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Моделирование невесомости подвешенной на тросах системы балок изменением сил натяжения

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Рассматривается проблема имитации невесомости систем балок, подвешенных на нерастяжимых тросах. Имитация невесомости означает обнуление или уменьшение какого-либо выбранного силового фактора (например, реакции опоры или момента в опоре или сочленении) и кинематического фактора (прогиба или угла поворота). Требуется подобрать усилия в тросах такими, чтобы сумма квадратов прогибов в точках упругой линии балки была минимальной.

Задача формулируется как задача нелинейного программирования, осуществляется поиск минимума целевой функции с ограничениями в виде уравнений равновесия. В общем виде все выписанные для геометрически изменяемой системы уравнения линейно-зависимы. Из системы уравнений выбираются параметры, при которых векторы вводятся в базис, а оставшиеся параметры считаются свободными и являются координатами целевой функции. Задача свелась к задаче квадратичного программирования без ограничений. Частные производные по координатам дают систему линейных алгебраических уравнений, позволяющую определить координаты, принятые как свободные параметры, а затем вычислить и координаты, введенные в базис. Опорный план нелинейных задач оптимизации может иметь локальные минимумы. Показано, что при любом начальном базисе, оптимальный план единственный.

Для вычисления прогибов балки применяется метод начальных параметров. В качестве начальных параметров рассматриваются прогиб, угол поворота, дополнительные углы поворота в шарнирных сочленениях, а также реакция и изгибающий момент. Континуальная задача переводится в дискретную ограничением количества точек, в которых вычисляются прогибы. Целевая функция имеет конечное число переменных. Определяется, какое количество выбранных точек на упругой линии балок является достаточным для обеспечения сходимости функций прогибов, углов поворота, изгибающих моментов и поперечных сил с целью приложения к практическим расчетам.

Выполнена оптимизация прогибов балки, шарнирно закрепленной, подвешенной на двух тросах с проверкой решений, сменой базисных переменных и исследованием сходимости в зависимости от выбора количества точек, в которых вычисляются прогибы.

Проанализировано деформирование систем двутавровых балок, соединенных шарнирами между собой, имеющими в условиях гравитации погонный вес. Для имитации невесомости системы подкрепляются тросами. Рассмотрены граничные условия: жесткое защемление; шарнирно-неподвижное опирание, скользящая заделка, свободный край. Модели систем трех балок при имитации невесомости в определенной степени эквиваленты. Вид граничного условия в большей мере влияет на первую балку. Силы натяжения тросов выравнивают деформированное и напряженное состояние в последующих балках. Любую из рассмотренных систем с представленными граничными условиями можно перевести в эквивалентную ей, изменив граничные силовые факторы, задав моменты или установив пружину с заданной жесткостью и корректировкой натяжения тросов.

Ключевые слова: прогибы балок, метод начальных параметров, нелинейное программирование, регулирование прогибов и внутренних сил, имитация невесомости, обезвешивание балок.

Modeling of suspended weightlessness on the cables of the beam system by changing the tension forces

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The problem of weightlessness simulation of beam systems suspended on inextensible cables is considered. Imitation of weightlessness means zeroing or reducing any selected force factor (for example, the reaction of the support or the moment in the support or joint), and the kinematic factor (deflection or angle of rotation). It is required to select the forces in the cables such that the sum of the squares of the deflections at the points of the elastic line of the beam is minimal.

The problem is formulated as a nonlinear programming problem; the search for the minimum of the objective function with constraints, in the form of equilibrium equations, is carried out. In general, all equations written out for a geometrically variable system are linearly dependent. Parameters are selected from the system of equations, the vectors at which are entered into the basis, and the remaining parameters are considered free and are the coordinates of the objective function. The problem was reduced to the problem of quadratic programming without restrictions. Partial derivatives of coordinates give a system of linear algebraic equations that allows you to determine the coordinates taken as free parameters, and then calculate the coordinates entered into the basis. The reference plan of nonlinear optimization problems can have local minima; it is shown that for any initial basis, the optimal plan is the only one.

To calculate the deflections of the beam, the method of initial parameters is used. Deflection, angle of rotation, additional angles of rotation in articulated joints are considered as initial parameters; as well as the reaction and bending moment. The continuum problem is transformed into a discrete one by limiting the number of points at which deflections are calculated. The objective function has a finite number of variables. It is determined which number of selected points on the elastic line of the beams is sufficient to ensure the convergence of the functions of deflections, angles of rotation, bending moments and transverse forces for the purpose of application to practical calculations.

Optimization of deflections of a beam pivotally fixed, suspended on two cables with verification of solutions, change of basic variables and convergence study depending on the choice of the number of points at which deflections are calculated is performed.

The deformation of systems of I-beams connected by hinges to each other, having linear weight in gravity conditions, is analyzed. To simulate weightlessness, the system is supported by six cables. The boundary conditions are considered: – rigid pinching; – hinge-fixed support, – sliding sealing; – free edge. Models of three-beam systems in the simulation of weightlessness, to a certain extent equivalent. The type of boundary condition affects the first beam to a greater extent; the tension forces of the cables equalize the deformed and stressed state in subsequent beams. Any of the considered systems with the presented boundary conditions can be converted into an equivalent one by changing the boundary force factors, setting torques or installing a spring with a given stiffness and adjusting the tension of the cables.

Keywords: deflections of beams, method of initial parameters, nonlinear programming, regulation of deflections and internal forces, simulation of weightlessness, de-hanging of beams.

Introduction

The problem of optimal and rational design of structures is relevant in aviation and aerospace engineering [1; 2]. Review, classification and design analysis of solar panels for spacecraft are considered in [3], where solar batteries made of rigid panels, flexible substrate solar panels, inflatable solar arrays, self-expanding solar panels and other structures.

The development of modern flexible solar cell carrier is given in [4]. In [5], the dynamic aspects of weight-free systems for large-sized transformable elements of spacecraft during deployment are considered. Numerous copyright certificates are known for the development of zero-gravity simulation stands, for example [6]. Schemes for cable weight-lifting of structures are common, affecting three-dimensional, two-dimensional and one-dimensional objects, for example, for antenna reflectors, each single-link spoke is considered as a beam, hinged at one end on a fixed base and pulled up by a cable [7].

In [8], a method for calculating the weightlessness of large-sized transformable elements of spacecraft during ground tests is considered using the example of a beam rigidly clamped at the end and suspended on cables. Deformation is not taken into account.

Thus, the weightless elements, in most cases, are considered to be infinitely rigid in bending and are pulled up by cables at the center of gravity so that secondary reactions do not occur at the articulation points.

For purposes of [3–5], we can conclude that it is necessary to master the issues of modeling the weightlessness of a structure, taking into account its deformation. This raises the problem of regulating stresses, deformations and deflections by additional tension (pre-tension) of certain parts of structures, in particular, tension by cables. The regulators will be the tension forces of the cables; the values of which should be determined from the condition that the sum of the squared deflections of the elastic line of the beam (beams) was minimal. As a result, the problem is reduced to a nonlinear programming problem.

The weightlessness factor, as an imitation of weightlessness, is considered as zeroing out any force parameter (reaction or torque). For example, the beam is rigidly clamped and the values of the forces in the cables are found. It is necessary to deweight so that there is no reaction in the support. To do this, we consider the calculation of a beam that has the ability to move in the direction of reaction with a zero angle of rotation. The calculated forces in the cables will ensure the weightlessness of the reaction. For example, we need to eliminate the moment, then we solve the problem with a hinged-fixed support. Comparing the angle of rotation found here with the bending moment under rigid clamping will allow us to select the spring stiffness to ensure equivalence.

The work considers linearly deformable systems. The cables have infinitely high tensile rigidity. The elastic line of the beam contains an infinite number of points, so a discrete problem with a finite number of points on the elastic line is analyzed. A nonlinear programming problem may have local minima. A test is performed to determine whether there is a single optimal plan depending on the choice of variables entered into the basis.

1. Formulation of the problem of minimizing deflections

Let us consider a multi-span beam, consisting of three beams of total length L, as a geometrically variable system (Fig. 1, *a*). To turn this system of beams into a statically determinate one, it is necessary to impose three additional constraints, since this system has three degrees of freedom. Let the beam (Fig. 1, *b*) be acted upon by active concentrated forces $P_1, P_2, ..., P_n$, distributed loads $q_1, q_2, ..., q_k$., and moment M_A. To balance these loads, we will apply de-weighting forces $N_1, N_2, ..., N_S$ to ensure the equilibrium of system. If the number of secondary forces $N_1, N_2, ..., N_S$ is less than the number of degrees of freedom of the beam, then the beam remains a geometrically variable system. If the number of secondary forces $N_1, N_2, ..., N_S$ is equal to the number of degrees of freedom of the beam, then the forces are calculated from the equilibrium equations (we assume that the forces are distributed correctly). In this case, there is nothing to optimize; the equilibrium of the beam and the equilibrium of its parts are satisfied. If the number of secondary forces $N_1, N_2, ..., N_S$ is greater than the number of degrees of freedom of the beam, then it is possible to vary the forces, achieving the required weight-loss parameters, for example, eliminating reactions in joints, reducing deflections at given points, regulating internal forces and stresses. The beam, which initially does not have a sufficient number of support connections, must be in equilibrium under the influence of secondary forces (naturally, there should be no signs of instantaneous variability).

The problem of modeling the deformation of beams suspended on cables with the condition of minimizing the sum of squares of its deflections by varying the tension forces leads to a nonlinear programming problem [9; 10]. Let us express the objective function F in terms of deflections at n points:

$$F(R_A,\theta_A,M_A,N_1,N_2,...,N_S)=\sum_{k=1}^{n}\left[v(z_k)\right]^2 \to \min\,,\; z_k=k(L/n),\; k=1,2,3,...,n\,. \tag{1}$$

Where

$$v(z_k)=f(R_A,\theta_A,M_A,N_1,N_2,...,N_S)\,, \tag{2}$$

the deflection function at a point with a coordinate z_k with unknown parameters $R_A,\theta_A,M_A,N_1,N_2,...,N_S$; k – point number (the distance between points is the same); θ_A – angle of rotation; R_A reaction at point *A* of the beam; M_A – bending moment at point *A*; $N_1,N_2,...,N_S$ – the desired forces.

