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ON THE FUNCTION OF TIME DISTRIBUTION OF A COMPLEX COMPUTING SYSTEM UPTIME

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Any space computing complex is a complicated system. A complicated system is understood as a set of functionally related heterogeneous devices designed to perform certain functions and solve problems facing the system. One of the important characteristics of a system is its uptime. This characteristic is often considered to be a random variable. However, such a mathematical model is quite limited, since the uptime depends on many characteristics (parameters) that describe a system. Therefore, the uptime can be assumed to be a continuous random field (that is, a random function of many variables). It is this approach that is used in this work. If there are certain restrictions on the uptime of a computing system, upper estimates are found for the distributions of a random number of system failures. Therefore, the problem of estimating Gaussian field distribution in Hilbert space arises.

Two theorems that allow calculating the probability of a Gaussian vector falling into a sphere of a given radius are proved in the paper.

The paper is devoted to the reliability of a computing system. The random number of a computing system failures v(r) is a characteristic of its reliability. The v(r) distribution is the distribution of the sum of a computing system random uptime. It is impossible to write down the distribution v (r) explicitly. Therefore, one has to look for an estimate of these distributions from above. Assuming that the uptime of a computing system is the sum of many variables, the authors of the paper obtained the following results: it is shown that the problem of estimating the distributions of a random number of system failures can be considered as the problem of estimating the convergence rate in the central limit theorem in Banach spaces; if there are certain restrictions on the uptime of a computing system, upper estimates are found for the distributions of a random number of system failures. The estimates obtained can be used for further research in the theory of computing systems reliability. Knowing these upper estimates, it is possible to predict the level of average costs for computer systems restoration, as well as for the development of special mathematical and algorithmic support for analysis systems, for management, decision-making and information processing tasks.

Keywords: computing system, distribution function, systems analysis.

О ФУНКЦИИ РАСПРЕДЕЛЕНИЯ ВРЕМЕНИ БЕЗОТКАЗНОЙ РАБОТЫ СЛОЖНОЙ ВЫЧИСЛИТЕЛЬНОЙ СИСТЕМЫ

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Любой космический вычислительный комплекс представляет собой сложную систему. Под сложной системой понимают совокупность функционально связанных разнородных устройств, предназначенных для выполнения определенных функций и решения стоящих перед системой задач. Одной из важных характеристик работы системы является время ее безотказной работы. Часто эту характеристику считают случайной величиной. Но такая математическая модель является довольно ограниченной, так как время безотказной работы зависит от многих характеристик (параметров), описывающих систему. Поэтому можно предполо-

жить, что время безотказной работы есть непрерывное случайное поле (то есть случайная функция многих переменных). Именно такой подход применяется в данной работе. При наличии определенных ограничений на время безотказной работы вычислительной системы найдены верхние оценки для распределений случайного числа отказов системы. Поэтому возникает вопрос оценки распределения гауссовского поля в гильбертовом пространстве.

В работе доказаны две теоремы, которые позволяют вычислить вероятность попадания гауссовского вектора в шар заданного радиуса.

Данная работа посвящена надежности работы вычислительной системы. Одной из характеристик надежности вычислительной системы является случайное число ее отказов ν(r). Распределение ν(r) есть распределение суммы случайных времен безотказной работы вычислительной системы. Записать распределение ν(r) в явном виде невозможно. Поэтому приходится искать оценку этих распределений сверху. В предположении, что время безотказной работы вычислительной системы есть сумма многих переменных, в данной работе получены следующие результаты: показано, что задачу оценки распределений случайного числа отказов системы можно рассматривать как задачу оценки скорости сходимости в центральной предельной теореме в банаховых пространствах; при наличии определенных ограничений на время безотказной работы вычислительной системы найдены верхние оценки для распределений случайного числа отказов системы. Полученные оценки могут быть использованы для дальнейших исследований в теории надежности вычислительных систем. Зная эти верхние оценки, можно прогнозировать уровень средних затрат на восстановление вычислительных систем, а также для разработки специального математического и алгоритмического обеспечения систем анализа, для задач управления, принятия решений и обработки информации.

Ключевые слова: вычислительная система, функция распределения, системный анализ.

