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METHOD OF EQUIVALENT STRENGTH CONDITIONS IN CALCULATIONS OF BODIES WITH INHOMOGENEOS REGULAR STRUCTURE

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Plates, beams and shells with a non-uniform and micro-uniform regular structure are widely used in aviation and rocket and space technology. In calculating the strength of elastic composite structures using the finite element method (FEM) it is important to know the error of the approximate solution for finding where you need to build a sequence of approximate solutions that is connected with the procedure of crushing discrete models. Implementation of the procedure for grinding (within the micro-pass) discrete models of composite structures (bodies) requires large computer resources, especially for discrete models with a microinhomogeneous structure. In this paper, we propose a method of equivalent strength conditions (MESC) for calculating elastic bodies static strength with inhomogeneous and microinhomogeneous regular structures, which is implemented via FEM using multigrid finite elements. The calculation of composite bodies' strength according to MESC is limited to the calculation of elastic isotropic homogeneous bodies strength using equivalent strength conditions, which are determined based on the strength conditions set for composite bodies. The MESC is based on the following statement. For all composite bodies V_0*, which are such a homogeneous isotropic body* V^b *and the number of* p *, if the safety factor* n_b *of the body* V^b *satisfies the equivalent conditions of strength* $pn_1(1+\delta_\alpha) \le n_b(1-\delta_\alpha^2) \le pn_2(1-\delta_\alpha)$*, the safety factor* n_0 *of the body* V_0 *meets the defined criteria for strength* $n_1 \le n_0 \le n_2$*, where* n_1*,* n_2 *specified, the safety factor* n_0 *(*n_b*) complies with the accurate (approximate) solution of elasticity theory problem is built for body* V_0 *(body* V^b*);* $\delta_\alpha < (n_2 - n_1)/(n_2 + n_1)$*;* δ_α *is the upper* δ_b *error estimation of the maximum equivalent body stress* V^b*, corresponding to approximate solution. When constructing equivalent strength conditions, i. e when finding the equivalence* p *coefficient, a system of discrete models is used, dimensions of which are smaller than the dimensions of the basic composite bodies models. The implementation of MESC requires small computer resources and does not use procedures for grinding composite discrete models. Strength calculations for bodies with a microinhomogeneous structure using MESC show its high efficiency. The main procedures for implementing the MESC are briefly described.*

Keywords: elasticity, composites, equivalent strength conditions, multigrid finite elements, plates, beams, shells.

МЕТОД ЭКВИВАЛЕНТНЫХ УСЛОВИЙ ПРОЧНОСТИ В РАСЧЕТАХ ТЕЛ С НЕОДНОРОДНОЙ РЕГУЛЯРНОЙ СТРУКТУРОЙ

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Пластины, балки и оболочки с неоднородной, микронеоднородной регулярной структурой широко применяются в авиационной и ракетно-космической технике. В расчетах на прочность упругих композитных конструкций с помощью метода конечных элементов (МКЭ) важно знать погрешность приближенного решения, для нахождения которой необходимо построить последовательность приближенных решений, что связано с применением процедуры измельчения дискретных моделей. Реализация процедуры измельчения (в рамках микроподхода) дискретных моделей композитных конструкций (тел) требует больших ресурсов ЭВМ, особенно для дискретных моделей с микронеоднородной структурой. В данной работе предложен метод эквивалентных *условий прочности (МЭУП) для расчета на статическую прочность упругих тел с неоднородной и микронеоднородной регулярной структурой, который реализуется с помощью МКЭ с применением многосеточных конечных элементов. Расчет на прочность композитных тел по МЭУП сводится к расчету на прочность упругих изотропных однородных тел с применением эквивалентных условий прочности, которые определяются на основе условий прочности заданных для композитных тел. В основе МЭУП лежит следующее утверждение. Для всякого композитного тела V_0 существуют такое изотропное однородное тело V^b и число p, что если коэффициент запаса n_b тела V^b удовлетворяет эквивалентным условиям прочности вида $pn_1(1+\delta_\alpha) \le n_b(1-\delta_\alpha^2) \le pn_2(1-\delta_\alpha)$, то коэффициент запаса n_0 тела V_0 удовлетворяет заданным условиям прочности $n_1 \le n_0 \le n_2$, где n_1, n_2 заданы, коэффициент запаса n_0 (n_b) отвечает точному (приближенному) решению задачи теории упругости, построенному для тела V_0 (тела V^b), $\delta_\alpha < (n_2 - n_1)/(n_2 + n_1)$, δ_α – верхняя оценка погрешности δ_b максимального эквивалентного напряжения тела V^b, отвечающего приближенному решению. При построении эквивалентных условий прочности, т. е. при нахождении коэффициента эквивалентности p, используется система дискретных моделей, размерности которых меньше размерностей базовых моделей композитных тел. Реализация МЭУП требует малых ресурсов ЭВМ и не использует процедуры измельчения композитных дискретных моделей. С помощью расчетов показано, что эквивалентные условия прочности, построенные для конкретного нагружения композитного тела, можно использовать для определенного вида его нагружений. Расчеты на прочность тел с микронеоднородной структурой с помощью МЭУП показывают высокую его эффективность. Кратко изложены основные процедуры реализации МЭУП.*

Ключевые слова: упругость, композиты, эквивалентные условия прочности, многосеточные конечные элементы, пластины, балки, оболочки.