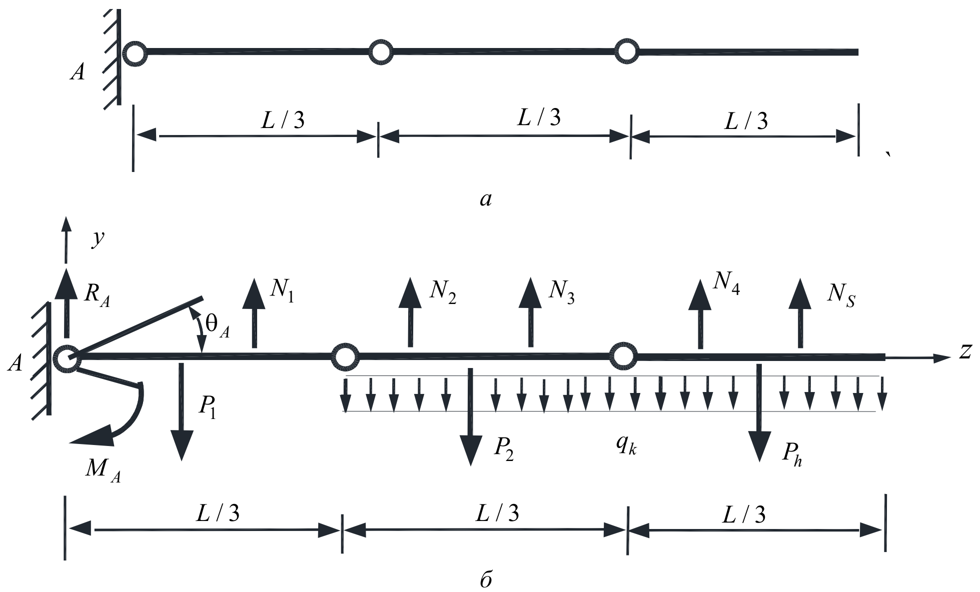


Рис. 1. Модель (расчетная схема) балки с тремя шарнирами:
а – балка как геометрически изменяемая система; *б* – балка с действующими на нее нагрузками

Fig. 1. Model (design scheme) of a beam with three hinges:
a – a beam as a geometrically variable system; *b* – a beam with loads acting on it

As restrictions, equilibrium equations are added to (1): $\sum y=0$ – the sum of the projections of forces on the *y*-axis; $\sum m_i=0$ – the sum of the moments of all forces relative to the point i. The system of restrictions has the form of equalities. From the compiled system of restrictions, we select basic variables, for example, R_A,N_1,N_2, which we substitute into (1). Now the objective function (1) will contain only free variables, i.e. parameters $\theta_A,M_A,...,N_S$:

$$F(\theta_A,M_A,...,N_S)=\sum_{k=1}^{n}\left[v(z_k)\right]^2\,, \tag{3}$$

which allows solving the optimization problem without restrictions [11]. Partial derivatives (3) with respect to free parameters (the parameters are often called coordinates):

$$\frac{\partial F}{\partial \theta_A} = 0\,,\ \frac{\partial F}{\partial M_A} = 0\,,\ \dots\,,\ \frac{\partial F}{\partial N_S} = 0 \tag{4}$$

give a system of linear algebraic equations for the required parameters that determine the optimal plan for calculating deflections (2) at discrete points.

Derivatives of the deflection function (2)

$$\theta(z) = dv(z)/dz\,;\ M(z) = -EJ\,d^2v(z)/dz^2\,;\ Q(z) = dM(z)/dz\,;\ q(z) = dQ(z)/dz \tag{5}$$

give functions of rotation angle, bending moment, shear force and load $q = q(z)$.

The basic solution of nonlinear optimization problems may have local minima, so the work will show convergence to the optimal plan for various combinations of basis and free variables. To calculate deflections (2), the method of initial parameters was used as a direct method of integrating a fourth order differential equation with discontinuous functions [12]. Derivatives of functions are calculated numerically [13].

Beams can be not only geometrically variable, as shown in Fig. 1, but also without supports, i.e. movable, which contradicts the concepts of kinematic analysis of structural mechanics [14]. However, the balance of the beams is achieved by tensioning the cables (Fig. 2). In Fig. 2, *a* we will show the design diagram of a beam without support connections with an active moment *M*. Here, equilibrium is ensured by a pair of equal forces. Let us present a design diagram of a composite beam consisting of four beams (Fig. 2, *b*). Having imagined the floor diagram of beams, we can see that the main beam is statically indeterminate; the secondary beam, statically determinate, rests on it. Both right beams, the third and fourth one, are supported by forces $N_4 - N_7$.

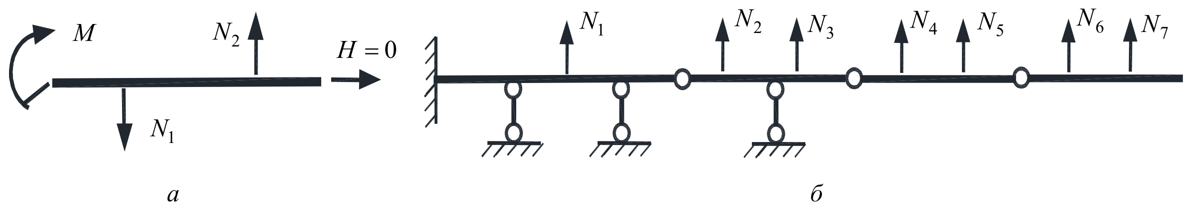


Рис. 2. Расчетные схемы балок:
а – опорных связей нет – равновесие поддерживается силами N_1, N_2;
б – составная балка – равновесие поддерживается реакциями опор и силами $N_1 - N_7$

Fig. 2. Design schemes of beams:
a – there are no support links – the equilibrium is maintained by forces;
b – composite beam – the balance is maintained by the reactions of the supports and forces $N_1 - N_7$

It should be noted that to calculate deflections (2) you can use the variational-difference method in the form of the finite difference method and FEM. However, the first method requires introducing additional contour nodes at all hinge joints, which complicates programming, and the second method requires defining deflections and rotation angles at all discrete points (we shall call them nodes), which increases the dimension of the problem. The method of initial parameters taking into account intermediate hinges requires calculating only deflections at discrete points, and at the joint nodes of beams the deflection and increment of the rotation angle are calculated.

Let us consider examples of beam calculations that show the unity of the optimal solution for a non-linear programming problem depending on the choice of variables introduced into the basis, and the convergence of solutions from the assigned number of points at which the deflection is calculated.

2. Optimization of deflections of the beam hinged-fixed and suspended on two cables

We consider an I-beam hinged at one end and supported by two cables (Fig. 3). The cables have infinitely high tensile rigidity. In Fig. 2 their action on the beam is shown by the forces and N_1 и N_2.

Initially, the beam is geometrically variable. Let us add balancing forces and create an objective function

$$F(R_A, \theta_A, N_1, N_2, q) = \sum_{k=1}^{n} \left[v(z_k) \right]^2 \to \min, \quad n = 24. \tag{6}$$

Let us write the deflection function based on the initial parameters method

$$v(z) = \theta_A z + \frac{1}{EJ}\left[\frac{R_A z^3}{3!} + \frac{N_1 (z - L/3)^3}{3!} H(z - L/3) + \frac{N_2 (z - 2L/3)^3}{3!} H(z - 2L/3) - \frac{q z^4}{4!} \right]. \tag{7}$$

Where $v(z)$ – deflection at the point z; θ_A – angle of rotation at the beginning of the beam (initial parameter at point *A*); R_A – reaction at point *A*; N_1 – force in the first cable; N_2 – force in the second cable (required parameters); q – uniformly distributed linear load; E – Young's modulus; J – axial moment of inertia of the beam cross-section; $H()$ – Heaviside function.

Beam length L = 6 m; rolled I-beam profile, hinged-fixed fastening at a point *A*; Young's modulus of the material $E = 2 \cdot 10^{11}$ Pa; axial moment of inertia in the bending plane $J = 200 \cdot 10^{-8}$ m^4; linear weight of the beam q = 100 n/m.

2.1. Checking solutions by changing basic variables

Let us consider solutions related to the peculiarities of beam bending as a discrete problem with a finite number of points at which deflections are calculated. We will search for the global minimum of the objective function.

We check that for different basic variables introduced into the objective function, there must be a single optimal solution.

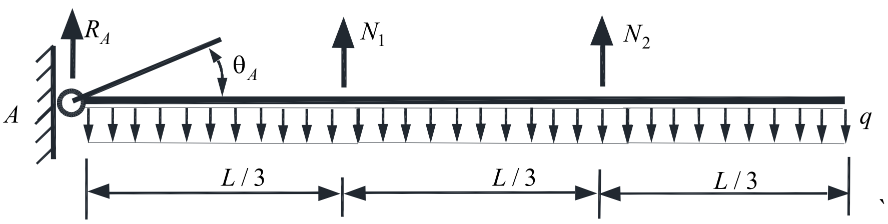


Рис. 3. Балка, прикрепленная шарниром и поддерживаемая двумя тросами

Fig. 3. A beam attached by a hinge and supported by two cables

As restrictions, we add equilibrium equations to the objective function (6)

$$N_1(L/3) + N_2(2L/3) - qL^2/3 = 0; \tag{8}$$

$$R_A + N_1 + N_2 - qL = 0. \tag{9}$$

Let us consider three options for applying basic variables.

