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Определение коэффициентов энергетической связи балок, соединенных под углом

А. П. Кравчуновский

Акционерное общество «Информационные спутниковые системы» имени академика М. Ф. Решетнева»
Российская Федерация, 662972, г. Железногорск Красноярского края, ул. Ленина, 52
E-mail: anton.kravchunovsky@yandex.ru

Использование статистического энергетического метода для анализа динамических систем предполагает, что коэффициенты энергетической связи подсистем должны быть известны. Коэффициенты энергетической связи показывают, какая часть энергии переходит из одной подсистемы в другую. Они входят в систему уравнений энергетического баланса и предварительно должны быть определены аналитически, экспериментально или численно. Наиболее перспективным из перечисленных методов является численный. В частности, в данной статье использован метод конечных элементов.

Целью настоящего исследования является определение коэффициентов энергетической связи двух подсистем в двух вариантах их относительного положения. За основу принята модель Г-образного соединения двух балок, которая довольно часто встречается в подобных исследованиях. Г-образное соединение частей конструкции часто встречается в строительных сооружениях, однако в других отраслях, таких как разработка космической и авиационной техники, зачастую элементы конструкции соединяются под углом, отличным от прямого. А поскольку энергетические методы могут применяться и для аэрокосмической отрасли, при разработке подходов к анализу конструкций с помощью таких методов будет полезным знание о том, как меняются энергетические параметры системы, в частности коэффициенты энергетической связи, в зависимости от того, под каким углом выполнено сопряжение их составных частей.

Рассмотрены две конфигурации системы: в первой – балки соединены под прямым углом, во второй – под углом 45°. Вычислены коэффициенты энергетической связи балок для обеих конфигураций системы. Сделаны выводы о возможности распространения полученного результата на более сложные конструкции, а именно конструкции космической техники.

Ключевые слова: энергетический метод, коэффициент энергетической связи, уравнение энергетического баланса, космический аппарат, колебания.

Determination of coupling loss factors for beams connected at an angle

A. P. Kravchunovskii

JSC “Information Satellite Systems” named after academician M.F. Reshetnev”
52, Lenin St., Zheleznogorsk, Krasnoyarsk region, 662972, Russian Federation
E-mail: anton.kravchunovsky@yandex.ru

The use of a statistical energy method for the analysis of dynamic systems assumes that coupling loss factors of subsystems must be known. Coupling loss factors show what part of energy moves from one subsystem to another. They are included in the system of energy balance equations and must first be determined analytically, experimentally or numerically. The most promising of the listed methods is a numerical one. In particular, this paper uses a finite element method.

The purpose of this study is to determine the coupling loss factors of two subsystems in two versions of their relative positions. The basis is the model of an L-shaped connection of two beams, which is quite common in such studies. L-shaped connections of structural parts are often found in building structures, but in other industries, such as the development of space and aviation technology, structural elements are often connected at an angle other than 90°. Since energy methods (EMs) can also be applied to aerospace industry, when developing approaches to analyzing structures using such methods, it is useful to know how the energy parameters of a system, in particular coupling loss factors, change depending on the fact at what angle their components are connected.

The paper considers two system configurations: in the first, the beams are connected at a right angle, in the second - at an angle of 45°. The coupling loss factors of the beams are calculated for both system configurations. Conclusions are drawn about the possibility of disseminating the results obtained to more complex structures, namely spacecraft structures.

Keywords: energy method, coupling loss factor, energy balance equation, spacecraft, oscillation.

Introduction

In modern practice, to analyze complex structures, along with a well-known finite element method (FEM), a slightly less common statistical energy method (SEM), or, as it is more often called in foreign literature, a SEA - Statistical Energy Analysis is used. It is called “statistical” since it is assumed that the systems under study belong to statistical populations with a known distribution of dynamic parameters [1–4].

Analysis of a system using a SEM, as well as using a FEM, involves breaking a system into simpler subsystems. However, a SEM allows for a much larger partition, which saves time and computational resources, i.e., it is enough to obtain such a composite unit of a system that vibrational waves of the same type propagate in it. At the same time, to analyze a complex system using a FEM (especially at high frequencies), it is necessary to greatly reduce the size of the final element, and thereby increase the dimension of the entire system in order to obtain a reliable result [5].

