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FLOW DYNAMICS IN THE RADIAL-ANNULAR CAVITY OF TURBOMACHINES

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This paper considers the problem of modeling a rotational flow in the radial-annular cavity of turbo machines with fixed walls. This case corresponds to the boundary conditions of the supply channel for a radial centripetal turbine. In the presented model, the flow is conventionally divided into radial and circumferential movement. The radial component of the velocity is determined by the mass flow rate from the continuity equation, the circumferential component is formed by the tangential channel supply. The main equation in the integration is the equation of the change in the momentum for the flow in the form of the Euler equation. In the case of the circumferential component of the velocity, the angular momentum law is used, assuming the potentiality of the flow and the constancy of the angular momentum within the integration step. As a result of the transformations of the motion equations, differential equations for the radial, circumferential component of velocity and static pressure are obtained, which represent a certain system of three equations in three unknowns. The system of equations allows integration under known boundary conditions at the inlet; as a result of integration, it is possible to obtain the field of distributions of velocities and pressures along the radius of the radial-annular cavity. The results of the study can be used in modeling the circumferential and radial forces on the rotor (impeller) of turbo machines.

Keywords: radial-annular cavity, turbo machine, flow dynamics, continuity equations, Euler equations, boundary conditions, impeller.

ДИНАМИКА ПОТОКА В РАДИАЛЬНО-КОЛЬЦЕВОЙ ПОЛОСТИ ТУРБОМАШИН

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В работе рассмотрена задача моделирования вращательного течения в радиально-кольцевой полости турбомашин с неподвижными стенками. Данный расчетный случай соответствует граничным условиям подводящего канала для радиальной центростремительной турбины. В представленной модели поток условно разделен на радиальное и окружное движение. Радиальная составляющая скорости определяется массовым расходом из уравнения неразрывности, окружная составляющая формируется тангенциальным канальным подводом. Основным уравнением при интегрировании является уравнение изменения количества движения для потока в форме уравнения Эйлера. В случае окружной составляющей скорости используется закон изменения момента количества движения при допущении потенциальности потока и постоянства момента количества движения в пределах шага интегрирования. В результате преобразований уравнений количества движения получены дифференциальные уравнения для радиальной и окружной составляющих скорости, а также для статического давления, представляющие определенную систему трех уравнений с тремя неизвестными. Система уравнений позволяет вести интегрирование при известных граничных условиях на входе, в результате интегрирования возможно получить поле распределений скоростей и давлений по радиусу радиально-кольцевой полости. Результаты исследования могут быть использованы при моделировании окружных и радиальных усилий на ротор (рабочее колесо) турбомашин.

Ключевые слова: радиально-кольцевая полость, турбомашина, динамика потока, уравнения неразрывности, уравнения Эйлера, граничные условия, рабочее колесо.

Introduction. Turbo machines of various types (pumps, compressors, turbines, expansion engines) are currently being used everywhere. While designing and constructing turbo machines, specialists often have to deal with the issue how to model the movement of liquid flow or gas flow in working cavities properly in order to assess the distribution fields of velocity and pressure, friction stresses, coefficients of losses and the overall energy efficiency of a turbo machine [1–4]. The problem of modeling is complicated by the fact that the flow has a complex spatial nature [5–6], for which it makes sense to decompose the main system of equations into two projections – radial and circumferential ones. The final form of the system of equations of motion depends on the design of the cavity of a turbo machine [7].

Herewith, we consider the problem of modeling a rotational flow in a radial-annular cavity of a turbo machine with fixed walls, which corresponds to the boundary conditions of the supply channel for a radial centripetal turbine. The approach presented in this work can be used to calculate the turbines of other types.

Research task description. The main task of the work is to obtain a system of equations that properly describe the fields of pressure and velocity distribution at the inlet into the impeller for the design case of flow in the supply device of a radial centripetal turbine. It is necessary to consider the following characteristic aspects [8–11]:

– the correct formation (without losses) of the velocity and pressure fields in front of the impeller mainly determines the value of the circumferential power of a turbine, and, as a consequence, the overall efficiency of a turbine.