Variant 1. We introduce variables into the basis N_1 and N_2. To do this, we obtain the forces from (8) and (9)

$$N_1 = qL/2 - 2R_A, \tag{10}$$

$$N_2 = qL/2 + R_A, \tag{11}$$

which, substituted into (6), give the following nonlinear programming problem:

$$F = F(R_A, \theta_A, q) \to \min \tag{12}$$

without a system of restrictions.

Derivatives with respect to free variables

$$\partial F / \partial R_A = 0, \quad \partial F / \partial \theta_A = 0$$

allow you to calculate the angle of rotation θ_A and reaction R_A. All parameters for calculating function (7) are defined.

Variant 2. Let us take N_1 and R_A as basic variables. Thereafter from (8) and (9) we derive the following

$$N_1 = 3qL/2 - 2N_2, \tag{13}$$

$$R_A = N_2 - qL/2, \tag{14}$$

which, when substituted into (6), give the search problem

$$F = F(N_2, \theta_A, q) \to \min. \tag{15}$$

Derivatives with respect to free variables

$$\partial F / \partial N_2 = 0, \quad \partial F / \partial \theta_A = 0$$

make it possible to calculate the angle of rotation θ_A and the tension force N_2. We calculate function (7).

Variant 3. As basic variables we take N_2 and R_A. Thereafter from (8) and (9) we obtain

$$N_2 = 3qL/4 - N_1/2, \tag{16}$$

$$R_A = qL/4 - N_1/2, \tag{17}$$

which, substituted into (6), give the objective function

$$F = F(N_1, \theta_A, q) \to \min. \tag{18}$$

Derivatives with respect to variables N_1 and θ_A

$$\partial F / \partial N_1 = 0, \quad \partial F / \partial \theta_A = 0$$

allow you to calculate the angle of rotation θ_A and tension force N_1. Function (7) is defined

In all three variants of the basic variables, we obtained exactly the same required parameters R_A, θ_A, N_1, N_2. The found parameters give solutions (diagrams), which we present in Fig. 4: diagram of deflections (Fig. 4, *a*); diagram of rotation angles (Fig. 4, *b*); diagram of bending moments (Fig. 4, *c*); diagram of transverse forces (Fig. 4, *d*). Fourth derivative of a function deflection $v(z)$ along the z coordinate gives a diagram $q(z) = -100$ кн/м $=$ const, which is a check (Fig. 4, *e*).

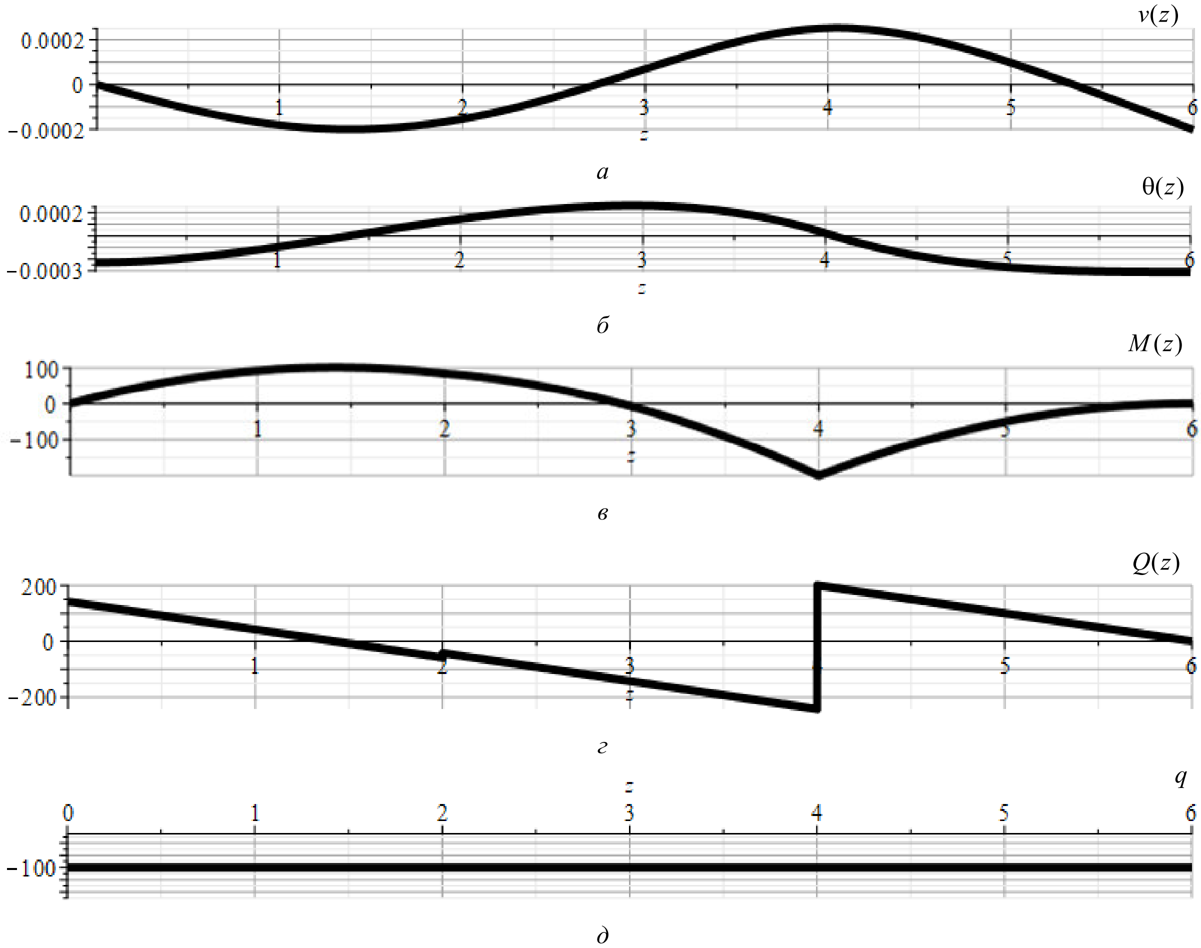


Рис. 4. Функции деформационных и внутренних силовых факторов:
а – прогиб; *б* – эпюра углов поворота; *в* – эпюра изгибающего момента; *г* – эпюра поперечных сил;
д – эпюра нагрузки $q = -100$ н/м

Fig. 4. Functions of deformation and internal force factors:
a – deflection; *b* – plot of rotation angles; *c* – the bending moment plot; *d* – the transverse forces plot;
e – the load plot $q = -100$ N/м

Thus, changing the basis did not affect the solution of the problem. In all three variants, calculating the programming parameters came to the single optimal solution.

2.2. Study of the convergence of solutions depending on the choice of the number of points in which deflections are calculated

A discrete problem with a finite number of points on the elastic line of a beam is considered. Initial data as in paragraph 2.1. The objective function is calculated using formula (6), and deflections are calculated using formula (7).

We check if the objective function is quadratic, the number of nodes on the elastic line of the beam is finite, and the method for calculating deflections is accurate, then convergence of deflections, rotation angles, and bending moments is expected from an increase in the number of discretization nodes.

We perform a numerical experiment for n = 6, n = 12, n = 24. The calculation results (diagrams) are shown in Fig. 5 and in Table 1.

In Fig. 5, *a* the deflection diagrams are shown. We find the following convergence of deflections. For example, at the end of the beam ($z = L$), the deflection increased from 0.2025 mm on a grid $n = 6$ to 0.2597 mm on a grid $n = 12$, which is 28%. Further, on the $n = 24$ grid, the deflection changed to a value of 0.2935 mm. Compared to a grid of $n = 12$, this is 13%. The next refinement of the grid should be a difference from the previous one of about 6%. Extrapolation gives the expected deflection of the beam end equal to 0.311 mm.