Introduction. Any space computing complex is a complicated system. A complicated system is understood as a set of functionally related heterogeneous devices designed to perform certain functions and solve problems facing the system. One of the important characteristics of a system is its uptime. This characteristic is often considered to be a random variable. However, such a mathematical model is quite limited, since the uptime depends on many characteristics (parameters) that describe a system.

Therefore, the uptime can be assumed to be a continuous random function of many variables. Such an assumption is used in the literature [1–15]. In this work we will also stick to it.

It is known that the characteristic functional of a random variable Y in the Hilbert space H is the functional

$$\varphi_Y(Z) = E\exp\{i(Z,X)\},$$

where $z \in H$, (Z, Y) is the scalar product in H, $i=\sqrt{-1}$, E is the sign of mathematical expectation. Let R be the covariant vector of a random variable H, $EY = A$. If Y is a Gaussian vector in H, then its characteristic functional has the form:

$$\varphi_Y(Z) = \exp\{i(A,Z) - \frac{1}{2}(RZ, XZ\}.$$

The converse is also true, that is, if Y has a characteristic functional that meets these requirements, then its distribution is Gaussian.

The covariant operator R of the Gaussian vector Y is a kernel and completely continuous one; therefore it has an orthonormal basis of eigenvectors e_k , k = 1,2,… Let us denote λ_k , that is, the eigenvalue of the operator R, corresponding to the eigenvector e_k. Let λ_k > 0 be random variables

$$\varepsilon_л = \frac{(Y - A, e_k)}{\sqrt{\lambda_k}}$$

which are independent, have a normal distribution and $E\varepsilon_k = 0$, $E\varepsilon_k^2 = 1$. Thus, the Gaussian vector Y can be written as

$$Y = A + \sum_{k=1}^{\infty} \sqrt{\lambda_k \varepsilon_k e_k}.$$

The representation of the Gaussian vector Y can be used in various calculations. In particular, if we denote $\alpha_k = (A, e_k)$, then the characteristic function for the real s of a random variable $|Y|_H^2$ has the form

$$\phi_{|Y|_h^2}(s) = E\exp\left\{is|A|_H^2\right\} \cdot \prod_{k=1}^{\infty} \frac{\exp\left\{\dfrac{2\lambda_k \alpha_k^2 s^2}{\sqrt{1-2si\lambda_k}}\right\}}{\sqrt{1-2si\lambda_k}}.$$

It is known that the distribution F (x) of a random variable is uniquely restored by the form of the characteristic function $\varphi(t)$:

$$F(y) - F(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{e^{-itx} - e^{-ity}}{it} \phi(t)dt,$$

then

$$\phi_{|Y|_h^2}(s) = \prod_{k=1}^{\infty} \frac{1}{\sqrt{1-2is\lambda_k}}.$$

Statement of the main results.

Theorem 1. Let λ_k be the eigenvalue of the covariant operator R of a Gaussian vector Y. Then the probability of Y falling into a sphere of radius r is

$$p(|Y|_H < r) = \frac{2}{\pi}\int_0^{+\infty} \frac{1}{t}\exp\left\{-\frac{1}{4}\sum_{k_2=1}^{\infty} \ln\left(1+(2t\lambda_\kappa)^2\right)\right\} \times$$

$$\times \cos\frac{tr^2 + \sum_{k=1}^{\infty} \mathrm{arctg}\, 2t\lambda_k + 2\pi k}{2} \sin\frac{tr^2}{2} dt.$$

Theorem 2. Let λ be the maximum eigenvalue of the covariant operator R of a Gaussian vector Y. Then the probability of Y falling into a sphere of radius r is equal to:

$$P(|Y|_H < r) \le \frac{2}{\pi}\int_0^{\frac{2\pi}{r^2}} \frac{1}{t\sqrt[4]{1+(2t\lambda)^2}}\sin\frac{tr^2}{2}dt .$$

