \qquad (31)$$

After replacement:

$$\delta_{t\varphi}^{**} = y,$$

$$D = \frac{\lambda}{J\varepsilon\rho C_p C^{\frac{2}{m+1}}}\left(\frac{\dfrac{xm}{(m+1)(m+2)} - \dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}},$$

we solve the equation (31) similarly to the case of rotational flow according to the law of a solid body for Pr < 1 by substitution $y = u\vartheta$, where

$$\vartheta = \frac{1}{R}, \quad u = \frac{D^{\frac{m+1}{m+3}}R^2}{2^{\frac{m+1}{m+3}}}. \qquad (32)$$

Then the energy loss thickness is determined as

$$\delta_{t\varphi}^{**} = \left(\frac{D}{2}\right)^{\frac{m+1}{m+3}} R, \qquad (33)$$

or, after substitution:

$$\delta_{t\varphi}^{**} = \left[\frac{\dfrac{\lambda}{J\varepsilon\rho C_p C^{\frac{2}{m+1}}} \times \left(\dfrac{\dfrac{xm}{(m+1)(m+2)} - \dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}}{2}\right]^{\frac{m+1}{m+3}} \cdot R. \qquad (34)$$

Substituting the equation for determining the energy loss thickness (34) in the equation of the heat transfer law (14), we obtain

$$\mathrm{St}=\frac{\dfrac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\dfrac{\dfrac{xm}{(m+1)(m+2)}-\dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}}{\left(\left[\dfrac{\lambda}{J\varepsilon C_p C^{\frac{2}{m+1}}}\times\left(\dfrac{\dfrac{xm}{(m+1)(m+2)}-\dfrac{\lambda(x-1)^2}{2x}}{\alpha_{л}^{m-1}\nu^{\frac{m-1}{2}}}\right)^{\frac{2}{m+1}}\Big/ 2^{\frac{m+1}{m+3}}\right]^{\frac{m+1}{m+3}}\cdot R\right)^{\frac{2}{m+1}}}. \tag{35}$$

After transformations and with regard to the Prandtl $\left(\mathrm{Pr}=\dfrac{\mu C_p}{\lambda}\right)$ and Reynolds criteria $\left(\mathrm{Re}=\dfrac{\rho U\varphi}{\mu}\right)$, we determine the local heat transfer coefficient in the form of the Stanton criterion for rotational flow according to the law of the free vortex for the case Pr < 1:

$$\mathrm{St}=\frac{1}{\mathrm{Pr}^{\frac{m+1}{m+3}}}\times$$

$$\times\left(\frac{2J\varepsilon}{\alpha_{л}^{m-1}\,\mathrm{Re}}\left(\frac{xm}{(m+1)(m+2)}-\frac{\lambda(x-1)^2}{2x}\right)\right)^{\frac{2}{m+3}}. \tag{36}$$

Thus, the equations for the local heat transfer coefficient are determined in the form of the Stanton criterion for various laws of the external flow for the power-law profile of the velocity distribution in the boundary layer for the case Pr < 1.

In fig. 3 shows the influence of the Prandtl criterion on friction and heat transfer according to various studies [29]. In the range of Prandtl number values Pr > 1, the obtained theoretical dependences for dimensionless heat transfer coefficients in the form of Stanton criteria, taking into account the integral relation of the energy equation, are in good agreement with the results of other authors. Cf represents the friction coefficient.

The heat transfer coefficient in the form of the Nusselt criterion is a product of the criteria of Stanton, Reynolds and Prandtl $Nu = St\,\mathrm{Re}\,\mathrm{Pr}$.

The distribution graph of the dimensionless heat transfer coefficient in the form of the Nusselt criterion for turbulent rotational flow according to the solid body law with the Prandtl criterion Pr = 0.7 is shown in fig. 4. The theoretical dependence determined by the model with a convective component gives the best agreement with the results of experimental studies and does not exceed 3.5 %. Also, the best convergence of the results determined by the model with the convective component is given by the theoretical dependence obtained by the affine-like model and the theoretical dependence obtained by J. M. Owen [30] and do not exceed 1.5 and 2.66 % respectively.