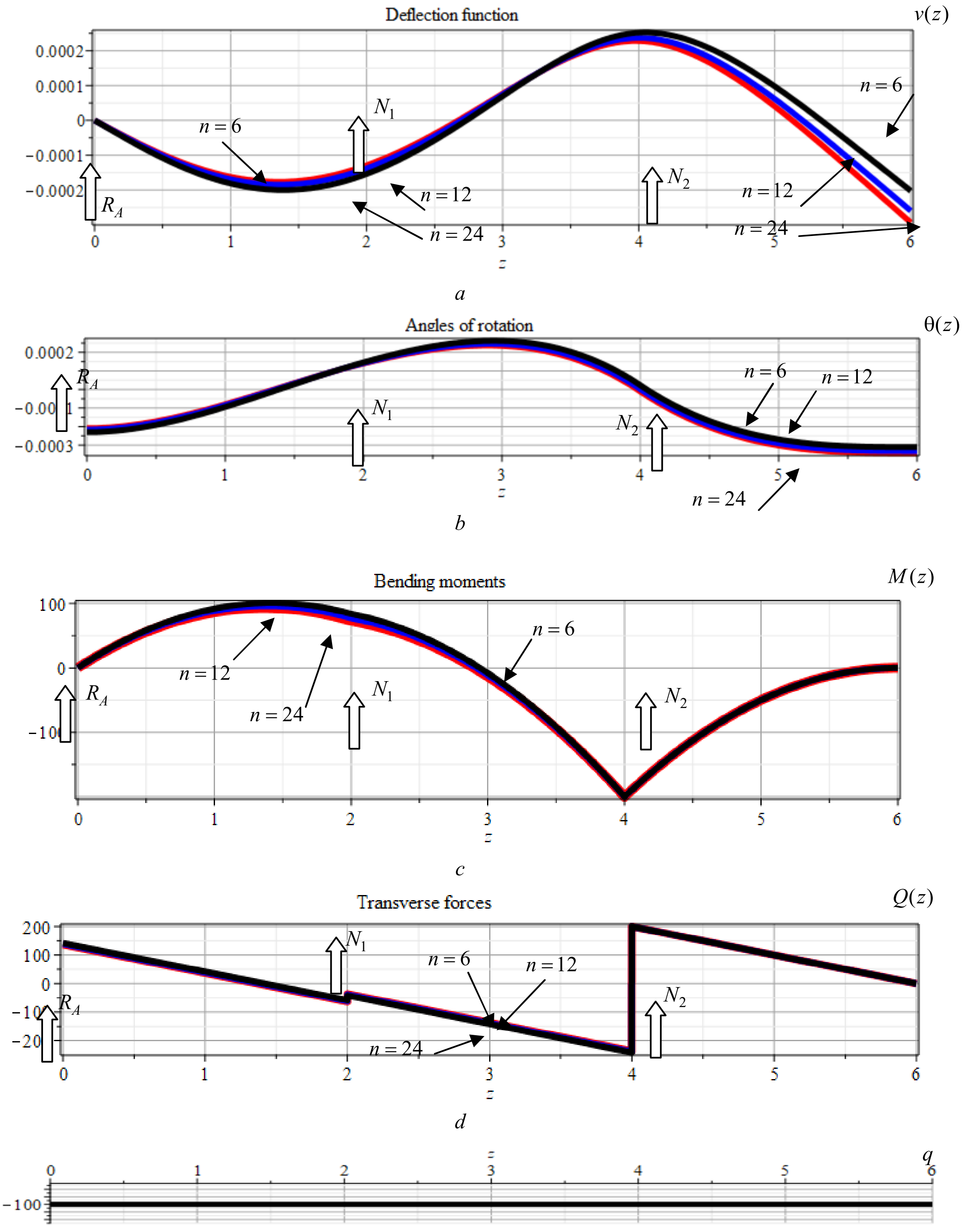


Рис. 5. Оптимальные параметры балки для сеток $n = 6$, $n = 12$ и $n = 24$:
а – функции прогибов; *б* – функции углов поворота; *в* – функции изгибающих моментов; *г* – функции поперечных сил; *д* – заданная распределенная нагрузка (вычислена, как четвертая производная функции прогиба)

Fig. 5. Optimal beam parameters for grids $n = 6$, $n = 12$ and $n = 24$:
a – deflection functions; *b* – rotation angle functions; *c* – bending moment functions; *d* – transverse force functions; *e* – a given distributed load (calculated as the fourth derivative of the deflection function)

We calculate the average values of the sum of squared deflections [15] using the formula

$$\delta_n = \sqrt{\sum_{k=1}^{n}\left[v(z_k)\right]^2 / n}\ , n = 6, n = 12, n = 24. \qquad (19)$$

The values are: $\delta_6 = 0{,}171$ мм, $\delta_{12} = 0{,}159$ мм, $\delta_{24} = 0{,}152$ мм. The difference is 7,4 % and 4,3 % respectively.

Fig. 5, *b* shows diagrams of the rotation angles of the beam sections, which coincided essentially.

The second derivatives of the deflection functions, that are bending moments (Fig. 5, *c*) calculated on different grids, are practically indistinguishable.

Diagrams of transverse forces (Fig. 5, *d*) coincide completely and they are represented by one broken line having breaks at the points of application of forces

Fig. 5, *d* shows the distributed load, calculated as the fourth derivative of the deflection function. This is a solution check. On all three diagrams we got the specified value $q = -100$ n/m.

To conclude this section, one could say that, based on the change in deflections depending on the grid refinement, grids from n = 12 to n = 24 are sufficient. If the prerogative is the need to calculate stresses, it is enough to assign the grid n = 6

We present the calculated variables in Table 1. Rows 1–3 of the table show the parameters for the grids $n = 6$, $n = 12$, $n = 24$. Lines 4–5 show the relative difference of the desired parameters. The following are written sequentially in the columns: $v(n)$ – deflections of the beam console at the point $z = L$; $\theta_A(n)$ – initial parameter, angle of rotation of the section; $R_A(n)$ – initial parameter, reaction at the beginning of the beam; $N_1(n)$ – calculated parameter, the first force; $N_2(n)$ – calculated parameter, the second force; $\delta(n)$ – root-mean-square deflection of the beam end.

Table 1

Optimal design parameters

Row number	Number of points	$v(n)$	$\theta_A(n)$	$R_A(n)$	$N_1(n)$	$N_2(n)$	$\delta(n)$
		m	rad	n	n	n	m
1	n = 6	–0.0002025	–0.00023013	141.956	16.087	441.956	0.00017097
2	n = 12	–0.0002597	–0.00021639	138.464	23.071	438.464	0.00015915
3	n = 24	–0.0002935	–0.00020861	136.454	27.091	436.454	0.00015247
	Percentage difference:						
4	n = 12/ n = 6	28 %	6.3 %	2.5 %	43 %	0.8 %	7.4 %
5	n = 24 / n =12	13 %	3.7 %	1.5 %	17.4 %	0.5 %	4.3 %

Note that the data in the rows 2 and 3 differ little compared to the parameters in the rows 1 and 2. Rows 4 and 5 can be understood as the rate of convergence of the desired parameters.

3. Suspension by cables of the system of three beams connected by hinges

Let us consider a system of I-beams hinged to each other (Fig. 6). Under gravity conditions, the system has its own linear weight (Fig. 6, *a*). To simulate weightlessness, we will consider four cases of fixing the system at point *A* and supporting it with six cables. The system is rigidly pinched at point *A* (Fig. 6, *b*); hinged-fixed support (Fig. 6, *c*), sliding embedding (Fig. 6, *d*); free edge (Fig. 6, *d*).

It is necessary to determine the tension forces of the cables so that the sum of the squares of the deflections at given points is minimal, i.e.

$$F = \sum_{k=1}^{n}\left[v(z_k)\right]^2 \to \min,\ z_k = k(L/6),\quad k = 1,2,3,...,18. \qquad (20)$$

In (20) F – the objective function; $v(z_k)$ – desired deflection; z_k – point coordinate; k – point number; $n = 18$ – amount of points; the lengths of all beams are the same and equal to L.

3.1. Rigid

To calculate deflections **restraint** using the initial parameters method with rigid clamping of the beam at point *A* (Fig. 6, *b*), the deflection function is written as:

$$v(z)=\theta_1(z-L)H(z-L)+\theta_2(z-2L)H(z-2L)+$$

$$+\frac{1}{6EJ}\left[3M_Az^2+R_Az^3+N_1\left(z-\frac{L}{3}\right)^3H\left(z-\frac{L}{3}\right)+N_2\left(z-\frac{2L}{3}\right)^3H\left(z-\frac{2L}{3}\right)+N_3\left(z-\frac{4L}{3}\right)^3H\left(z-\frac{4L}{3}\right)+\right.$$

$$+N_4\left(z-\frac{5L}{3}\right)^3H\left(z-\frac{5L}{3}\right)+N_5\left(z-\frac{7L}{3}\right)^3H\left(z-\frac{7L}{3}\right)+$$

$$\left.+N_6\left(z-\frac{8L}{3}\right)^3H\left(z-\frac{8L}{3}\right)-\frac{qz^4}{4}\right],z\in[0,3L]. \tag{21}$$

Where θ_1 – the rotation angle (initial parameter at point *B*); θ_2 – angle of rotation (initial parameter at point *C*); M_A – fixing moment (point *A*); R_A – reaction acting on the beam at point *A*; $N_i,(i=1-6)$ – forces in the cables (required parameters); $q=\text{const}$ – uniformly distributed linear load; E – Young's modulus; J – axial moment of inertia of the beam cross-section; $H()$ – Heaviside function.

Formula (21) gives the parameters θ_1 and θ_2, which are the angles of rotation of the beams adjacent to the intermediate hinges [16], i.e. $\theta_i=\theta_i^{\text{справа}}-\theta_i^{\text{слева}}$, $i=1,2$. $\theta_i^{\text{справа}}$ – the angle of rotation at the point i on the right («справа») of the hinge, and $\theta_i^{\text{слева}}$ – the angle of rotation at the point i on the left («слева») of the hinge. The rotation angles θ_i are additional unknown variables similar to initial parameters v_A and θ_A.

To equations (21) we add the equilibrium equations

$$\Sigma m_C^{\text{справа}}=0,\ \Sigma m_B^{\text{справа}}=0,\ \Sigma m_A^{\text{справа}}=0, \tag{22}$$

representing the set of constraints. From (22) we select the basic variables N_4, N_5, N_6

$$\begin{Bmatrix}1\\0\\0\end{Bmatrix}N_4+\begin{Bmatrix}0\\1\\0\end{Bmatrix}N_5+\begin{Bmatrix}0\\0\\1\end{Bmatrix}N_6=\begin{Bmatrix}qL/2\\2qL-N_3/3\\9qL/2-N_1/3-2N_2/3-4N_3/3\end{Bmatrix}, \tag{23}$$

which we add to the objective function (20). Next observe that $F=F(M_A,\theta_1,\theta_2,N_1,N_2,N_3)$. The equilibrium equation $\Sigma y=0$ – redundant and serves to check solutions. Partial derivatives F with respect to coordinates $M_A,\theta_1,\theta_2,N_1,N_2,N_3$ give a system of six equations regarding the moment, two angles of rotation and three forces in the cables (note the symmetry of the matrix of the system of equations). The problem was reduced to a quadratic programming problem without restrictions. The minimum objective function (20) determined the optimal solution.

The found parameters $M_A,\theta_1,\theta_2,N_1,N_2,N_3$, substituted into equations (21), give according to formulas (5) the functions of deflections, rotation angles, bending moment and shear force, respectively. Fig. 7–10 show the functions for rigid embedding; solutions for a hinged-fixed edge are considered; diagrams for sliding embedding are shown; functions for the free edge at point *A* of the beam system are considered.