The proof of Theorem 1. Since $P(Y_H < r) = P(Y_H < r^2)$ then by the inversion formula:

$$P(|Y|_H^2 \le r^2) = F_{|Y|_h^2}(r^2) - F_{|Y|_H^2}(0) =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{e^{-it0} - e^{-itr^2}}{it}\phi_{|Y|_H^2}(t)dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1-e^{-itr^2}}{it}\prod_{k=1}^{\infty}\frac{1}{\sqrt{1-2it\lambda_k}}dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1-e^{-itr^2}}{it}\frac{1}{\prod\limits_{л=1}^{\infty}\sqrt{1-2it\lambda_k}}dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1-(\cos tr^2 - i\sin tr^2)}{it}e^{-\ln\prod\limits_{k=1}^{\infty}\sqrt{1-2it\lambda_k}}dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1-(\cos tr^2 - i\sin tr^2)}{it}e^{-\frac{1}{2}\sum\limits_{k+1}^{\infty}\ln(1{,}2it\lambda_k)}dt.$$

Let us simplify the form of the function using the properties of the functions of the complex variable [4]:

$$\ln(1-2it\lambda_k) = \ln|1+2it\lambda_k| + i(\arg(1-2it\lambda_k)+2\pi k)$$

$$\arg(1-2it\lambda_k) = \mathrm{arctg}2t\lambda_k + 2\pi k .$$

Substituting, we obtain

$$P(|Y|_H^2 \le r^2) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{i\cos tr^2 + \sin tr^2 - i}{t}\times$$

$$\times e^{\sum\limits_{k-1}^{\infty}\ln(1+(2t\lambda_k)^2)} e^{-\frac{1}{2}i\sum\limits_{k=1}^{\infty}(\mathrm{arctg}2t\lambda_k 2\pi k)}.$$

Let us use the properties of the functions of the complex variable again:

$$e^{-\frac{1}{2}i(\sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_2+2\pi k)} =$$

$$= \cos\frac{\sum\limits_{k=1}^{\infty}(\mathrm{arctg}2t\lambda_k + 2\pi k)}{2} + i\sin\frac{\sum\limits_{k=1}^{\infty}(\mathrm{arctg}2t\lambda_k + 2\pi k)}{2}.$$

In the future, for the convenience of calculations we introduce the following notation:

$$C = \cos\frac{\sum\limits_{k=1}^{\infty}(\mathrm{arctg}2t\lambda_k + 2\pi k)}{2},$$

$$S = \sin\frac{\sum\limits_{k=1}^{\infty}(\mathrm{arctg}2t\lambda_k + 2\pi k)}{2}.$$

Substituting the introduced notation in the formula, we obtain

$$P(|Y|_H^2 \le r^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-\frac{1}{4}\sum\limits_{k=1}^{\infty}\ln(1+(2t\lambda_k)^2)}}{t} =$$

$$= C\sin tr^2 - iS\sin tr^2 + iC\cos tr^2 + S\cos tr^2 - iC - S)dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-\frac{1}{4}\ln(1+(2t\lambda_k)^2)}}{t}(C\sin tr^2 + S\cos tr^2 - S)dt +$$

$$+\frac{1}{2\pi}i\int_{-\infty}^{\infty}\frac{e^{-\frac{1}{4}\sum\limits_{k=1}^{\infty}\ln(1+(2t\lambda_k)^2)}}{t}(C\cos tr^2 - C - S\sin tr^2)dt.$$

The imaginary part is 0, since the integrand is odd and is considered on the entire axis $(-\infty;+\infty)$. Then

$$P(|Y|_H^2 \le r^2) =$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-\frac{1}{4}\ln(1+(2t\lambda_k)^2)}}{t}(C\sin tr^2 + S\cos tr^2 - S)dt.$$

We take into account the following:

$$\sin tr^2\cos\frac{\sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_k + 2\pi k}{2} +$$

$$+\cos tr^2\sin\frac{\sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_k + 2\pi k}{2} =$$

$$= \cos\left(tr^2 + \frac{\sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_k + 2\pi k}{2}\right) -$$

$$-\sin\frac{\sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_k + 2\pi k}{2} =$$

$$= 2\cos\frac{tr^2 + \sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_k + 2\pi k}{2}\sin\frac{tr^2}{2}.$$

As a result, the formula will take the form:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-\frac{1}{4}\ln(1+(2t\lambda_k)^2)}}{t}(C\sin tr^2 + S\cos tr^2 - S)dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-\frac{1}{4}\ln(1+(2t\lambda_k)^2)}}{t}\times$$

$$\times\left(2\cos\frac{tr^2 + \sum\limits_{k=1}^{\infty}\mathrm{arctg}2t\lambda_k + 2\pi k}{2}\sin\frac{tr^2}{2}\right)dt =$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{4}\ln(1+(2t\lambda_k)^2)}}{t} \times$$

$$\times \left(\cos \frac{tr^2 + \sum_{k=1}^{\infty} \operatorname{arctg} 2t\lambda_k + 2\pi k}{2} \sin \frac{tr^2}{2} \right) dt$$