Discrepancy of theoretical data obtained by theoretical dependence using a model with a convective component with a dependence obtained by I. V. Shevchuk [31] accounted for 9.5 %. A discrepancy with the dependence obtained L. A. Dorfman [32] is 16.7 %.

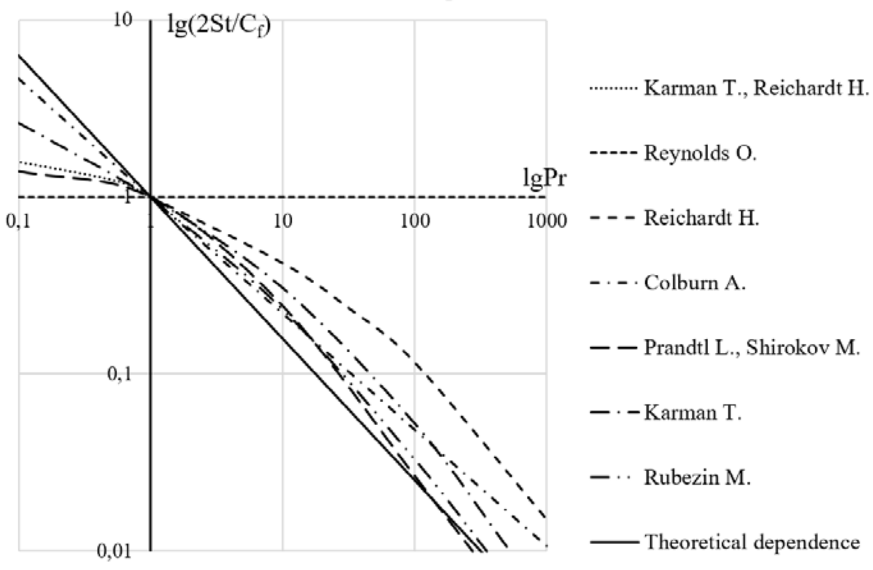

Fig. 3. Comparison of various theories of the analogy between friction and heat transfer in turbulent flows at Re = 10^7

Рис. 3. Сравнение различных теорий аналогии между трением и теплообменом в турбулентных потоках при Re = 10^7

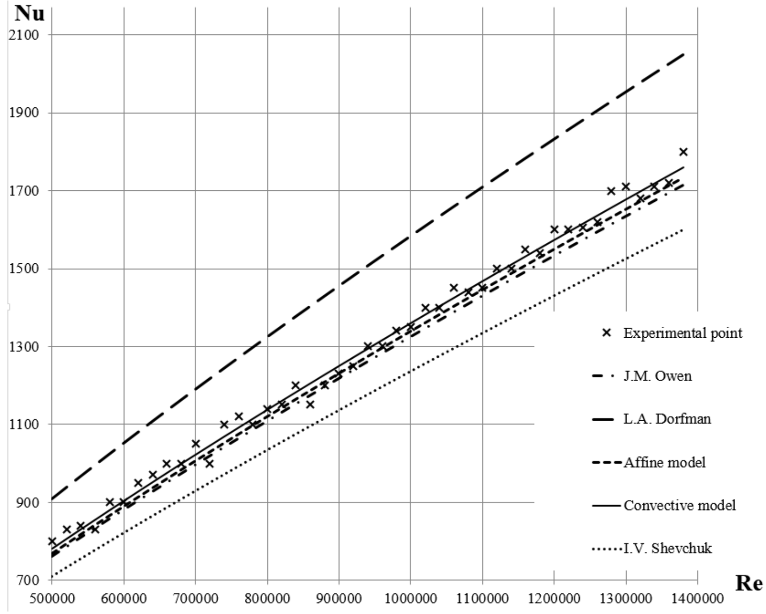

Fig. 4. The dependence of the dimensionless heat transfer coefficient
of turbulent rotational flow according to the law of a solid body at Pr = 0.7

Рис. 4. Зависимость безразмерного коэффициента теплоотдачи турбулентного
вращательного течения по закону твердого тела при Pr = 0,7