3.2. Hinged-fixed support

For the system of beams with hinged support at point *A* (Fig. 6, *c*), the formula for the initial parameters method is as follows:

$$v(z) = \theta_A z + \theta_1 (z-L)H(z-L) + \theta_2 (z-2L)H(z-2L) +$$

$$+\frac{1}{6EJ}\left[R_A z^3 + N_1\left(z-\frac{L}{3}\right)^3 H\left(z-\frac{L}{3}\right) + N_2\left(z-\frac{2L}{3}\right)^3 H\left(z-\frac{2L}{3}\right) + N_3\left(z-\frac{4L}{3}\right)^3 H\left(z-\frac{4L}{3}\right)+\right.$$

$$\left.+N_4\left(z-\frac{5L}{3}\right)^3 H\left(z-\frac{5L}{3}\right) + N_5\left(z-\frac{7L}{3}\right)^3 H\left(z-\frac{7L}{3}\right) + N_6\left(z-\frac{8L}{3}\right)^3 H\left(z-\frac{8L}{3}\right) - \frac{qz^4}{4}\right], z \in [0,3L]. \quad (24)$$

Where θ_A – angle of rotation at point A.

Additional equations (22) allow you to introduce variables N_4, N_5, N_6, into the basis, as in (23). The reaction for formula (24) is determined from the equilibrium equation $\Sigma y = 0$. The functions of deflections, rotation angles, bending moment, and shear force are shown in Fig. 7–10.

3.3. Sliding sealing

We consider the sliding sealing at point *A*, a fragment of which is shown in Fig. 6, *d*. The following equilibrium equations are compiled:

$$\Sigma m_C^{\text{справа}} = 0,\ \Sigma m_B^{\text{справа}} = 0,\ \Sigma y = 0. \quad (25)$$

We introduce vectors for the variables N_4, N_5, N_6, into the basis, and after calculating the derivatives with respect to free variables from the equilibrium equation $\Sigma m_A = 0$, we express the deflection functions M_A.

Now, the deflection function is the following:

$$v(z) = v_A + \theta_1 (z-L)H(z-L) + \theta_2 (z-2L)H(z-2L) +$$

$$+\frac{1}{6EJ}\left[3M_A z^2 + N_1\left(z-\frac{L}{3}\right)^3 H\left(z-\frac{L}{3}\right) + N_2\left(z-\frac{2L}{3}\right)^3 H\left(z-\frac{2L}{3}\right) + N_3\left(z-\frac{4L}{3}\right)^3 H\left(z-\frac{4L}{3}\right)+\right.$$

$$\left.+N_4\left(z-\frac{5L}{3}\right)^3 H\left(z-\frac{5L}{3}\right) + N_5\left(z-\frac{7L}{3}\right)^3 H\left(z-\frac{7L}{3}\right) + N_6\left(z-\frac{8L}{3}\right)^3 H\left(z-\frac{8L}{3}\right) - \frac{qz^4}{4}\right], z \in [0,3L]. \quad (26)$$

All solutions are presented in Fig. 7–10.

3.4. Free edge

In the case of a free edge, there are no connections at point *A* of the beam (Fig. 6, *d*), there are 10 unknown parameters. Then observe that $v(z) = f(v_A, \theta_A, \theta_1, \theta_2, N_1, N_2, ..., N_6)$. Each beam disk in a plane has three degrees of freedom. There are 9 of them in total. Each simple hinge takes away two degrees of freedom; 4 in total are eliminated. For a system of three beams, 5 degrees of freedom remain. To turn the beam system under consideration into a geometrically unchangeable one, 5 connections should be added. One degree of freedom is subtracted, since there is no displacement in the direction of the longitudinal force. You can write 4 equilibrium equations, but they are linearly dependent.

We compose three equilibrium equations. The deflection function will look like this:

$$v(z) = v_A + \theta_A z + \theta_1 (z-L)H(z-L) + \theta_2 (z-2L)H(z-2L) +$$

$$+\frac{1}{6EJ}\left[N_1\left(z-\frac{L}{3}\right)^3 H\left(z-\frac{L}{3}\right)+N_2\left(z-\frac{2L}{3}\right)^3 H\left(z-\frac{2L}{3}\right)+N_3\left(z-\frac{4L}{3}\right)^3 H\left(z-\frac{4L}{3}\right)+\right.$$

$$\left.+N_4\left(z-\frac{5L}{3}\right)^3 H\left(z-\frac{5L}{3}\right)+N_5\left(z-\frac{7L}{3}\right)^3 H\left(z-\frac{7L}{3}\right)+N_6\left(z-\frac{8L}{3}\right)^3 H\left(z-\frac{8L}{3}\right)-\frac{qz^4}{4}\right], z\in[0,3L]. \quad (27)$$

We display the solutions in Fig. 7–10.

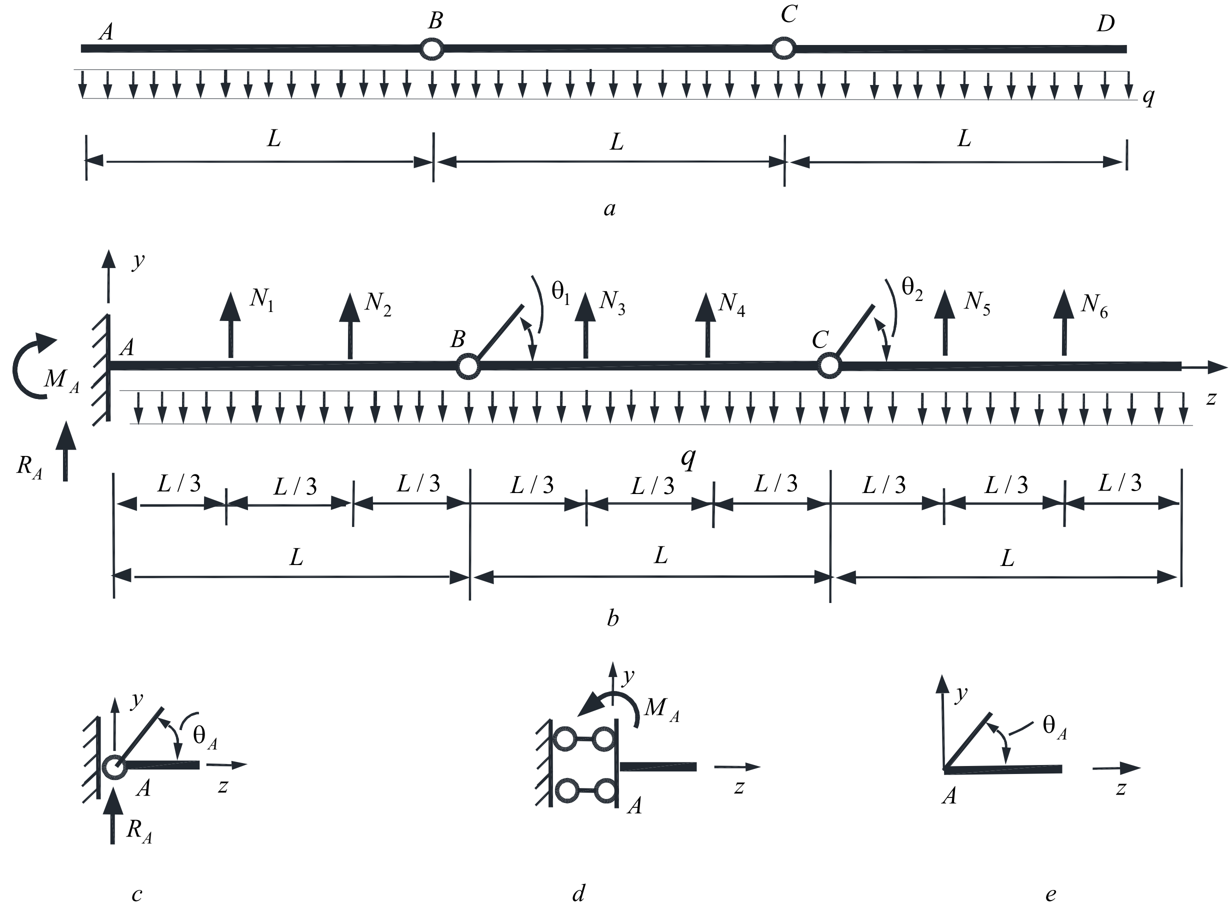


Рис. 6. Система трех балок, соединенных шарнирами:
а – система в условиях гравитации; *б* – моделирование невесомости системы жестким защемление в точке *A*; *в* – фрагмент шарнирно-неподвижного опирания системы балок в точке *A*; *г* – фрагмент скользящей заделки системы балок в точке *A*; *д* – фрагмент свободного края системы балок в точке *A*

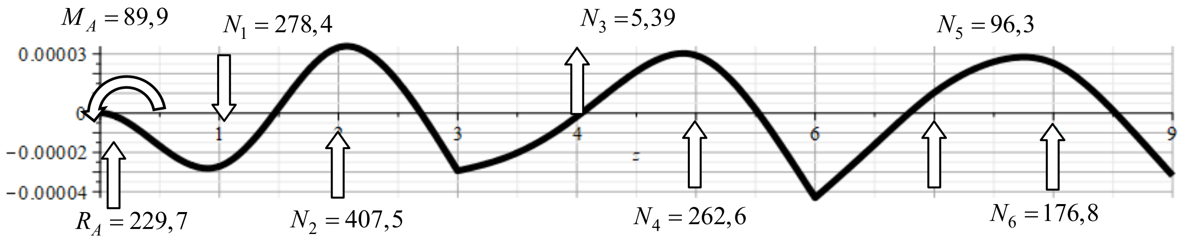
Fig. 6. A system of three beams connected by hinges:
a – the system under gravity conditions; *b* – modeling of the weightlessness of the system by rigid pinching at point A; *c* – a fragment of the hinge-fixed support of the beam system at point A; *d* – a fragment of the sliding sealing of the beam system at point A; *e* – a fragment of the free edge of the beam system at point A

In table 2 we present the tension forces of the cables for a system of beams with four types of boundary conditions. We compare the tension forces for a rigidly clamped and hingedly supported beam. On the first beam, the tension forces of the cables are equal: $N_1^{\text{заделка}}=-278{,}4$ н («заделка» – sealing), $N_1^{\text{шарнир}}=-124{,}02$ н («шарнир» – hinge) – the difference is 220%; $N_2^{\text{заделка}}=407{,}5$ н, $N_2^{\text{шарнир}}=336{,}07$ н – the difference is 21%. On the second beam, the greatest tensile forces are: $N_4^{\text{заделка}}=262{,}6$ н, $N_4^{\text{шарнир}}=251{,}9$ н – the difference is 4.2%. On the third beam, the tension forces of the cables are almost equal: $N_5^{\text{заделка}}=96{,}3$ н, $N_5^{\text{шарнир}}=100{,}7$ н – the difference is 4.5 %. In the extreme cable: $N_6^{\text{шарнир}}=174{,}6$ Н and $N_6^{\text{заделка}}=176{,}8$ Н – the difference is 0.13 %.