Introduction. Structure strength calculation is one of the most important stages in the outline design of a structure based on a structure project feasibility study. As a rule, calculations for static strength, elastic structure (body) of a certain class (for example, elements or aircraft and rocket-space structures) are carried out according to safety requirements [1–3], and limited to the equivalent structure stress determination. In this case for the body V_0 the given strength conditions are $n_1 \le n_0 \le n_2$, where n_1, n_2 are given, n_0 is the body safety factor, V_0, $n_0 = \sigma_T / \sigma_0$, σ_T is the yield stress [1], σ_0 is the maximum equivalent stress corresponding to the exact solution of the elasticity problem (constructed for the body V_0). If the safety factor n_0 satisfies the given strength conditions, then it is suggested that the body V_0 does not collapse during operation. It should be noted that construction of analytical solutions of the three-dimensional problem of elasticity theory for composite bodies is associated with great difficulties. If the maximum equivalent stresses of the bodies is approximate, then in this case the corrected strength conditions are used [4], which pass the stress error. In the analysis of the stress-strain state (SSS), the finite element method (FEM) is widely used [5; 6]. Basic discrete models of bodies, accounting for their inhomogeneous and micro-inhomogeneous structures within the micro-approach [7], have a very high dimension. Implementation of FEM for such discrete models is very difficult, since it requires large computer resources. In addition, to determine the error in the solution, a sequence of approximate solutions constructed using refinement (within the micro approach) of discrete models is used. The grinding procedure is difficult to implement; it leads to a sharp increase in the discrete models size, making implementation of FEM challenging. To determine the SSS of composite bodies, the method of multigrid finite elements (MFEM) [8–14] is effectively applied, which generates discrete models, dimensions of which are $10^3 \div 10^6$ times less than the base models dimensions. It should be noted that FEM is a special case of MFEM. If when solving boundary value problems by FEM, multigrid finite elements (MgFE) are used [8–22], then MFEM is implemented in this case.

In this work, for calculating the strength of solid composite bodies using equivalent strengths, the method of equivalent strength (MESC) is proposed, which means calculating the strength of isotropic homogeneous bodies using equivalent strengths [23]. In this paper in contrast to [24], a theorem is formulated and proved, which underlies the MESC. In addition, the following should be noted: equivalent strength conditions are based on specified strength conditions using the equivalence coefficient *p*. In fact, the construction of equivalent strength islimited to determining the coefficient *p*, which is determined for a given composite body loading. However, it is important to note that the equivalent strength conditions constructed using the coefficient *p* can be used in composite body strength calculations for a certain type of its loading.

To find the coefficient *p*, a system of homogeneous and composite discrete models is used, dimensions of which are less than the dimensions of composite bodies models. The analysis of SSS in discrete models is carried out using the MFEM, which generates discrete models of small dimension. The advantages of the MESC are that its implementation requires small computer resources and does not use the procedure for refining discrete models of composite bodies. The use of MESC in strength calculations of bodies with a micro-inhomogeneous regular structure shows its effectiveness.

1. Equivalent strength conditions and equivalent strength structures. Suppose two elastic structures V_1 and V_2 have the same shape, geometrical dimensions, fixings and static loading, but differ in elasticity modulus.

Suppose strength conditions n_1, n_2 are given for the safety factors, respectively of structures V_1, V_2

$$n_a^1 \le n_1 \le n_b^1, \tag{1}$$

$$n_a^2 \le n_2 \le n_b^2, \tag{2}$$

where $n_a^1, n_a^2 > 1$; $n_a^1, n_a^2, n_b^1, n_b^2$ – are given; safety factors n_1 (n_2) complie with the precise solution of elasticity theory, built for structures V_1 (V_2).

For structures V_1, V_2 the following two definitions are introduced:

Definition 1. Fulfillment of conditions (2) for the coefficient n_2 implies fulfillment of conditions (1) for the coefficient n_1 and vice versa, if the fulfillment of conditions (1) for the coefficient n_1 implies the fulfillment of conditions (2) for the coefficient n_2, then the strength conditions (1), (2) will be called equivalent strength conditions for structures V_2, V_1, respectively.