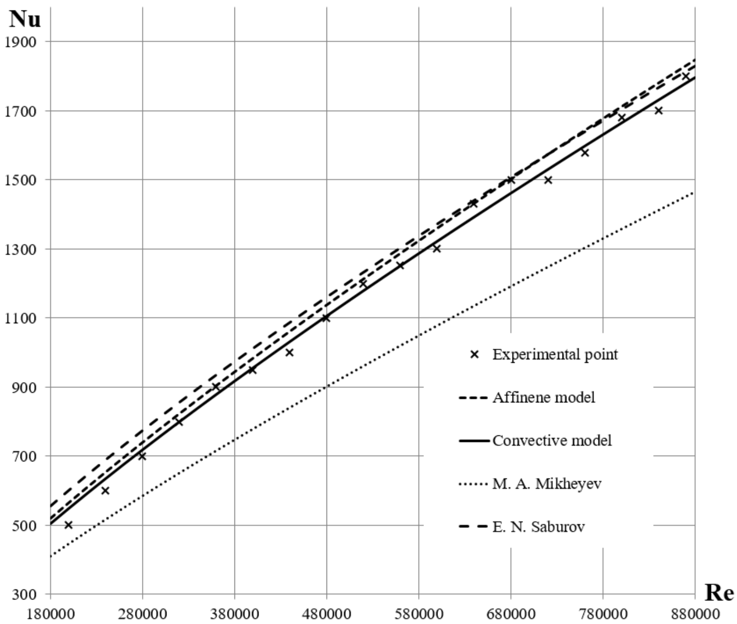

Fig. 5. The dependence of the dimensionless heat transfer coefficient
of a turbulent rotational flow according to the free vortex law at Pr = 0.7

Рис. 5. Зависимость безразмерного коэффициента теплоотдачи турбулентного
вращательного течения по закону свободного вихря при Pr = 0,7

In fig. 5 shows a distribution graph of the dimensionless heat transfer coefficient in the form of the Nusselt criterion for the turbulent potential rotational flow according to the law of the free vortex under the Prandtl criterion Pr = 0.7. The theoretical dependence, which was determined by the model with a convective component, is in good agreement with the results of experimental studies and does not exceed 5 %.

The discrepancy between the theoretical data obtained from the theoretical dependence using the model with the convective component and the model with affine-like profiles does not exceed 3.5 %. The theoretical dependences obtained from the distribution models of the temperature and dynamic boundary layers with a convective component and with affine-like profiles at Pr = 0.7 give fairly close results, due to the close similarity of the distribution of the temperature and dynamic layers, and are close to the case Pr = 1.

From the fig. 5 graph we can conclude that the obtained theoretical dependences are in the region determined by other authors, but the comparison of the results is not correct, since the theoretical dependences were obtained for other boundary conditions of the potential rotational flow according to the law of the free vortex.

The obtained theoretical dependencies and the dependences of other authors are in the same range and are suitable for engineering calculations and data analysis. It should be noted that the boundary conditions of flow and heat transfer, such as speed, viscosity, density and temperature gradient of the working fluid and heat transfer surface, significantly affect the dimensionless heat transfer coefficient in the form of the Nusselt criterion.

Conclusion. The integral relationship of the energy equation of the spatial boundary layer temperature, allowing integration over the surface of any shape, which is necessary to determine the energy loss thickness, was obtained. The equations for determining the energy loss thickness of the spatial boundary layer temperature are necessary to determine the local heat transfer coefficients for typical flow cases taking into account heat transfer.

The equations for determining the local heat transfer coefficient in the form of the Stanton criterion for a rectilinear uniform flow, a rotational flow according to the law of a solid body, and a free vortex rotational flow of a power-law profile of the dynamic and temperature boundary layer distribution parameters for the case Pr < 1 were obtained analytically.

The analytical equations for the heat transfer coefficients are in good agreement with the experimental data and dependences of other authors.

References

1. Voiskunski J. I., Faddeev, Y. I., Fedyaevsky K. K. *Gidromekhanika* [Hydromechanics]. Leningrad, Sudostroenie Publ., 1982, 456 p.

2. Stoll J., Straub J. Film cooling and heat transfer in nozzles. *J. Turbomach.* 1988, No. 110, P. 57–64.

3. Dellimore K. Modeling and Simulation of Mixing Layer Flows for Rocket Engine Film Cooling (Ph. D. thesis). University of Maryland at College Park, 2010.

4. Jansson L. S., Davidson L., Olsson E. Calculation of steady and unsteady flows in a film-cooling arrangement using a two-layer algebraic stress model. *Numer. Heat. Transf.* 1994, Part A 25, P. 237–258.