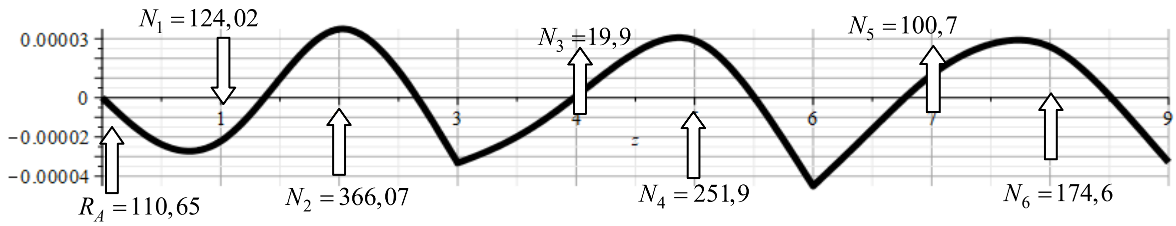
The type of fastening (hinged or sealing) affects the first beam from fastening. The tension forces of the cables, to a certain extent, level out the deformed and a tense state. In subsequent beams, the tension forces of the cables are practically the same, i.e., they do not depend on the type of fastening of the first beam. The same conclusion can be drawn for beams with sliding sealing.

Models of three beam systems when simulating weightlessness are equivalent to a certain extent. So, for example, if a model of a beam system with hinged support at point A is given a moment (for example, by a mover) equal to $89{,}9\ \text{н}\cdot\text{м}$ or a spring is set to stiffness $G_A = M_A / \theta_A = 89{,}9\ \text{нм} / 5{,}78\cdot 10^{-5}\,\text{рад} = 1{,}5\ \text{кн/рад}$, we obtain a model of deformation with rigid pinching. Accordingly, it is necessary to change the tension forces of the cables.

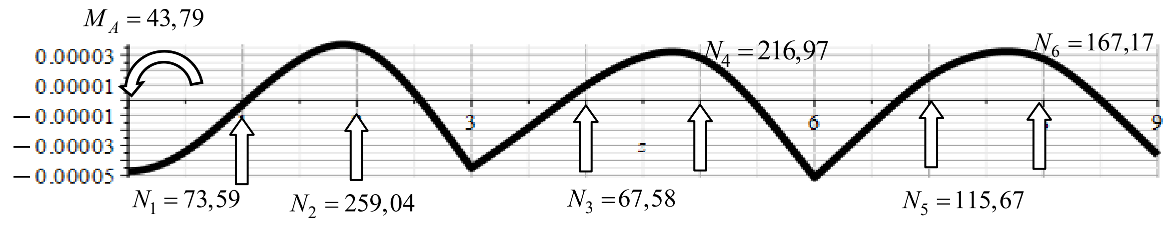
In table 3, to the cable tension forces from table 2, rotation angles were added at the beginning of the beam system (point A) and additional rotation angles at the hinges, root-mean-square deflections and maximum deflections. When moving from rigid fastening to hinged fastening, then from sliding fastening to the free edge, the rigidity of the system decreases. The root mean square deflections increase: $\delta_{\text{заделка}} = 2{,}301\cdot 10^{-5}$ м («заделка» – sealing); $\delta_{\text{шарнир}} = 2{,}396\cdot 10^{-5}$ м («шарнир» – hinge); $\delta_{\text{скользящая}} = 2{,}741\cdot 10^{-5}$ м («скользящая» – sliding); $\delta_{\text{свободный край}} = 3{,}282\cdot 10^{-5}$ м («свободный край» – free edge), i.e. by 4.1, 14.4 and 19.7%. Maximum deflections in the first three cases occur at the point where the hinge connecting the second and third beams, respectively, is installed, $(4{,}31 \div 4{,}5 \div 5{,}14)\cdot 10^{-5}$ м and in the middle of the system of beams with a free edge – $6{,}07\cdot 10^{-5}$ м.



а



б



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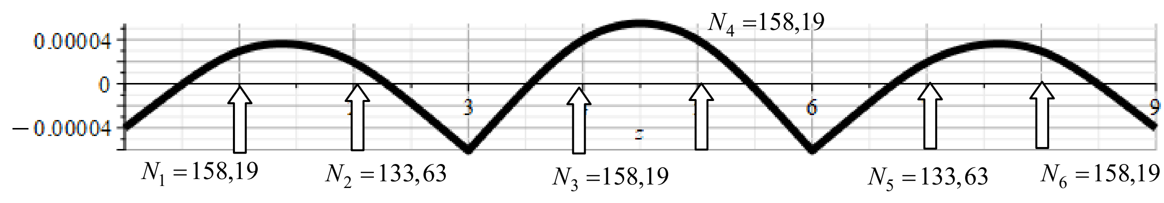


г

Рис. 7. Функции прогибов и силы натяжения тросов для граничных условий слева:
а – жесткой заделки; *б* – шарнирного опирания; *в* – скользящей заделки; *г* – свободного края

Fig. 7. Functions of deflections and tension forces of cables for boundary conditions on the left:
a – rigid sealing; *b* – hinged support; *c* – sliding sealing; *d* – free edge

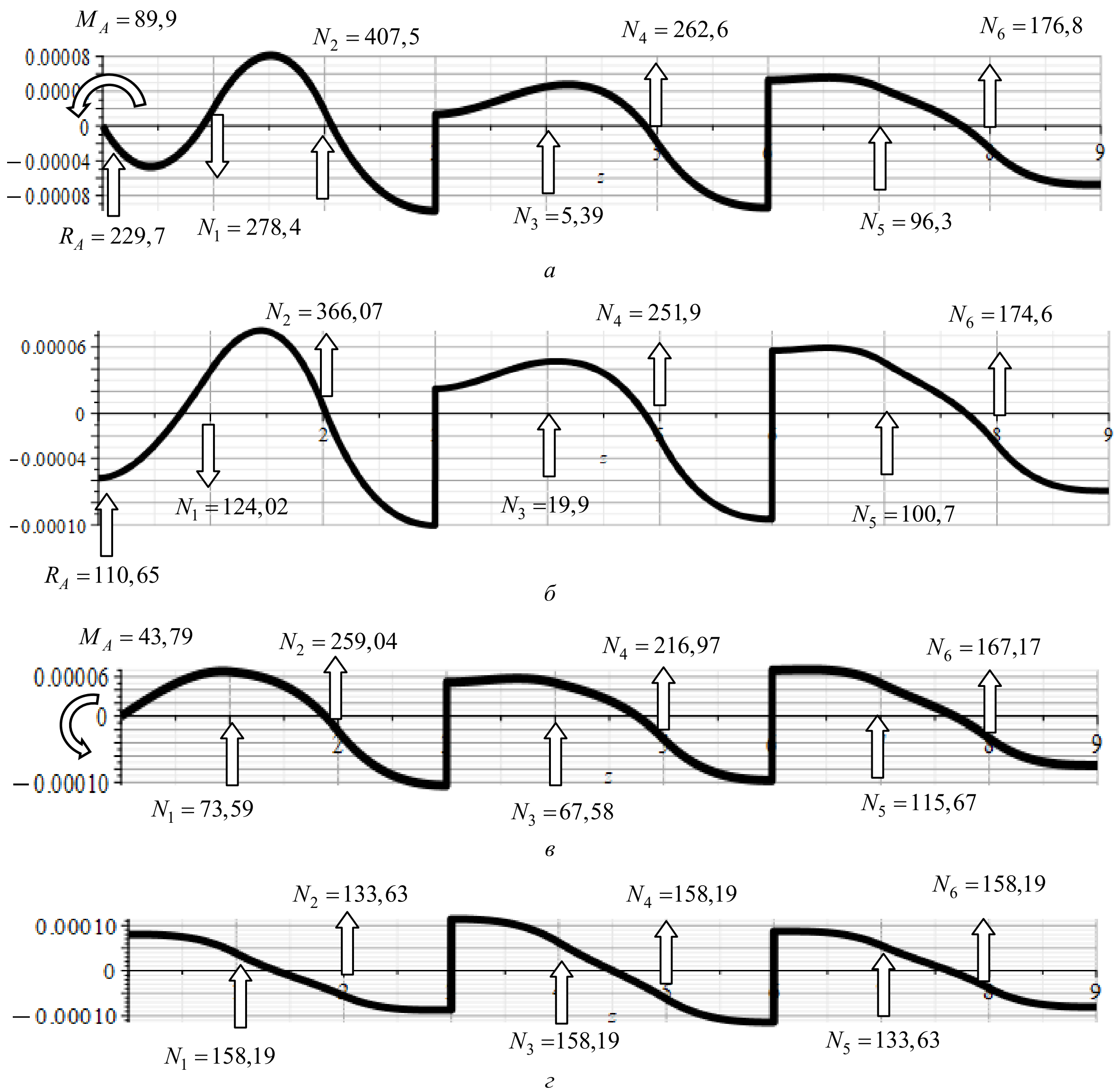


Рис. 8. Функции углов поворота и силы натяжения тросов для граничных условий слева:
а – жесткой заделки; *б* – шарнирного опирания; *в* – скользящей заделки; *г* – свободного края