Definition 2. Suppose the structures V_1, V_2, for which respectively condition (2), (1) is equivalent to strength conditions do not collapse under the same operating conditions. Then the structures V_1, V_2 will be called strength equivalent.

In practice, the equivalence in strength of structures V_1, V_2 means that V_2 structure can be used instead of a working structure V_1, and vice versa. It should be noted that of the two structures equivalent in strength, it is advisable to use such a structure that is more technologically advanced in manufacturing, meets the specified technical requirements and more cost effective for manufacturing and operation.

2. Provisions of the method of equivalent strength conditions MESC are used to calculate the strength of structures (bodies) that satisfy the following:

Provision 1. Linearly elastic three-dimensional isotropic homogeneous bodies and bodies with an inhomogeneous, micro-inhomogeneous regular structure, which consist of plastic materials, have smooth boundaries and static loading are considered. The body loading functions are smooth functions. Solid boundaries do not degenerate into points.

Provision 2. Composite bodies consist of isotropic homogeneous bodies of different modulus, connections between which are ideal, that is, on common boundaries of homogeneous bodies of different modulus, the functions of displacements and stresses are continuous.

Provision 3. Displacements, deformations and stresses of heterogeneous isotropic homogeneous bodies correspond to the Cauchy relations and Hooke's law of the three-dimensional linear problem of elasticity theory [25]. Equivalent stresses for bodies are determined according to the 4th theory of strength [1].

Provision 4. The maximum equivalent stress of the basic discrete model of a composite body (which consists of a first-order FE of the cube shape, takes into account the inhomogeneous structure of the composite body and generates a three-dimensional uniform mesh) shows a small difference with the exact solution. It should be noted that due to the convergence of the FEM, such basic discrete models for composite bodies always exist.

Provision 5. For the typical dimensions of a composite body and its regular cell, the condition $d / B << 1$ is fulfilled, where *d* is the maximum typical size of the regular cell of the composite body, *B* is the minimum typical size of the composite body.

It should be noted that positions 4, 5, as a rule, are fulfilled for bodies with micro-inhomogeneous regular structure.

3. The main theorem of the method of equivalent strength conditions. Without losing shared judgments, we consider bodies with an inhomogeneous regular fibrous structure, which are widely used in practice. The MESC is based on the following theorem:

Theorem. Suppose the strength conditions of the form 3 are given to the safety factor of a composite body n_0 (fibrous structure).

$$n_1 \le n_0 \le n_2, \tag{3}$$

where n_1, n_2 – are given, $n_1 > 1$, $n_0 = \sigma_T / \sigma_0$, σ_T – fiber yield stress, σ_0 – the maximum equivalent stress of the body V_0, which corresponds to the exact solution of the problem of the elasticity theory, constructed for the body V_0.

Then there is such an isotropic homogeneous body V^b and such a number $p > 0$ (equivalence coefficient) that if the body V^b safety factor n_b satisfies the corrected equivalent strength conditions

$$\frac{pn_1}{1-\delta_\alpha} \le n_b \le \frac{pn_2}{1+\delta_\alpha}, \tag{4}$$

then, safety factor n_0 of the structure V_0 meets the strength requirements (3), where $n_b = \sigma_T / \sigma_b$, σ_b – the maximum equivalent stress of the body V^b, which corresponds to the approximate solution of the theory of elasticity problem, constructed for the body V^b,

$$\delta_\alpha < \frac{n_2 - n_1}{n_2 + n_1}, \tag{5}$$

δ_α – upper bound on relative error, δ_b pressure σ_b of body V^b, $|\delta_b| \le \delta_\alpha$.

Deduction.

First, let us prove the existence of equivalent strength conditions for linearly elastic composite bodies. Suppose an elastic homogeneous isotropic body V^b and a composite body V_0 have the same shape, size, fixation and loading, but differ in elastic moduli. Suppose the elastic moduli of the body V^b and fiber be the same. The safety factors n_0, n_b^0 respectively bodies V_0, V^b are found by the formulas

$$n_0 = \frac{\sigma_T}{\sigma_0}, \tag{6}$$

$$n_b^0 = \frac{\sigma_T}{\sigma_b^0}, \tag{7}$$

where σ_T – fiber yield strength [1–3]; σ_b^0 – maximum equivalent body stress V^b, corresponding to the exact solution of the elasticity theory problem.