5. Cruz C. Experimental and Numerical Characterization of Turbulent Slot Film Cooling (Ph.D. thesis). University of Maryland at College Park, 2008.

6. Cruz C., Marshall A. Surface and gas measurements along a film cooled wall. *J. Thermophys. Heat. Transf.* 2007, No. 21, P. 181–189.

7. Betti B., Martelli E. Heat flux evaluation in oxygen/ methane thrust chambers by RANS approach. *Proceedings of the 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, AIAA*. 2010, P. 2010–6721.

8. Ilinkov A. V., Gabdrakhmanov R. R., Takmovtsev V. V., Shchukin A. V. [Effect of centrifugal mass forces on heat transfer during air flow around a concave surface with transverse protrusions]. *Vestnik Moskovskogo aviatsionnogo institute*. 2018, Vol. 25, No. 1, P. 39–48 (In Russ.).

9. Gorelov Yu. G., Strokach E. A. [Analysis of regularities of calculation of the heat transfer coefficient from gas at the inlet edges of the nozzle blades of high-pressure turbines]. *Vestnik Moskovskogo aviatsionnogo institute*. 2016, Vol. 23, No 1, P. 80–85 (In Russ.).

10. Shcherbakov M. A., Vorobyev D. A., Maslakov S. A., Ravikovich Yu. A. [Determination of the heat transfer coefficient on the turbine blade at off-design operating conditions]. *Vestnik Moskovskogo aviatsionnogo institute*. 2013, Vol. 20, No 3, P. 95–103 (In Russ.).

11. Dreytser G. A., Isayev S. A., Lobanov I. E. [Calculation of convective heat transfer in a pipe with periodic projections]. *Vestnik Moskovskogo aviatsionnogo institute*. 2004, Vol. 11, No. 2, P. 28–35 (In Russ.).

12. Knuth E. L. The mechanism of film cooling, (Ph.D. thesis). California Institute of Technology. 1954.

13. Rannie W. D. Heat transfer in turbulent shear flow. *J. Aeronaut. Sci.* 1956, No. 23, P. 485–489.

14. Turcotte D. L. A sublayer theory for fluid injection into the incompressible turbulent boundary layer. *J. Aeronaut. Sci.* 1960, No. 27, P. 675–678.

15. Stechman R. C., Oberstone J., Howell J. C. Design criteria for film cooling for small liquid-propellant rocket engines. *J. Spacecr. Rocket.* 1969, No. 6, P. 97–102.

16. Bartz D. R. A simple equation for rapid estimation of rocket nozzle convective heat transfer coefficients, Jet. Propuls. *ARS J.* 1957, No. 27, P. 49–51.

17. Shine S. R., Kumar S. S., Suresh B. N. A new generalised model for liquid film cooling in rocket combustion chambers. *Int. J. Heat. Mass Transf.* 2012, No. 55, P. 5065–5075.

18. Elhefny A., Liang G. Stress and deformation of rocket gas turbine disc under different loads using finite element modelling. *Propulsion and Power Research.* 2013, Vol. 2, Iss. 1, P. 38–49.

19. Grigoryev V. A., Zagrebelnyy A. O., Kuznetsov S. P. [On the estimation of the mass of the power plant in the problem of optimization of the working process parameters of the aircraft turboprop engine]. *Vestnik Moskovskogo aviatsionnogo institute*. 2015. Vol. 22, No 3. P. 103–106 (In Russ.).

20. Kishkin A. A., Chernenko D. V., Chernenko E. V. [The equation of impulses of a three-dimensional boundary layer]. *Izv. vuzov. Tekhnicheskiye nauki*. 2007, No. 4, P. 35–41 (In Russ.).

21. Romanenko P. N. *Teplomassoobmen i treniye pri gradiyentnom techenii zhidkostey* [Heat and mass transfer and friction at gradient flow of liquids]. Moscow, Energy Publ., 1971, 568 p.

22. Shlikhting G. *Teoriya pogranichnogo sloya* [The theory of the boundary layer]. Moscow, Nauka Publ., 1974, 712 p.

23. Shine S. R., Shri S. Nidhi. Review on film cooling of liquid rocket engines. *Propulsion and Power Research*. 2018, No. 7, Iss. 1, P. 1–18.