Fig. 8. Functions of rotation angles and cable tension force for boundary conditions on the left:
a – rigid sealing; *b* – hinged support; *c* – sliding sealing; *d* – free edge

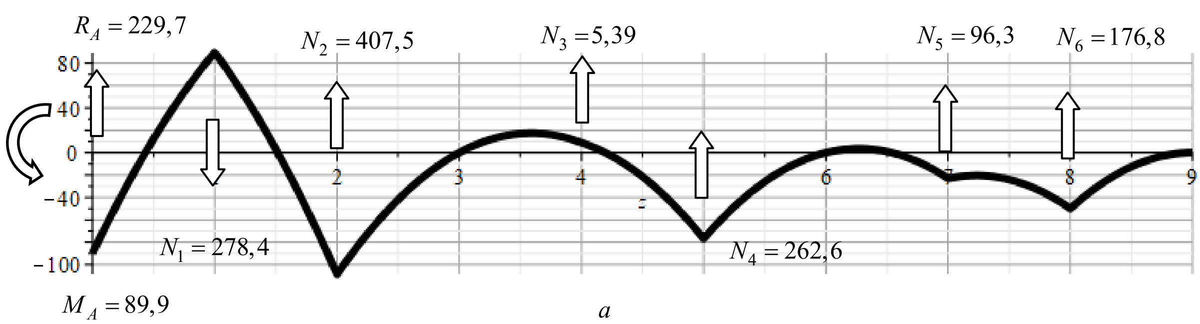
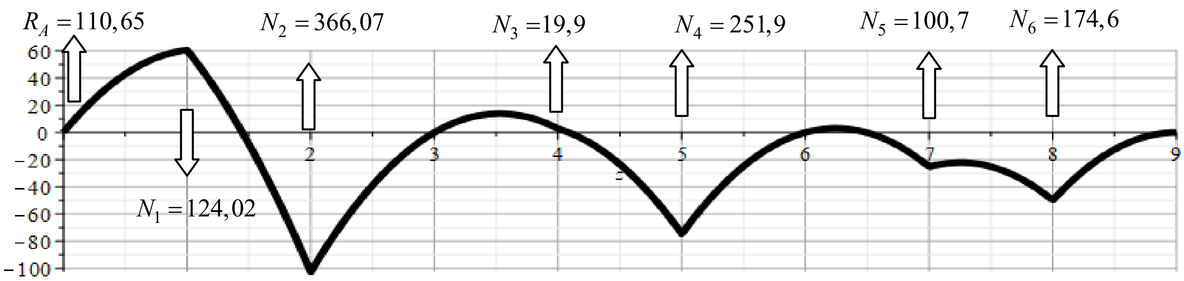
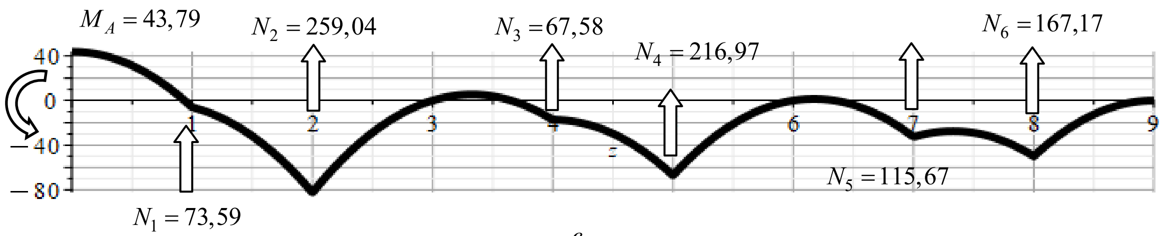


Рис. 9. Функции изгибающих моментов и силы натяжения тросов для граничных условий слева:
а – жесткой заделки; *б* – шарнирного опирания; *в* – скользящей заделки; *г* – свободного края (Начало)

Fig. 9. Functions of bending moments and tension forces of cables for boundary conditions on the left:
a – rigid sealing; *b* – hinged support; *c* – sliding sealing; *d* – free edge (The beginning)



б



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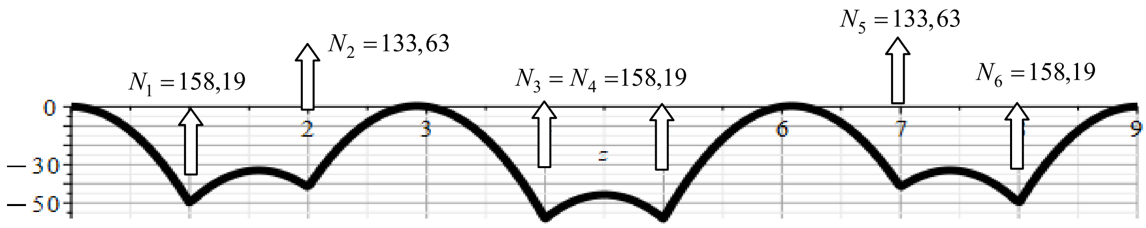
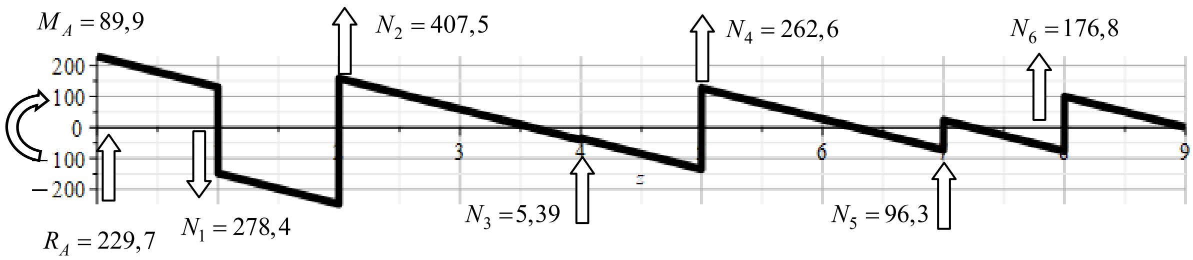


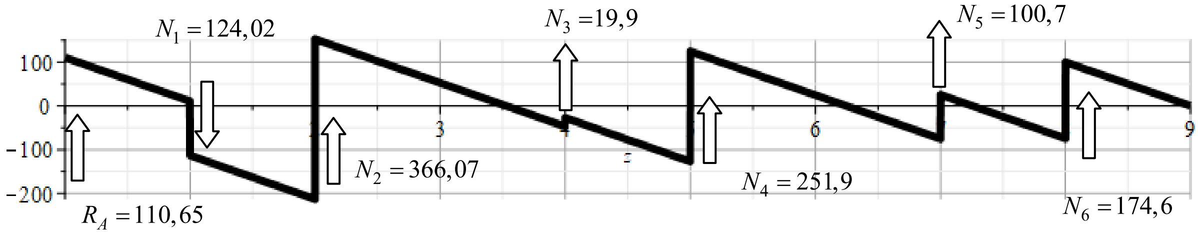
г

Рис. 9. Окончание

Fig. 9. The ending



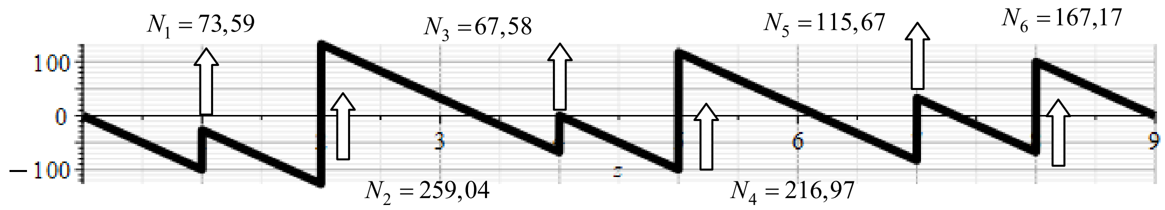
а



б

Рис. 10. Функции перерезывающих сил и силы натяжения тросов для граничных условий слева: *а* – жесткой заделки; *б* – шарнирного опирания; *в* – скользящей заделки; *г* – свободного края (Начало)

Fig. 10. Functions of shearing forces and cable tension forces for boundary conditions on the left: *a* – rigid sealing; *b* – hinged support; *c* – sliding sealing; *d* – free edge (The beginning)



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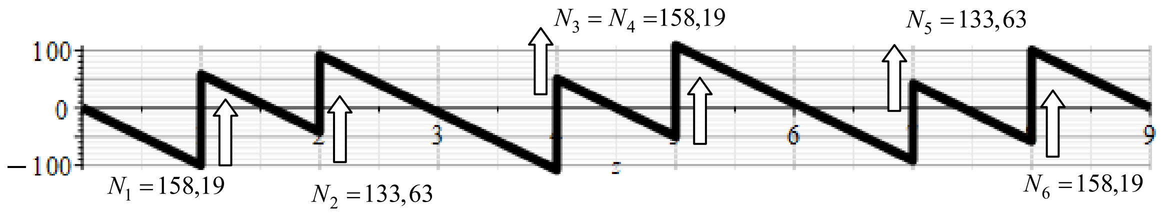

г

Рис. 10. Окончание

Fig. 10. The ending

Table 2

Cable tension forces

Row numbers	Type of fastening at point A	*N1*	*N2*	*N3*	*N4*	*N5*	*N6*
		n	n	n	n	n	n
1	sealing	–278.4	407.5	5.39	262.6	96.3	176.8
2	hinged support	–124.02	336.07	19.9	251.9	100.7	174.6
3	sliding sealing	73.59	259.04	67.58	216.97	115.67	167.17
4	free edge	158.19	133.63	158.19	158.19	133.63	158.19

Table 3

Reaction forces and geometric characteristics of the system

Row numbers	Type of fastening at point A	M_A	R_A	θ_A	θ_1	θ_2	δ	$v(z)$
		n·m	n	10^{-5} rad	10^{-5} rad	10^{-5} rad	10^{-5} m	10^{-5} m
1	sealing	–89.9	229.7	0	11,1	14.7	2.301	$v(6\ \text{м}) = 4{,}31$
2	hinged support	0	110.65	5.78	12.3	15.2	2.396	$v(6\ \text{м}) = 4{,}50$
3	sliding sealing	–43.79	0	0	15.6	16.7	2.741	$v(6\ \text{м}) = 5{,}14$
4	free edge	0	0	8.01	–20.1	20.1	3.282	$v(4{,}5\ \text{м}) = 6{,}07$

The dimensions of physical quantities are described according to [17].