A SEM is based on the law of energy conservation, according to which the total energy of a closed-loop system is conserved [6]. To solve the problem, the balance of energies introduced into a system and absorbed by it, as well as those moving from one subsystem to another, is compiled [7].

The process of energy flow between subsystems is mathematically described by the so-called coupling loss factors of subsystems. They can be found analytically, experimentally or numerically. Analytical dependencies are not always applicable due to the complexity of real structures being modeled; experimental ones are often too labor-intensive. The most promising for these purposes are numerical methods.

Problem statement

In a SEM, a system is divided into several related subsystems that combine modes of the same type (flexural, torsional or longitudinal). The method is statistical in the sense that the subsystems under consideration should conditionally unite groups of identical objects that should have similar dynamic properties [1]. The method is based on calculating the energy transferred from one subsystem to another. The energy of each subsystem can be transferred to adjacent subsystems [8] or dissipated through damping, which is expressed by the loss factor. The coupling loss factor shows what part of the energy passes from one subsystem to another. Thus, the variable in the analysis is energy. It is assumed that energy is concentrated only on resonant modes. In this case, the total energy of each subsystem is the sum of the energies of each mode.

The basis of the SEM is the system of energy balance equations (EBES), which has the following form:

$$\begin{bmatrix} \eta_1+\sum\limits_{i=2}^{n}\eta_{1i} & -\eta_{21} & -\eta_{31} & \cdots & -\eta_{n1} \\ -\eta_{12} & \eta_2+\sum\limits_{\substack{i=1\\ i\neq 2}}^{n}\eta_{2i} & -\eta_{32} & \cdots & -\eta_{n2} \\ -\eta_{13} & -\eta_{23} & \eta_3+\sum\limits_{\substack{i=1\\ i\neq 3}}^{n}\eta_{3i} & \cdots & -\eta_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\eta_{1n} & -\eta_{2n} & -\eta_{3n} & \cdots & \eta_n+\sum\limits_{\substack{i=1\\ i\neq n}}^{n}\eta_{ni} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_n \end{Bmatrix} = \begin{Bmatrix} \frac{S_1}{\omega} \\ \frac{S_2}{\omega} \\ \frac{S_3}{\omega} \\ \vdots \\ \frac{S_n}{\omega} \end{Bmatrix}.$$

Here η_i are the factors of internal losses of subsystems; η_{ij} are coupling loss factors of subsystems; E_i are vibrational energies of subsystems; S_i is energy introduced into the subsystems; ω is angular frequency; n is the number of subsystems.

The right side of the EBES determines the energies introduced into the subsystems, while the left side determines the vibrational energies of the system. The number of equations included in the EBES must be equal to the number of subsystems. In the vector-column of the right side of the EBES, only the components of those subsystems into which energy is introduced are not equal to zero.

The vibrational energy used in the calculation is the total mechanical energy, which is the sum of the kinetic and potential energies of each subsystem and is unknown in the equations. It is assumed that the input energies are pre-calculated, and the loss factors and coupling factors of the subsystems are adopted by analogy with the already carried out calculations of identical structures and are known when compiling the EBES.

As it was noted above, analytical and experimental methods for determining coupling loss factors are not universal, and preference in this case should be given to numerical methods. The use of a FEM for these purposes is a promising and convenient approach [9–13].

Thus, in [14] an L-shaped connection of two beams is considered, through which coupling loss factors were found during energy transition from the first beam to the second using a FEM.

L-shaped connection of structural parts is indeed often found in building structures, but in other industries, such as the development of space and aviation technology, structural elements are often connected at an angle other than 90º. Since EMs can also be applied to aerospace industry, when developing approaches to structural analysis using EMs, it is useful to know how the energy parameters of the system, in particular coupling loss factors, change depending on the angle at which their components are connected.

For example, a cell of a mesh isogrid (or anisogrid) structure of a spacecraft load-carrying structure implies the connection of edges at arbitrary angles.

In this study, to control the design parameters, we take the beam model from [14] as a basis, but we change some conditions to suit the analysis tasks.

Let us consider a system represented by two pin-ended beams, rigidly connected to each other (Fig. 1). For convenience of research, the boundary conditions are chosen in such a way that only bending vibrations occur in the beams. The length of the first beam is 1.1 m, the length of the second beam is 0.9 m. The modeling is carried out with two-dimensional finite elements of the same length (1 cm). The cross-sections of the beams are 0.001×0.005 m. The beams are connected to each other along the long side of the section. The material is steel. The internal loss coefficient for both beams is 0.01.