24. Keys V. M. *Konvektivnyy teplo- i massoobmen* [Convective heat and mass transfer]. Moscow, Energy Publ., 1972, 448 p.

25. Kishkin A. A., Zuev A. A., Leonov V. P. [Local heat transfer in the boundary conditions of turbomachines]. *Izv. vuzov. Mashinostroyeniye*. 2015, No (658), P. 3–10 (In Russ.).

26. Tolstopyatov M. I., Zuev A. A., Kishkin A. A., Zhuikov D. A., Nazarov V. P. [Straight-line uniform flow of gases with heat transfer in power plants of aircraft]. *Vestnik SibGAU.* 2012, No. 4(44), P. 134–139 (In Russ.).

27. Kishkin A. A., Zhuykov D. A., Tolstopyatov M. I. [Flow with heat transfer in the cavities of rotation of power plants of space and aircraft]. *Vestnik SibGAU.* 2011, No. 7(40), P. 63–68 (In Russ.).

28. Kishkin A. A., Zuev A. A., Chernenko E. V., Smirnov P. N. [The rotation of the liquid over a fixed base according to the law of a solid]. *Izv. vuzov. Tekhnicheskiye nauki.* 2011, No. 1, P. 126–131 (In Russ.).

29. Chapman D. R. Kester R. H. Measurements of turbulent skin friction on cylinders in axial flow at subsonic and supersonic velocities. *JAS*. 1953, Vol. 20, P. 441–448.

30. Owen J. M., Rogers R. H. Flow and heat transfer in rotating disc systems. Rotor-stator systems. Taunton: Research Studies Press, 1989, 302 p.

31. Shevchuk I. V. Convective Heat and Mass Transfer in Rotating Disk Systems. Springer, 2009, 235 p.

32. Dorfman L. A. *Gidrodinamicheskoye soprotivleniye i teplootdacha vrashchayushchikhsya tel* [Hydrodynamic resistance and heat transfer of rotating bodies]. Moscow, Fizmatgiz Publ., 1960, 260 p.

Библиографические ссылки

1. Войткунский Я. И., Фаддеев Ю. И., Федяевский К. К. Гидромеханика. Л. : Судостроение, 1982. 456 с.

2. Stoll J., Straub J. Film cooling and heat transfer in nozzles // J. Turbomach. 1988. No. 110. P. 57–64.

3. Dellimore K. Modeling and Simulation of Mixing Layer Flows for Rocket Engine Film Cooling (Ph. D. thesis). University of Maryland at College Park, 2010.

4. Jansson L. S., Davidson L., Olsson E. Calculation of steady and unsteady flows in a film-cooling arrangement using a two-layer algebraic stress model // Numer. Heat. Transf. 1994. Part A 25. P. 237–258.

5. Cruz C. Experimental and Numerical Characterization of Turbulent Slot Film Cooling (Ph.D. thesis). University of Maryland at College Park, 2008.

6. Cruz C., Marshall A. Surface and gas measurements along a film cooled wall // J. Thermophys. Heat. Transf. 2007. No. 21. P. 181–189.

7. Betti B., Martelli E. Heat flux evaluation in oxygen/ methane thrust chambers by RANS approach // Proceedings of the 46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, AIAA. 2010. P. 2010–6721.

8. Влияние центробежных массовых сил на теплоотдачу при обтекании потоком воздуха вогнутой поверхности с поперечными выступами / А. В. Ильинков, Р. Р. Габдрахманов, В. В. Такмовцев, А. В. Щукин // Вестник Московского авиационного ин-та. 2018. Т. 25, № 1. С. 39–48.

9. Горелов Ю. Г., Строкач Е. А. Анализ закономерностей расчета коэффициента теплоотдачи от газа на входных кромках сопловых лопаток турбин высокого давления // Вестник Московского авиационного ин-та. 2016. Т. 23, № 1. С. 80–85.

10. Щербаков М. А., Воробьев Д. А., Маслаков С. А., Равикович Ю. А. Определение коэффициента теплоотдачи на пере лопатки турбины на нерасчётных режимах работы // Вестник Московского авиационного ин-та. 2013. Т. 20, № 3. С. 95–103.