– the partiality, nonuniformity of the velocity and pressure fields in this area determine the value of axial load on the turbine rotor, which reduces the resource of rolling units [12].

To solve the problem, it is necessary to transform the equations of motion together with the continuity equation in order to obtain expressions for numerical integration for the flow rate and circumferential velocity components, as well as static pressure, taking the mass velocity into consideration as an initial parameter.

Basic assumptions and design flow diagram. The radial-annular cavities of turbo machines with multidirectional flows relative to the radius R can form a confusor or diffuser flow. This flow is asymmetric, therefore, the solution is considered in cylindrical coordinates with the condition $\partial/\partial x = 0$.

There are two possible cases of flow: purely radial $C = V_R$ and radial-circumferential $C = (V_R^2 + U^2)^{0,5}$. In both cases, we use the momentum conservation equation for the mass fluid flow for the analysis. The design diagram of the radial-annular cavity is shown in figure.

The flow area in the radial direction F_R is defined as follows:

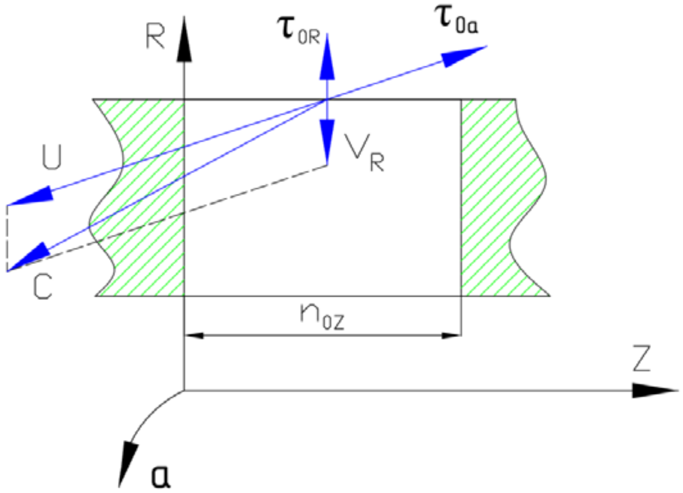
$$F_R = 2\pi R \cdot n_{oz}, \quad (1)$$

where R is the radius of the cavity, n_{0z} is the axial clearance in the direction of the z coordinate.

The incrementation of the flow area in the radial direction:

$$dF_z = 2\pi R \cdot dR. \quad (2)$$

The equations (1) and (2) determine the geometric parameters of the flow area at the integration step, which make it possible to find the radial velocity at a known mass flow rate of the actuation fluid.



Design diagram of a radial-annular cavity:
R, Z, a – coordinates; C, U, V_R – velocity components; τ_{0R}, τ_{0a} – friction stresses in the radial and circumferential direction; n_{0z} – normal gap clearance

Расчетная схема радиально-кольцевой полости:
R, Z, a – координаты; C, U, V_R – компоненты скорости; τ_{0R}, τ_{0a} – напряжения трения в радиальном и окружном направлении;
n_{0z} – нормальный зазор

Mathematical model of the flow. The incrementation in the change in the momentum for the flow in differential form is determined by the equation:

$$\dot{m}\frac{dV_R}{dR}\cdot dR=\frac{d\left(p\cdot F_R\right)}{dR}\cdot dR+2dF_z\tau_{oR},\qquad(3)$$

where $\dot{m}$ is a mass flow; V_R is a radial velocity component; τ_{0R} is the circumferential component of the friction stress on the wall, p is a pressure quantity.

Taking (1) into consideration, the equation for the radial velocity component has the following form:

$$V_R=\frac{\dot{m}}{p\cdot F_R}=\frac{\dot{m}}{p\cdot 2\pi Rn_{oz}}.\qquad(4)$$

We change the equation (3) taking (2) into consideration:

$$\dot{m}\frac{dV_R}{dR}=F_R\frac{dp}{dR}+p\frac{dF_R}{dR}+\tau_{oR}\cdot 4\pi RdR.\qquad(5)$$