Suppose coefficient n_0 meets the requirements (3). Applying (6) to (3) we obtain

$$n_1 \le \frac{\sigma_T}{\sigma_0} \le n_2 . \quad (8)$$

There is a number $p > 0$,

$$p = \frac{\sigma_0}{\sigma_b^0} . \quad (9)$$

Considering (9) in (8), we obtain

$$pn_1 \le \frac{\sigma_T}{\sigma_b^0} \le pn_2 \quad (10)$$

Applying (7) in (10), we obtain

$$pn_1 \le n_b^0 \le pn_2 . \quad (11)$$

So, the safety factor n_b^0 of an isotropic homogeneous body V^b satisfies conditions (11). Conversely, suppose body V^b safety factor n_b^0 satisfy the strength conditions (11). Applying (7) in (11) considering (9), we obtain $pn_1 \le \frac{p\sigma_T}{\sigma_0} \le pn_2$. Whence, taking into account (6), follows the fulfillment of the strength conditions for the safety factor n_0 of the composite body V_0 (3). It is shown that each coefficient $n_b^0 \in (pn_1, pn_2)$ corresponds to a single coefficient $n_0 \in (n_1, n_2)$ found by formula (6), and vice versa. Further limiting cases are considered. Suppose $n_b^0 = pn_1$. Using relation (7) in the latter equation we obtain $p\sigma_T / \sigma_0 = pn_1$. Whence, taking into account (6) it follows $n_0 = n_1$. Similarly, one can show that if $n_b^0 = pn_2$, then $n_0 = n_2$. Suppose $n_0 = n_1$. Using (6), (9) in the latter equation, we obtain $\sigma_T / \sigma_b^0 = pn_1$. Now then, taking into account (7), it follows that $n_b^0 = pn_1$. Similarly, one can show that if $n_0 = n_2$, then $n_b^0 = pn_2$. Hence it follows that conditions (11), according to Definition 1, are equivalent strength conditions for a body V_0 .

Suppose for the body V^b the maximum equivalent stress has been defined as σ_b, corresponding to the approximate solution of the elasticity theory problem, such that

$$|\,\delta_b\,| \le \delta_\alpha < C_\alpha = \frac{n_2 - n_1}{n_1 + n_2} , \quad (12)$$

where n_1 , n_2 – are given; $n_1 > 1$, $n_2 > n_1$, δ_b – relative stress error σ_b, i. e.

$$\delta_b = \frac{\sigma_b - \sigma_b^0}{\sigma_b^0} , \quad (13)$$

where δ_α – upper bound for error δ_b .

From (13) it follows that $\sigma_b = (1 + \delta_b)\, \sigma_b^0$. Hence obtain

$$n_b^0 = (1 + \delta_b) n_b . \quad (14)$$

Let us note that in (12) $C_\alpha < 1$. Suppose δ_0 is such that $\delta_0 = \;|\,\delta_b\,|$. Then due to (12) obtain

$$0 \le \delta_0 = \;|\,\delta_b\,|\; \le \delta_\alpha < 1 . \quad (15)$$

Assuming in (14) consecutively $\delta_b = -\delta_0$, $\delta_b = \delta_0$, apply coefficients

$$n_1^r = (1 - \delta_0) n_b , \quad n_2^r = (1 + \delta_0) n_b , \quad (16)$$

Then due to (14), (16) obtain

$$n_b^0 = n_1^r \quad \text{or} \quad n_b^0 = n_2^r . \quad (17)$$

Apply coefficients n_1^d , n_2^d according to formulas

$$n_1^d = (1 - \delta_\alpha) n_b , \quad n_2^d = (1 + \delta_\alpha) n_b . \quad (18)$$

Due to $0 \le \delta_\alpha < 1$, $n_b > 0$, from (18) it follows that

$$n_2^d \ge n_1^d . \quad (19)$$

Equivalent strength conditions that take into account stress error, i. e., corrected equivalent strength conditions (4) are presented in the form

$$pn_1 (1 + \delta_\alpha) \le n_b (1 - \delta_\alpha^2) \le pn_2 (1 - \delta_\alpha) , \quad (20)$$

where $n_b = \sigma_T / \sigma_b$, σ_T – fiber yield strength.

Suppose for coefficient n_b strength conditions are met (20), i. e. suppose $pn_1 \le (1 - \delta_g) n_b$, $(1 + \delta_g) n_b \le pn_2$. Hence for the coefficient n_1^d , n_2^d , taking into account (18), (19) inequation is done

$$pn_1 \le n_1^d \le n_2^d \le pn_2 . \quad (21)$$

Comparing (16), (18) with respect to (15), equations $n_1^d \le n_1^r$, $n_2^r \le n_2^d$ follow. Hence, considering that according to (16) $n_1^r \le n_2^r$, we obtain