Knowing that the function is symmetric with respect to the origin of coordinates, we can write

$$P(|Y|_H^2 \le r^2) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{4}\ln(1+(2t\lambda_k)^2)}}{t} \times$$

$$\times \left(\cos \frac{tr^2 + \sum_{k=1}^{\infty} \operatorname{arctg} 2t\lambda_k + 2\pi k}{2} \sin \frac{tr^2}{2} \right) dt.$$

The theorem is proved.

The proof of Theorem 2. The practical use of Theorem 1 is difficult, since, firstly, the knowledge of all eigenvalues of λ_k is assumed, and secondly, the integral sign contains infinite sums. Therefore, one has to confine oneself to estimates from above the studied probability. It's obvious that

$$\exp\left\{ -\frac{1}{2} \sum_{k=1}^{\infty} \ln(1 + (2t\lambda_k)^2) \right\} = \frac{1}{4} \cdot \frac{1}{\prod_{k=1}^{\infty} \sqrt[4]{1 + (2t\lambda_k)^2}}.$$

Let $\lambda = \max_{k \ge 1} \lambda_k$, then

$$\frac{1}{t} \cdot \frac{1}{\prod_{k=1}^{\infty} \sqrt[4]{1 + (2t\lambda_k)^2}} \le \frac{1}{t} \cdot \frac{1}{\sqrt[4]{1 + (2t\lambda_k)^2}}.$$

$$\frac{1}{t} \cdot \frac{1}{\prod_{k=1}^{\infty} \sqrt[4]{1 + (2t\lambda_k)^2}} \le \frac{1}{t} \cdot \frac{1}{\sqrt[4]{1 + (2t\lambda_k)^2}}.$$

Consequently

$$P(|Y|_H < r) \le \frac{2}{\pi} \int_0^{\infty} \frac{1}{t\sqrt[4]{1 + 4t^2\lambda^2}} \sin \frac{tr^2}{2} dt.$$

Let us consider the integrand

$$p(t) = \frac{1}{t\sqrt[4]{1 + 4t^2\lambda^2}} \sin \frac{tr^2}{2}.$$

It is obvious that $\lim_{t \to 0} p(t) \to \frac{r^2}{2}$, and it means the function $p(t)$ is a bounded one. $p(t)$ =0 at $t = \frac{2\pi k}{r^2}$. The function graph is a sinusoid $t \to \infty, p(t) \to 0$.

Hence

$$P(|Y|_H < r) \le \frac{2}{\pi} \int_0^{\frac{2\pi}{r^2}} \frac{1}{t\sqrt[4]{1 + (2t\lambda)^2}} \sin \frac{tr^2}{2} dt.$$

The theorem is proved.

Thus, one can find the numerical values of the upper probability estimates $P(|Y|_H < r)$ depending on the radius of the sphere r and the maximum eigenvalue λ of the covariant operator R. Table was compiled for some values of r and λ.

Upper numerical estimates in the case of Gaussian uptime of a computing system. Let

$$J(\lambda, r) = \frac{2}{\pi} \int_0^{\frac{2\pi}{r^2}} \frac{1}{t\sqrt[4]{1 + 4t^2\lambda^2}} \sin \frac{tr^2}{2} dt.$$

If the random uptime of the computing system is a normal random field, then its distribution determined by the norm of the space *C* (*K*) can be estimated numerically from above. To do this, one only needs to know the value of the maximum eigenvalue λ of the covariance operator *R*. Then using the embedding inequality [5] we have:

$$P(|Y|_c < r) \le P(a|Y|_h < r) = P\left(|Y|_H < \frac{r}{a} \right) \le J\left(\lambda, \frac{r}{a} \right),$$

the constant a is equal to a fixed number, which is determined exactly depending on the number of variables *d* of the random field and the space *H* that is embedded in *C* (*K*).