We determine the derivative of the radial velocity component using the equation (4):

$$\frac{dV_R}{dR}=\frac{d}{dR}\left(\frac{\dot{m}}{p\cdot 2\pi Rn_{oz}}\right)=\frac{-\dot{m}}{p\cdot 2\pi R^2n_{oz}},$$

$$\frac{dV_R}{dR}=\frac{V_R}{R}.\qquad(6)$$

The derivative of the flow area F $_R$, considering (1), is determined by the following equation:

$$\frac{dF_R}{dR}=\frac{d}{dR}\left(2\pi\cdot Rn_{oz}\right)=2\pi n_{oz}.\qquad(7)$$

Taking (6) and (7) into consideration, we rewrite the equation (5) in the form:

$$\frac{-\dot{m}V_R}{R}=F_R\frac{dp}{dR}+2\pi n_{oz}\cdot p+\tau_{oR}\cdot 4\pi R.$$

Let us distinguish the derivative dp/dR in the equation:

$$\frac{dp}{dR}=\frac{-\dot{m}}{F_RR}-\frac{2\pi n_{oz}}{F_R}\cdot p-\frac{4\pi R}{F_R}\cdot\tau_{0R}.\qquad(8)$$

Taking (1) and (4) into consideration we obtain:

$$\frac{dp}{dR}=-\frac{pV_R^2}{R}-\frac{p}{R}-\frac{2\tau_0R}{n_{oz}}\cdot\tau_{oR},\qquad(9)$$

or finally:

$$\frac{dp}{dR}=\frac{-\dot{m}^2}{p\cdot 4\pi^2\cdot n_{oz}^2\cdot R^3}-\frac{p}{R}-\frac{2\tau_{0R}}{n_{oz}},\qquad(10)$$

It will be recalled that that when flowing to the center of coordinates ($-V_R$), the flow is confusor and vice versa.

For the radial flow, the system of equations (4) and (9) is sufficient, with V_R = const. For the radial flow with a circumferential component, the formula for the peripheral velocity U is required. For the potential flow $rot\ U=0$, the following formula is satisfied:

$$U\cdot R=C_u=\text{const}.\qquad(11)$$

The formula (11) completely determines the function $U=f(R)$, however, when integrating over the radius, it is necessary to take into consideration the influence of the circumferential component of the friction stress on the wall τ_{0a}; this friction stress reduces the values of C_U for any direction of the radial velocity V_R [13].

Next we find the formula for the circumferential component of the velocity of the radial-circumferential flow. We use the law of changing the angular momentum [14–15]:

$$\dot{m}\frac{d(U\cdot R)}{dR}\cdot dR=dF_{mp}\cdot R,\qquad(12)$$

where dF_{fr} is the friction force on the elementary volume $2\pi n_{0z}\cdot dR$.

Friction force on two surfaces (see fig.):

$$dF_{mp}=4\cdot\tau_{oa}\cdot\pi R\cdot dR,\qquad(13)$$

We take the derivative of the equation (12) and take into consideration (13):

$$\frac{dU}{dR}=-\frac{U}{R}+\frac{4\pi\tau_{oa}R}{\dot{m}},$$

$$\dot{m}\left(R\frac{dU}{dR}+U\right)=4R\cdot\tau_{oa}\cdot 4\pi R.$$

We express the derivative:

$$\frac{dU}{dR}=-\frac{U}{R}+\frac{4\pi\tau_{oa}R}{\dot{m}}.\qquad(14)$$

Or taking $U=\omega R$ into consideration we obtain the equation:

$$\frac{d\omega}{dR}=-\frac{2\omega}{R}+\frac{4\pi\tau_{oa}}{\dot{m}}.\qquad(15)$$

Taking (11) into consideration, we finally obtain:

$$\frac{dC_u}{dR}=\frac{4\pi R^2\tau_{oa}}{\dot{m}}=\frac{2R\tau_{oa}}{n_{oz}\cdot p\cdot V_R}.\qquad(16)$$

The equations (14), (15), (16) can be integrated autonomously without knowing the change in the pressure field p.

Next, we find the formula for the static pressure for the radial-circumferential flow. We use the equation for changing the momentum for absolute speed (see fig.):

$$C=\sqrt{V_R^2+U^2},\qquad(17)$$

Where V_R and U are determined by the equations (4) and (11).