$$n_1^d \le n_1^r \le n_2^r \le n_2^d . \quad (22)$$

Then, due to (21), (22) inequations are done

$$pn_1 \le n_1^r \le n_2^r \le pn_2 . \quad (23)$$

From (23) taking into account (17), i. e. from meeting for the body V^b safety factor n_b (corresponding to the approximate solution) of the corrected equivalent strength conditions (20), that is (4), it follows that strength conditions (11) for the safety factor n_b^0 of the body V^b (corresponding to the exact solution) are met, therefore, satisfying the given strength conditions (3) for the safety factor n_0 of the composite body V_0 (corresponding to the exact solution). Constraints on the parameter δ_α are found from the assumption of strength conditions existence (4), i. e. suppose inequation $pn_1 (1 + \delta_\alpha) \le pn_2 (1 - \delta_\alpha)$ is done. Whence it follows that

$$\delta_\alpha < C_\alpha = \frac{n_2 - n_1}{n_1 + n_2} . \quad (24)$$

It should be noted that, since $n_2 > n_1 \geq 1$, then from (24) it follows that $0 < C_\alpha < 1$. If $\delta_\alpha = C_\alpha$, then the range for varying values of the coefficient n_0 is zero, which is difficult to perform in practice. Now then $\delta_\alpha < C_\alpha$, it is possible to meet the equivalent strength conditions (11) for the coefficient n_b^0 applying corrected equivalent strength conditions (4) and the approximate solution that generates an error δ_b for the stress σ_b that $|\delta_b| \leq \delta_\alpha$. Note that meeting conditions (11) implies the fulfillment of the specified strength conditions (3). The theorem is proved.

Note that it follows from the theorem that if the safety factor n_b of the body V^b satisfies the corrected equivalent strength conditions (4), then this means that the error δ_b of the maximum equivalent stress σ_b of the body V^b is not greater than δ_α, i. e. $|\delta_b| \leq \delta_\alpha$.

4. Procedures for implementing the method of equivalent strength conditions. Implementation of the MESC is reduced to construction of equivalent strength conditions (4) applying the MFEM, that is, to determination of the equivalence coefficient p, and to determination of the maximum equivalent stress σ_b for the body V^b with an error $|\delta_b| < \delta_\alpha$, $n_b = \sigma_T / \sigma_b$. The coefficient p is determined by the formula (9), i. e.

$$p = \frac{\sigma_0}{\sigma_b^0}. \tag{25}$$

Without losing shared judgments, for convenience and clarity of presentation, we will consider the basic procedures for the implementation of MESC using the example of calculating the strength of a composite beam (body) V_0 with dimensions $H \times L \times H$, where $H = 128h$, $L = 1536h$, h – is given, Fig. 1. The body V_0 is reinforced with continuous longitudinal fibers of constant cross-section with dimensions $h \times h$. The fibers have the same modulus of elasticity. When $y = 0$ the body is fixed and has loading $q_z(x, y)$ on the surface $z = H$. The inhomogeneous structure of the body V_0 is represented by regular cells G_0 with $8h \times 8h \times 8h$ size, fig. 2, the sections of 16 fibers are painted over. It is believed [26] that if the fiber thickness is less than 0.5 mm, then such fibers form a micro-inhomogeneous structure. Suppose $L = 600$ мм, $H = 50$ мм, then $h = 0.3906$ мм. In this case, the body V_0 has a micro-inhomogeneous regular structure.

It should be noted that since the filling factor of the composite body V_0 is small (equal to 0.25), it is difficult to determine the effective elastic moduli for the body V_0. The case when the filling coefficient is close to one was considered in [23].

Suppose the strength conditions (3) are given for the safety factor n_0 of the composite body V_0. The basic discrete V_0 body model $\mathbf{R}_0$ consists of finite elements (FE) of the 1st order of a cube shape with a side h [6], in which a three-dimensional SSS is realized, accounting for the inhomogeneous structure of the beam and generates a basic uniform mesh with a step h with dimension $129 \times 1537 \times 129$.

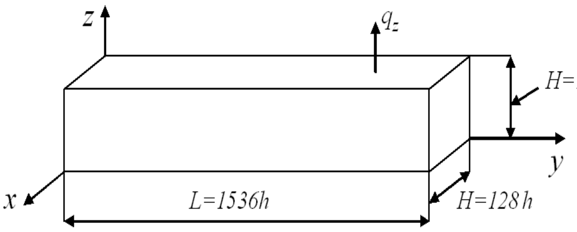

Fig. 1. The characteristic sizes of the beam (body) V_0

Рис. 1. Характерные размеры балки (тела) V_0

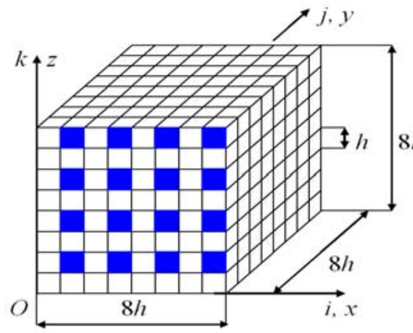

Fig. 2. Regular cell (body) G_0

Рис. 2. Регулярная ячейка (тело) G_0

Fig. 2 shows the basic grid G_0 of a regular dimension cell $9 \times 9 \times 9$; $i, j, k = 1, ..., 9$. The model $\mathbf{R}_0$ has $N_0 = 76681728$ nodal unknown FEM, system tape width of FEM equations is $b_0 = 50316$. The basic model $\mathbf{R}_0$ takes into account the micro-inhomogeneous structure of the body V_0 with high dimension, therefore we can assume that this model satisfies position 4. However, it is difficult to apply the discrete model $\mathbf{R}_0$ in calculations, since the implementation of the FEM for the $\mathbf{R}_0$ model requires essential computer resources.