Let us calculate the coupling loss factors for two connection options of beams (Fig. 1). For each calculation option, a harmonic load of 1 N is applied in turn to each beam at a distance of 0.1 m from the edge, always perpendicular to the beam.

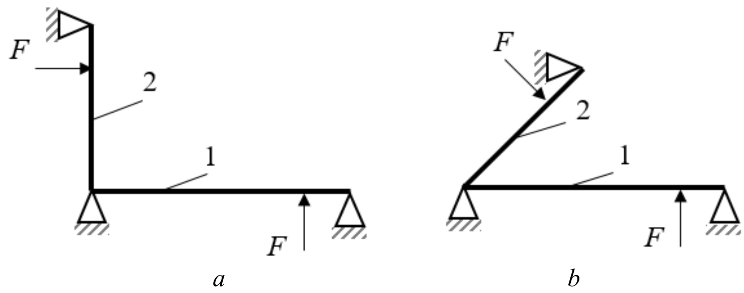


Рис. 1. Варианты соединения двух балок:
а – угол равен 90°; *б* – угол равен 45°

Fig. 1. Variants of two beams connection:
a – 90° angle; *b* – 45° angle

Let us compose the EBES for this system. For convenience, for E we introduce a second index so that the notation E_{12} will mean the energy of the first subsystem when energy is introduced into the second one [15]. Likewise for the remaining E_{11}, E_{21}, E_{22}. After which the EBES in matrix form will look like this:

$$\begin{bmatrix} \eta_1+\eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2+\eta_{21} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix},$$

and the equations of the system will take the form

$$\eta_1 E_{11} + \eta_{12} E_{11} - \eta_{21} E_{21} = \frac{S_1}{\omega},$$

$$\eta_2 E_{21} + \eta_{21} E_{21} - \eta_{12} E_{11} = 0,$$

$$\eta_1 E_{12} + \eta_{12} E_{12} - \eta_{21} E_{22} = 0,$$

$$\eta_2 E_{22} + \eta_{21} E_{22} - \eta_{12} E_{12} = \frac{S_2}{\omega}.$$

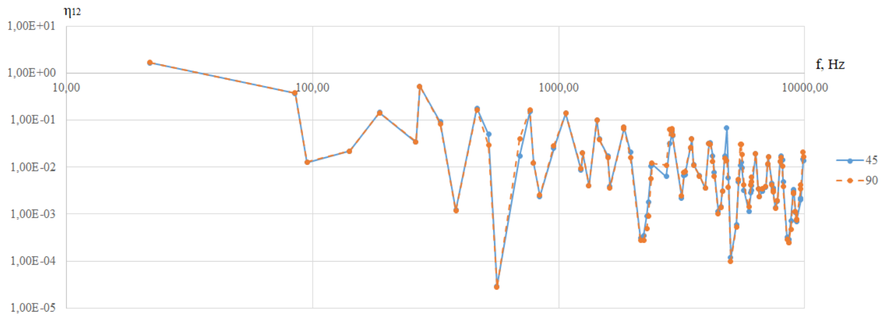
Knowing the internal loss coefficients of beams, we can use FEM calculations to find the total vibration energies of the subsystems. To do this, it is necessary to carry out two calculations, alternately introducing energy into each beam.

Thus, the coupling loss factors η_{12} and η_{21} can be calculated from the following expressions:

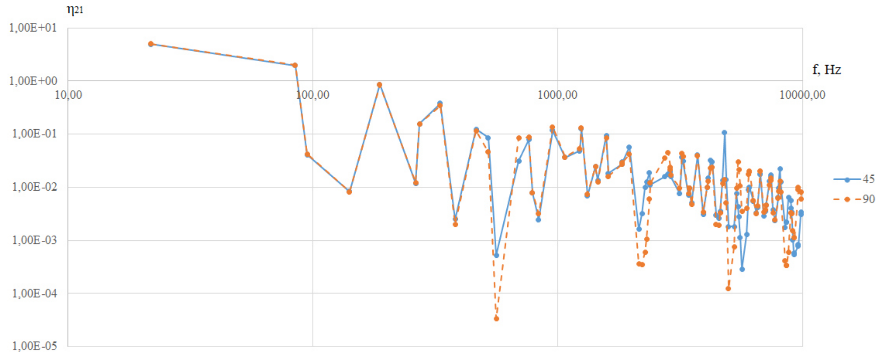
$$\eta_{12} = \frac{E_{21}\left(\eta_2 E_{22} + \eta_1 E_{12}\right)}{E_{11}E_{22} - E_{12}E_{21}},$$

$$\eta_{21} = \frac{E_{12}\left(\eta_1 E_{11} + \eta_2 E_{21}\right)}{E_{11}E_{22} - E_{12}E_{21}}.$$

The results of calculating the coupling loss factors for two options for connecting beams are presented in Fig. 2 and 3.