Conclusion

In a quadratic programming problem, given different basis variables introduced into the objective function, there is a single optimal solution. Changing the basis does not affect the solution of the problem.

Analysis of the solution to discrete problems (with a finite number of points on the elastic line of beams) showed that grids from $n = 12$ to $n = 24$ points are sufficient. The difference in weighted average deflections is 4.3%.

The models of three beam systems are equivalent to a certain extent when simulating weightlessness. Any of the considered systems with the presented boundary conditions can be converted into an equivalent one by changing the boundary force factors. For example, if in a model with hinged support we specify a moment or install a spring with a given stiffness, we obtain a deformation model with rigid pinching. Accordingly, it is necessary to adjust the tension forces of the cables

The type of boundary condition has a greater effect on the first beam; the tension forces of the cables level out the deformed and stressed state. In subsequent beams, the tension forces of the cables are practically the same.

When moving from rigid fastening to hinged fastening, then from sliding fastening to the free edge, the rigidity of the systems decreases. The root-mean-square deflections increase.

Simulating the weightlessness of a system with the condition of minimizing the sum of squared deflections can be useful in preparing physical experiments.

It is possible to generalize the formulation of the problem of regulating the stress-strain state for systems of suspended plates and panels; it is possible to install springs in hinged joints; the forces in the cables can be additionally distributed with weight factors.

Библиографические ссылки

1. Феодосьев В. И. Основы техники ракетного полета. М. : Наука, 1979. 496 с.

2. Строительная механика летательных аппаратов / И. Ф. Образцов, Л. А. Булычев, В. В. Васильев и др. М. : Машиностроение, 1986. 536 с.

3. Анализ конструкций солнечных батарей космических аппаратов / З. А. Казанцев, А. М. Ерошенко, Л. А. Бабкина, А. В. Лопатин // Космические аппараты и технологии. 2021. Т. 5, № 3 (37). С. 121–136.

4. Волков М. В., Двирный В. В. Каркас солнечной батареи из труб треугольного сечения // Космические аппараты и технологии. 2021. Т. 5, № 3 (37). С. 137–145.

5. Автоматическая система обезвешивания крупногабаритных трансформируемых конструкций при раскрытии / А. Г. Верхогляд, С. Н. Макаров, В. М. Михалкин и др. // Изв. вузов. Приборостроение. 2016. Т. 59, № 2. С. 134–142.

6. А.с. 1467418 СССР, МКИG01M13/02, F16H 21/16. Стенд для моделирования невесомости двухзвенных механизмов / А. В. Медарь, В. Б. Бурыкин, Я. Ф. Гайденко, Д. С. Михайлов, В. М. Бажанов, В. П. Кравченко, С. В. Балошин, Е. В, Морозов, С. М. Осохин (СССР). № 4238824/25-28 ; заявл. 04.05.87 ; опубл. 32.03.89. Бюл. № 11. 2 с.

7. Звонцова К. К. Исследование зависимости угла раскрытия спицы от перемещения мачты при моделировании процессов стендовых испытаний механических устройств рефлекторов антенн больших диаметров // Технологии Microsoft в теории и практике программирования : сб. тр. XIII Всеросс. науч.-практ. конф. студентов, аспирантов и молодых уч. Томск, 2016. С. 48–50.

8. Методика расчета системы обезвешивания крупногабаритных трансформируемых элементов космических аппаратов при наземных испытаниях / А. С. Беляев, А. А. Филипас, А. В. Цавнин, А. В. Тырышкин // Сибирский аэрокосмический журнал. 2021. Т. 22, № 1. С. 106–120.

9. Базара М., Шетти К. Нелинейное программирование. Теория и алгоритмы. М. : Мир. 1982. 584 с.

10. Кузнецов Ю. Н., Кузубов В. И., Волощенко А. Б. Математическое программирование. М. : Высш. школа, 1980. 300 с.

11. Химмельблау Д. Прикладное нелинейное программирование. М. : Мир.1975. 536 с.

12. Биргер И. А., Мавлютов Р. Р. Сопротивление материалов. М. : Наука, 1986. 560 с.

13. Матросов А. В. Maple 6. Решение задач высшей математики и механики. СПб. : БВХ-Петербург, 2001. 528 с.

14. Строительная механика. Стержневые системы / А. Ф. Смирнов, А. В. Александров, Б. Я. Лащеников, Н. Н. Шапошников. М. : Стройиздат, 1981. 512 с.

15. Микеладзе Ш. Е. Численные методы математического анализа. М. : Гос. изд-во техн.-теорет. лит-ры, 1953. 528 с.

16. Писаренко Г. С., Яковлев А. П., Матвеев В. В. Справочник по сопротивлению материалов. Киев : Наука. 1975. 400 с.

17. Чертов А. Г. Международная система единиц измерений. М. : Высшая школа, 1967. 288 с.

References

1. Feodosiev V. I. *Osnovy tekhniki raketnogo poleta* [Fundamentals of rocket flight technology]. Moscow, Nauka Publ., 1979, 496 p.

2. Obraztsov I. F., Bulychev L. A., Vasiliev V. V. et al. *Stroitel'naya mekhanika letatel'nykh apparatov* [Structural mechanics of aircraft]. Moscow, Mashinostroenie, 1986,536 p.

3. Kazantsev Z. A., Eroshenko A. M., Babkina L. A., Lopatin A. V. [Analysis of spacecraft solar array designs]. *Kosmicheskie apparaty i tekhnologii*. 2021, Vol. 5, No. 3 (37), P. 121–136 (In Russ.).

4. Volkov M. V., Dvirny V. V. [Solar battery frame from tubes of triangular section]. *Kosmicheskie apparaty i tekhnologii*. 2021, Vol. 5, No. 3 (37), P. 137–145 (In Russ.).

5. Verkhoglyad A. G., Makarov S. N., Mikhalkin V. M., Stupak M. F., Shevlyakov A. V. [Automatic weight loss system for large transformable structures during opening]. *Instrumentation.* 2016, Vol. 59, No. 2, P. 134–142 (In Russ.).

6. Medar A.V. et al. *A. S. 1467418 USSR, MKIG01M13/02, F16H 21/16. Stend dlya modelirovaniya nevesomosti dvuhzvennyh mekhanizmov* (USSR) [A.s. 1467418 USSR, MKIG01M13/02, F16H 21/16. Stand for determining the weightlessness of two-link supports]. No. 4238824/25-28. 2 p.

7. Zvontsova K. K. [Investigation of the dependence of the opening angle of the spoke on the movement of the mast when modeling the processes of bench testing of mechanical devices of reflectors of antennas of large diameters]. *Tekhnologii Microsoft v teorii i praktike programmirovaniya: sbornik trudov XIII Vserossijskoj nauchno-prakticheskoj konferencii studentov, aspirantov i molodyh uchenyh*. Tomsk, 2016. P. 48–50 (In Russ.).

8. Belyaev A. S. et al. [Methodology for calculating the dewatering system of large-sized transformable elements of spacecraft during ground tests]. *Sibirskiy aerokosmicheskiy zhurnal*. 2021, Vol. 22, No. 1, P. 106–120 (In Russ.).

9. Bazara M., Shetty K. *Nelinejnoe programmirovanie. Teoriya i algoritmy* [Nonlinear programming. Theory and algorithms]. Moscow, Mir Publ., 1982, 584 p.

10. Kuznetsov Yu. N., Kuzubov V. I., Voloshchenko A. B. *Matematicheskoe programmirovanie* [Mathematical programming]. Moscow, Higher. School Publ., 1980, 300 p.

11. Himmelblau D. *Prikladnoe nelinejnoe programmirovanie* [Applied non-linear programming]. Moscow, Mir Publ., 1975,536 p.

12. Birger I. A., Mavlyutov R. R. *Soprotivlenie materialov* [Resistance of materials]. Moscow, Nauka Publ., 1986, 560 p.

13. Matrosov A.V. *Maple 6. Reshenie zadach vysshej matematiki i mekhaniki* [Maple 6. Solving problems of higher mathematics and mechanics]. St. Petersburg, BVH-Petersburg Publ., 2001, 528 p.

14. Smirnov A. F. et al. *Stroitel'naya mekhanika. Sterzhnevye sistemy* [Building mechanics. Rod systems] Moscow, Stroyizdat Publ., 1981, 512 p.

15. Mikeladze Sh. E. *CHislennye metody matematicheskogo analiza* [Numerical methods of mathematical analysis]. Moscow, State. publishing house of technical and theoretical literature Publ., 1953, 528 p.

16. Pisarenko G. S., Yakovlev A. P., Matveev V. V. *Spravochnik po soprotivleniyu materialov* [Resistance Handbook materials]. Kyiv, Nauka Publ., 1975, 400 p.

17. Chertov A. G. *Mezhdunarodnaya sistema edinic izmerenij* [International system of units of measurements]. Moscow, Higher School Publ., 1967, 288 p.

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