According to the MESC, introduced is an isotropic homogeneous body V^b such that the bodies V^b, V_0 have the same shape, dimensions, specified fixing and loading, but differ in elastic moduli. The elastic moduli of the body V^b are equal to the elastic moduli of the body V_0 fiber. For the body V^b we define a discrete model V_n^b, which consists of an FE $V_e^{(n)}$ of the 1st order of a cube shape with a side h_n [6] and has a uniform mesh with a step h_n with dimension $n_1^{(n)} \times n_2^{(n)} \times n_3^{(n)}$, where

$$n_1^{(n)} = 8n + 1, \; n_2^{(n)} = 12 \times 8n + 1,$$

$$n_3^{(n)} = 8n + 1, \quad n = 1, 2, 3, \ldots \tag{26}$$

The steps of the fine mesh of the model V_n^0 along the axeses Ox, Oy, Oz equal $h_x^{(n)} = H/(8n)$, $h_y^{(n)} = L/(96n)$, $h_z^{(n)} = H/(8n)$. Since $L = 12H$, then $h_n = h_x^{(n)} = h_y^{(n)} = h_z^{(n)}$. Due to (26) we obtain $h_n = \beta_n h$, where β_n – scale factor, $\beta_n = 16/n$, $n = 1,2,3,...$. Under $n = 1,...,15$ we have $\beta_n > 1$, i. e. $h_n > h$. Under $n \to 16$ we have $\beta_n \to 1$, $\beta_{16} = 1$, $h_{16} = h$. Discrete model V_n^b of a finite number of bodies of the same shape G_n^b with dimensions $8h_n \times 8h_n \times 8h_n$, $n = 1,2,3,...$. The body and the regular cell G_0 have the same shape (cube shape), but differ in characteristic dimensions.

Let us introduce a composite body G_n^0 (cube shaped) with dimensions $8h_n \times 8h_n \times 8h_n$. Suppose the composite body G_n^0 consist of FE $V_e^{(n)}$ cube-shaped with the side h_n. The composite body G_n^0 is of fibrous structure, the same number of fibers (16 longitudinal fibers with a square cross section $h_n \times h_n$, the distance between the fibers equals h_n) and the same mutual arrangement as in the regular cell G_0 (the cell G_0 has 16 with dimensions $h \times h$, the distance between them equals h, fig. 2). $n = 1,2,3,...$. Inhomogeneous structure in the composite body G_n^0 is taken into account using FE $V_e^{(n)}$. Fibers and matrices of the bodies G_n^0, G_0 have the same elastic moduli. The bodies G_n^0, G_0 in fact differ only in scale, they can formally be written as $G_n^0 = (\beta_n)^3 G_0$. Under $n = 16$ we obtain $\beta_{16} = 1$, i. e. $G_{16}^0 = G_0$.

Using the bodies G_n^0 instead of the bodies G_n^b in the discrete model V_n^b we obtain a composite discrete model R_n^0, $n = 1,2,3,...$, which accounts for inhomogeneous structure. Composite body G_n^0 is, in fact, a regular cell for the model R_n^0, $n = 1,2,3,...$. Discrete model R_n^0 has the same uniform grid with step h_n and dimensions V_n^b. Under $n = 16$ the discrete models V_{16}^b, R_{16}^0 and $\mathbf{R}_0$ have the same shape, characteristic size and dimensions. Since $G_{16}^0 = G_0$, then under $n = 16$ models R_{16}^0 and $\mathbf{R}_0$ coincide, i. e. $R_{16}^0 = \mathbf{R}_0$. Thus, the discrete models V_n^0, R_n^0 possess the same shape , characteristic size and dimensions, the same fixing and loading, like a body (beam) V_0, but differ only in elastic moduli $n = 1,2,3,...$. It is important to note the following:

1. Dimensions of discrete models V_n^0, R_n^0 under $n = 1,...,15$, due to (26), are less than the dimensions of the basic discrete model $\mathbf{R}_0$ of a composite body V_0.