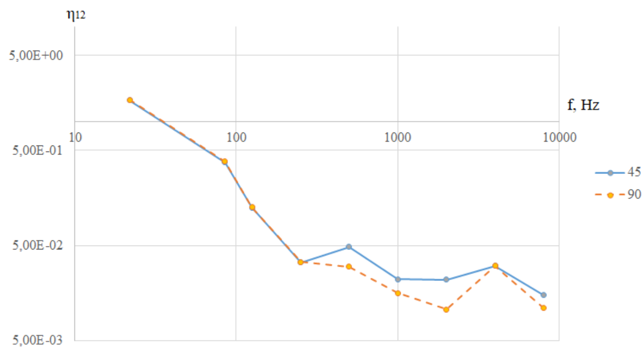
a



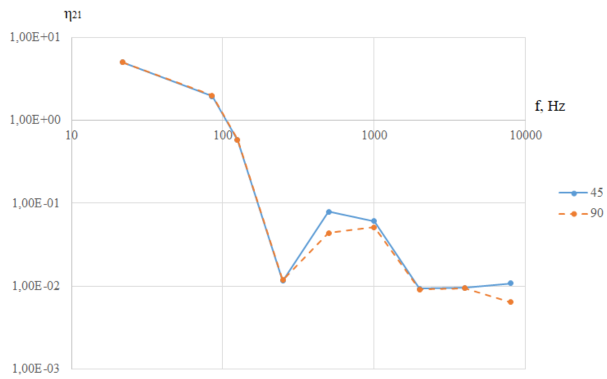
b

Рис. 2. Коэффициент энергетической связи в зависимости от частоты: *а* – η_{12}; *б* – η_{21}

Fig. 2. Coupling loss factor depending on the frequency: *a* – η_{12}; *b* – η_{21}



a



b

Рис. 3. Коэффициенты энергетической связи по третьоктавным частотным диапазонам: *а* – η_{12}; *б* – η_{21}

Fig. 3. Coupling loss factors in 1/3-octave frequency bands: *a* – η_{12}; *b* – η_{21}

Results

From the graphs shown in Fig. 2 it can be seen that the values of the coupling loss factors of two beams during the energy transition from the first to the second η_{12} and, conversely, from the second to the first η_{21}, increase slightly as the angle between the beams decreases. At the same time, the difference in the values of η_{21} for the cases of connecting beams at angles of 90° and 45° is more noticeable at certain natural frequencies.

However, if we sum up the values of the coupling loss factors in one-third octave frequency bands, then such conclusions become more difficult to draw. This indicates, that averaging results should be approached with caution and not limited to the analysis of averaged values.

Conclusion

The values of the coupling loss factors of two beams are obtained according to the results of two calculation options. In the first option, the beams are connected at right angles, in the second, the angle formed by the two beams is equal to 45°. A comparative analysis of the calculations showed that a decrease in the angle between the beams has little effect on the values of the coupling loss factors of the two beams. The results obtained suggest that for more complex systems, such as isogrid (or anisogrid) load-carrying structures of spacecraft, formed by the intersection of ribs at a certain angle, the coupling loss factors also change slightly when the angle of ribs intersection changes. This would make it possible not to carry out additional calculations and measurements due to the use of already known coupling loss factors in the calculations. This assumption requires further confirmation using the example of analysis of more complex systems.

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Кравчуновский Антон Павлович – начальник группы; акционерное общество «Информационные спутниковые системы» имени академика М. Ф. Решетнева». E-mail: anton.kravchunovsky@yandex.ru.

Kravchunovky Anton Pavlovich is the Head of the Group; Joint-Stock Company "Information Satellite Systems" named after academician M. F. Reshetnev". E-mail: anton.kravchunovsky@yandex.ru.