11. Дрейцер Г. А., Исаев С. А., Лобанов И. Е. Расчет конвективного теплообмена в трубе с периодическими выступами // Вестник Московского авиационного ин-та. 2004. Т. 11, № 2. С. 28–35.

12. Knuth E. L. The mechanism of film cooling, (Ph.D. thesis). California Institute of Technology. 1954.

13. Rannie W. D. Heat transfer in turbulent shear flow // J. Aeronaut. Sci. 1956. No. 23. P. 485–489.

14. Turcotte D. L. A sublayer theory for fluid injection into the incompressible turbulent boundary layer // J. Aeronaut. Sci. 1960. No. 27. P. 675–678.

15. Stechman R. C., Oberstone J., Howell J. C. Design criteria for film cooling for small liquid-propellant rocket engines // J. Spacecr. Rocket. 1969. No. 6. P. 97–102.

16. Bartz D. R. A simple equation for rapid estimation of rocket nozzle convective heat transfer coefficients, Jet. Propuls // ARS J. 1957. No. 27. P. 49–51.

17. Shine S. R., Kumar S. S., Suresh B. N. A new generalised model for liquid film cooling in rocket combustion chambers // Int. J. Heat. Mass Transf. 2012. No. 55. P. 5065–5075.

18. Elhefny A., Liang G. Stress and deformation of rocket gas turbine disc under different loads using finite element modelling // Propulsion and Power Research. 2013. Vol. 2, Iss. 1. P. 38–49.

19. Григорьев В. А., Загребельный А. О., Кузнецов С. П. К вопросу оценки массы силовой установки в задаче оптимизации параметров рабочего процесса авиационного турбовинтового двигателя // Вестник Московского авиационного ин-та. 2015. Т. 22, № 3. С. 103–106.

20. Кишкин А. А., Черненко Д. В., Черненко Е. В. Уравнение импульсов трехмерного пограничного слоя // Изв. вузов. Северо-Кавказский регион. Техн. науки. 2007. № 4. С. 35–41.

21. Романенко П. Н. Тепломассообмен и трение при градиентном течении жидкостей. М. : Энергия, 1971. 568 с.

22. Шлихтинг Г. Теория пограничного слоя М. : Наука, 1974. 712 с.

23. Shine S. R., Shri S. Nidhi. Review on film cooling of liquid rocket engines // Propulsion and Power Research. 2018. Vol. 7, Iss. 1. P. 1–18.

24. Кейс В. М. Конвективный тепло- и массообмен : пер. с англ. М. : Энергия, 1972. 448 с.

25. Кишкин А. А., Зуев А. А., Леонов В. П. Локальная теплоотдача в граничных условиях турбомашин // Изв. вузов. Машиностроение. 2015. № 1 (658). С. 3–10.

26. Прямолинейное равномерное течение газов с теплоотдачей в энергетических установках летательных аппаратов / М. И. Толстопятов, А. А. Зуев, А. А. Кишкин и др. // Вестник СибГАУ. 2012. № 4(44). С. 134–139.

27. Течение с теплоотдачей в полостях вращения энергетических установок космических и летательных аппаратов / А. А. Зуев, А. А. Кишкин, Д. А. Жуйков, М. И. Толстопятов // Вестник СибГАУ. 2011. № 7(40). С. 63–68.

28. Вращение жидкости над неподвижным основанием по закону твердого тела / А. А. Кишкин, А. А. Зуев, Е. В. Черненко, П. Н. Смирнов // Изв. вузов. Северо-Кавказский регион. Техн. науки. 2011. № 1. С. 126–131.

29. Chapman D. R. Kester R. H., Measurements of turbulent skin friction on cylinders in axial flow at subsonic and supersonic velocities // JAS. 1953. Vol. 20. P. 441–448.

30. Owen J. M., Rogers R. H. Flow and heat transfer in rotating disc systems. Rotor-stator systems. Taunton: Research Studies Press, 1989. 302 p.

31. Shevchuk I. V. Convective Heat and Mass Transfer in Rotating Disk Systems. Springer, 2009. 235 p.

32. Дорфман Л. А. Гидродинамическое сопротивление и теплоотдача вращающихся тел. М. : Физматгиз, 1960. 260с.

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