2. When constructing composite discrete models $\{R_n^0\}_{n=2}^{15}$, the procedure of grinding composite discrete models is not applied.

To reduce the dimensions of the models V_n^b, R_n^0 MgFE are used [8–22]. Since the models R_{16}^0, V_{16}^b have the same high dimension as the basic discrete body model $\mathbf{R}_0$, which has 76681728 nodal unknown FEM, we believe that the maximum equivalent stress σ_{16}^0 (stress σ_{16}^b) of the model R_{16}^0 (model V_{16}^b) differs a little from the exact stress σ_0 (σ_b^0). Therefore, we assume $\sigma_0 = \sigma_{16}^0$, $\sigma_b^0 = \sigma_{16}^b$.

We find the equivalence coefficient p by formula (25) accounting for the latter 2 equations, i. e.

$$p = \sigma_{16}^0 / \sigma_{16}^b. \tag{27}$$

Taking into account in the formula $p_n = \sigma_n^0 / \sigma_n^b$, where σ_n^0 (σ_n^b) is the maximum equivalent stress of the model R_n^0 (model), which at $n \to 16$ we have $\sigma_n^0 \to \sigma_{16}^0$, $\sigma_n^b \to \sigma_{16}^b$, due to (27) we have $p_n \to p$ at $n \to 16$. Suppose p_n quickly converge to p. Let the value $\delta_n = |p_n - p_{n-1}|/p_n$ be small, where then we accept hat $p = p_n$. Applying the found coefficient p and parameter δ_α (δ_α specified and satisfies condition (5)) n_1 and n_2 specified in representation (4), we determine the corrected equivalent strength conditions, which accounts for the stress error. Suppose σ_n^b quickly converge to σ_b^0. Let the small value $\delta_n^\sigma = |\sigma_n^b - \sigma_{n-1}^b|/\sigma_n^b$ and $|\delta_n^b| \le \delta_\alpha$, where δ_n^b is the relative voltage error, σ_n^b δ_α is given, $\delta_\alpha < C_\alpha$ $n = 2,3,...,$. Then we accept that $\sigma_b = \sigma_n^b$, i. e., the maximum equivalent body V^b stress σ_b is found. Suppose the found safety factor n_b (where $n_b = \sigma_T / \sigma_b$, i. e. $n_b = \sigma_T / \sigma_n^b$) of an isotropic homogeneous body V^b (corresponding to an approximate solution) satisfy the constructed equivalent strength conditions (4). Then the safety factor n_0 of the composite body V_0 (which corresponds to the exact solution) satisfies the given strength conditions (3).

When calculating the composite bodies strength according to MESC, it is advisable to use MgFE [24]. In this case, the implementation of MESC requires small computer resources.

5. Application of the corrected equivalent strength conditions in the calculations of composite bodies with a certain type of loading. The calculations given below show that the corrected equivalent strength conditions (4), constructed for a specific body loading, can be used in the strength calculations of a composite body V_0 (fig. 1), for which a certain type of loading is specified.

In [24], an example of a cantilever beam V_0 (fig. 1) strength analysis according to MESC using three-mesh FE is considered in detail. The beam is reinforced with longitudinal fibers. The regular cell of the beam is shown in fig. 2. Under $y = 0$, $u = v = w = 0$, i. e in the xOz

plane the beam is fixed. For the safety factor n_0 of the beam, the given strength conditions have the form

$$1.3 \le n_0 \le 3.2 . \tag{28}$$

In the calculations of the beam the following data were used:

$$h = 0.3906 ;\ \sigma_T = 5 ;\ E_v = 10 ,\ E_c = 1 ,$$
$$\nu_c = \nu_v = 0.3 ,\ q_z = 0.0018, \tag{29}$$

where E_c, E_v (ν_c, ν_v) – Young's moduli (Poisson's ratios) of the binder and fibers, respectively, σ_T is the yield stress q_z of the fiber, the load acts on the surface $z = H$, $0.5L \le y \le L$, fig. 1.

The equivalence factor p for the composite beam V_0 is determined using the procedure described above. Discrete models V_n^b, R_n^0, $n = 9, 11, 12$ are constructed using 3sFE (the construction procedure of which is described in detail in [24]) on the basis of basic regular partitions, respectively of dimensions: $73 \times 865 \times 73$, $89 \times 1057 \times 89$ and $97 \times 1153 \times 97$. The coefficients p_n are found by the formula $p_n = \sigma_n / \sigma_n^b$, where σ_n, σ_n^b are the maximum equivalent stresses, respectively of the models R_n^0, V_n^b, $n = 9, 11, 12$. As a result of calculations we get: $p_9 = 3.002$, $p_{11} = 3.000$, $p_{12} = 2.999$. The relative errors for the found coefficients p_9, p_{11}, p_{12} are

$$\delta_1(\%) = 100\ \% \times | p_{11} - p_9 | / p_{11} =$$
$$= 100\ \% \times | 3.002 - 3.000 | / 3.000 = 0.066\ \%,$$

$$\delta_2(\%) = 100\ \% \times | p_{12} - p_{11} | / p_{12} =$$
$$= 100\ \% \times | 3.000 - 2.999 | / 2.999 = 0.033\ \%.$$