Conclusion. This paper is devoted to the reliability of the computing system. One of the characteristics of the computing system reliability is the random number of its failures *v* (*r*). The distribution *v* (*r*) is the distribution of the sum of random times X_i (*t*) of the failure-free operation of the computing system, *i* = 1, ..., *n*. It is impossible to write down the distribution *v* (*r*) explicitly. Therefore, one has to look for an estimate of these distributions from above. Assuming that the uptime X_i (*t*) of the computing system is the sum of many variables, the authors of the paper obtained following results:

– it is shown that the problem of estimating the distributions of a random number of system failures can be considered as the problem of estimating the rate of convergence in the central limit theorem in Banach spaces;

– if there are certain restrictions on the uptime *X* (*t*) of the computing system, upper estimates are found for the distributions F_n (*r*) of a random number of system failures. These estimates can be written as

$$F_n(r) \le \left(\frac{r - Tn}{a\sqrt{n}} \right) + cn^{\beta} (\ln n)^{\gamma},$$

where *N* (*r*) is the normal distribution, the constants *a, c* are determined earlier, the exponents β, γ are determined by the conditions on *X* (*t*).

Numerical upper estimates of the form $N(r) \le J(\lambda, r)$ are found for the normal distribution *N*(*r*).

$$P(|Y|_H < r) \le \frac{2}{\pi} \int_0^{\frac{2\pi}{r^2}} \frac{1}{t\sqrt[4]{1 + (2t\lambda)^2}} \sin \frac{tr^2}{2} dt.$$

The estimates obtained can be used for further research in the theory of computing systems reliability.

Upper probabilities for $P(|Y|_C < r)$

r	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	r	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$
0.1	0.0583	0.0415	0.0340	2.6	0.7586	0.6693	0.6034
0.2	0.1139	0.0817	0.0671	2.7	0.7659	0.6807	0.6161
0.3	0.1169	0.1200	0.0993	2.8	0.7724	0.6914	0.6282
0.4	0.2171	0.1580	0.1306	2.9	0.7783	0.7013	0.6397
0.5	0.2647	0.1941	0.1610	3,0	0.7836	0.7106	0.6506
0.6	0.3096	0.2289	0.1905	3.1	0.7884	0.7193	0.6610
0.7	0.3520	0.2624	0.2191	3.2	0.7926	0.7273	0.6708
0.8	0.3917	0.2945	0.2469	3.3	0.7965	0.7348	0.6801
0.9	0.4289	0.3253	0.2737	3.4	0.7999	0.7418	0.6893
1.0	0.4637	0.3549	0.2997	3.5	0.8029	0.7482	0.6972
1.1	0.4960	0.3831	0.3248	3.6	0.8057	0.7542	0.7051
1.2	0.5260	0.4101	0.3490	3.7	0.8082	0.7598	0.7125
1.3	0.5538	0.4358	0.3724	3.8	0.8104	0.7650	0.7195
1.4	0.5795	0.4603	0.3949	3.9	0.7124	0.7698	0.7261
1.5	0.6031	0.4836	0.4166	4.0	0.8142	0.7742	0.7323
1.6	0.6248	0.5057	0.4374	4.1	0.8159	0.7783	0.7382
1.7	0.6446	0.5266	0.4574	4.2	0.8173	0.7821	0.7437
1.8	0.6628	0.5465	0.4766	4.3	0.8187	0.7856	0.7489
1.9	0.6793	0.5653	0.4951	4.4	0.8199	0.7889	0.7538
2.0	0.6943	0.5830	0.5127	4.5	0.8210	0.7919	0.7584
2.1	0.7079	0.5997	0.5296	4.6	0.8219	0.7947	0.7627
2.2	0.7202	0.6154	0.5458	4.7	0.8228	0.7973	0.7668
2.3	0.7313	0.6302	0.5612	4.8	0.8224	0.7997	0.7706
2.4	0.7414	0.6441	0.5759	4.9	0.8237	0.8019	0.7742
2.5	0.7504	0.6571	0.5900	5.0	0.8251	0.8040	0.7776

The average number of system failures is $H(r) = \sum_{n-1}^{\infty} F_n(r)$, therefore, knowing the upper estimates for *Fn* (*r*), one can obtain upper estimates for *H* (*r*) and predict the level of average costs for the restoration of computing systems.

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