After transformations we obtain:

$$\dot{m}\frac{dC}{dR}\cdot dR=\frac{d\left(p\cdot F_R\right)}{dR}\cdot dR+2\tau_{oR}\cdot dF_z,\qquad(18)$$

where $dF_z=4\pi RdR$ is the double lateral surface of the elementary volume $dV=2\pi Rn_{0z}\cdot dR$, $F_R=2\pi Rn_{0z}$ is the flow area.

We define the derivatives of the velocities. The derivative dV_R/dR, according to the equation (6):

$$\frac{dV_R}{dR}=-\frac{V_R}{R}.\qquad(19)$$

Considering the equation (11), the derivative of the circumferential component on the elementary volume $dV = 2\pi R n_{0z} \cdot dR$ can be defined as follows:

$$\frac{dU}{dR} = -\frac{C_u}{R^2}. \quad (20)$$

The derivative dC/dR according to the equation (17) is defined as:

$$\frac{dC}{dR} = \frac{d}{dR}\left(V_R^2 + U^2\right)^{0.5} = \frac{1}{2}\left(V_R^2 + U^2\right)^{-0.5} \cdot \frac{d}{dR}\left(V_R^2 + U^2\right)^{0.5} =$$
$$= \frac{1}{2}\left(V_R^2 + U^2\right)^{-0.5} \cdot \left(2V_R \frac{dV_R}{dR} + 2U \frac{dU}{dR}\right).$$

We take into consideration the equations (19) and (20) and continue the transformation:

$$\frac{dC}{dR} = \frac{1}{2}\left(V_R^2 + U^2\right)^{-0.5} \cdot \left(-2V_R \frac{dV_R}{dR} - 2U \frac{dU}{dR}\right),$$

$$\frac{dC}{dR} = -\frac{1}{R}\sqrt{\left(V_R^2 + U^2\right)} = -\frac{C}{R}. \quad (21)$$

Considering (21) we transform the equation (18) into the following form:

$$-\frac{C\dot{m}}{R} = -F_R \frac{dp}{dR} + p\frac{dF_R}{dR} + 2\tau_{oR} \cdot dF_z. \quad (22)$$

After substitution of the equations (1) and (2) we obtain, respectively:

$$\frac{dF_R}{dR} = 2\pi n_{oz}; \qquad 2dF_z = 4\pi R dR; \quad F_R = 2\pi R n_{oz};$$

Thereupon:

$$\frac{-\dot{m}C}{R} dR = \left(F_R \frac{dp}{dR} + p\frac{F_R}{R}\right) dR + 4\tau_{oR} \cdot \pi \cdot R \cdot dR.$$

We simplify the last equation by dR, as a result we obtain:

$$\frac{-\dot{m}C}{R} = F_R \frac{dp}{dR} + p\frac{F_R}{R} + 4 \cdot \pi \cdot R \cdot \tau_{oR}. \quad (23)$$

We express the pressure derivative:

$$-\frac{dp}{dR} = -\frac{\dot{m}}{RF_R} - p\frac{1}{R} + \frac{4 \cdot \pi \cdot R \cdot \tau_{oR}}{F_R}. \quad (24)$$

Considering $V_R = \dot{m}/\rho F_R$, we obtain:

$$\frac{dp}{dR} = -\frac{\dot{m} \cdot C}{2\pi R^2 n_{oz}} - \frac{p}{R} - \frac{2 \cdot \tau_{0R}}{n_{oz}}. \quad (25)$$

The equation (25) combined with the equations (4) and (17) form a closed system of equations for determining the velocity and pressure fields in the radial-annular cavity of turbo machines.

Conclusion. The mathematical model obtained in this work can be used at complex modeling of radial centripetal turbo machines for calculating the flow dynamics in inlet and outlet devices. The model determines the fields of velocity and pressure at the inlet and outlet of the rotor, which is a condition that forms the vector of radial and axial forces that determine the dynamics of the rotor, the load on the rolling units, and, as a consequence, the resource of a turbo machine in general. The approach presented in this work can be used to calculate the turbines of other types.

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