Since $p_9 > p_{11} > p_{12}$ and δ_2 is the smallest value, we consider, equivalent coefficient equals $p = p_{12} = 2.999$. Applying to (4) $\delta_\alpha = 0.15$, $n_1 = 1.3$, $n_2 = 3.2$, we obtain the corrected equivalent strength conditions expressed in terms of the equivalence coefficient p

$$1.5288 p \le n_b \le 2.7805 p . \tag{30}$$

Applying to (30) $p = 2.999$, we obtain the following corrected equivalent strength conditions $4.584 \le n_b \le 8.339$, which in practice, in order to take into account the error of computer calculations, is used in the following modified form

$$4.65 \le n_b \le 8.25 . \tag{31}$$

Table 1 shows the results of calculations for five loadings q_z^n of the beam V_0, for which the equivalence coefficients p^n are found, where x, y, z are the coordinates of the points of the beam surface, on which a constant load q_z^n, $n = 1,...,5$ is applied. Loads q_z^n, $n = 1, 4, 5$ provide direct bending of the beam, loads q_z^2, q_z^3 – oblique bending of the beam. The relative error $\delta_n(\%)$ for the equivalence coefficient p_n, presented in table, is determined by the formula

$$\delta_n(\%) = 100\ \% \times | p - p_n | / p , \tag{32}$$

where $p = 2.999$, $n = 1,...,5$.

Analysis of the calculation results shows that the equivalence coefficients p^n, $n = \overline{1.5}$ differ from the equivalence coefficient $p = 2.999$ by small values, which are $0.35\ \%$ less (see formula (32), tab. 1). According to (30), the corrected equivalent strength conditions for the equivalence coefficient p^n, $n = 1,...,5$ have the form

$$1.5288 p^n \le n_b \le 2.7805 p^n . \tag{33}$$

Since the coefficients p^n, $n = 1,...,5$ have minor difference with p (see formula (32), fig. 1), then equivalent strength conditions (33) will differ a little from the equivalent strength conditions (30); moreover, we have

$$1.5288 p^n \le 4.65 \le n_b \le 8.25 \le 2.7805 p^n , \tag{34}$$

where $n = 1,...,5$.

Fulfillment of (34) implies that the equivalence coefficients p^n, $n = \overline{1.5}$, in fact, generate corrected equivalent strength conditions (31).

Consequently, the results of the calculations show that when calculating the strength of a composite beam V_0 under the action of piecewise constant loads q_z^n on the surface $z = H$, $n = 1,...,5$ it is possible to use the corrected equivalent strength conditions (31) constructed for a beam V_0 with loading $q_z = 0.0018$ on the surface $0.5L \le y \le L$, $z = H$ i. e., constructed using the equivalence coefficient $p = 2.999$.

The results of calculations of the beam V_0

n	x	y	z	q_z^n	p^n	$\delta_n(\%)$
1	$0 \le x \le H$	$0 \le y \le L$	H	0.0078	2.997	0.066 %
2	$0 \le x \le H/2$	$0 \le y \le L/2$	H	0.543	2.991	0.267 %
3	$0 \le x \le H/2$	$0 \le y \le L$	H	0.125	2.989	0.333 %
4	$0 \le x \le H$	$0{,}998L \le y \le L$	H	2.8000	2.999	0.000 %
5	$0 \le x \le H$ $0 \le x \le H$	$0 \le y \le L/2$ $0{,}5L \le y \le L$	H	0.0145 0.0034	2.994	0.167 %

Now then, if a piecewise constant load q_z acts on the upper surface of the beam V_0, which provides direct or oblique bending of the beam, then when calculating the strength of the beam V_0, you can use the corrected equivalent strength conditions (31).

Given in [24] example of calculating the strength of a cantilever beam (having a micro-inhomogeneous regular fibrous structure) using the MESC shows its high efficiency.

Conclusion. The method of equivalent strength conditions is proposed for calculating the static strength of elastic bodies with an inhomogeneous, micro-inhomogeneous regular structure under given strength conditions. The proposed method is implemented applying FEM using multigrid finite elements and is limited to calculation of isotropic homogeneous bodies strength using equivalent strength conditions that account for solution errors. In the process of implementation, the method of equivalent strength conditions requires little time or computer resources and is exceptionally effective when calculating the strength of bodies that have a micro-inhomogeneous regular fibrous structure.

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