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## Динамические параметры потока в естественных криволинейных координатах для линии тока во вращающемся канале

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Особый интерес к теме математического анализа протекания процессов переноса теплоты определяется научной значимостью и практическим применением при разработке, проектировании и производстве ракетно-космических аппаратов и установок. Обоснование разработанных методик и моделирование данных, полученных в ходе эксперимента с применением 3д технологий процесса, дает преимущество. Точность и достоверность результатов расчетов играют ключевую роль в обеспечении безопасности и надежности ракетно-космических систем. Регулярная проверка и верификация результатов также необходимы для обеспечения высокой степени надежности и безопасности. Представленный в статье комплексный анализ течения потока жидкости в межлопаточном канале рабочего колеса малорасходного центробежного насоса с построением энергетических характеристик рабочего колеса может быть использован для уточнения числа лопаток. Разработанная методика расчета состоит из четырех частей: во-первых, получено выражение для определения проекции градиента давления на продольную ось  $\varphi$ , во-вторых, получено выражение для определения проекции градиента давления на поперечную ось  $\psi$ , в-третьих, определена производная продольной скорости в поперечном направлении и, в-четвертых, представлены результаты численной и экспериментальной визуализации (баланс мощностей, зависимость напора и коэффициента влияния конечного числа лопаток от расхода малорасходного центробежного насоса). На основе результатов теоретических исследований были разработаны алгоритм и программа расчета, позволяющие рассчитывать локальные значения. Рассматриваемый подход подтверждается верификацией результатов математического моделирования графической визуализацией течения и измерением баланса мощностей малорасходного центробежного насоса. Полученные выражения для проекций градиента давления, определение производной продольной скорости и экспериментальная визуализация играют важную роль при расчете и анализе работы центробежных насосов. Однако существует необходимость в дальнейшей проработке метода для приведения его к виду, позволяющему рассчитывать трехмерное течение рабочего тела в канале произвольной формы.

**Ключевые слова:** центробежный насос, рабочее колесо, напор, оптимизация, градиент скорости, градиент давления, баланс мощностей.

## Dynamic flow parameters in natural curvilinear coordinates for a current line in a rotating channel

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*A special interest in the topic of mathematical analysis of the flow of heat transfer processes is determined by the scientific significance and practical application in the development, design and production of rocket and space vehicles and installations. Substantiation of the developed techniques and modeling of the data obtained during the experiment using 3D process technologies gives an advantage. The accuracy and reliability of the calculation results play a key role in ensuring the safety and reliability of rocket and space systems. Regular verification and verification of the results are also necessary to ensure a high degree of reliability and safety. The comprehensive analysis of the fluid flow in the inter-vane channel of the impeller of a low-flow centrifugal pump presented in the article, with the construction of the energy characteristics of the impeller, can be used to clarify the number of vanes. The developed calculation method consists of four parts: firstly, an expression is obtained to determine the projection of the pressure gradient on the longitudinal axis  $\varphi$ , secondly, an expression is obtained to determine the projection of the pressure gradient on the transverse axis  $\psi$ , thirdly, the derivative of the longitudinal velocity in the transverse direction is determined, and fourthly, the results are presented numerical and experimental visualization: the power balance, the dependence of the pressure and the coefficient of influence of a finite number of vanes on the flow rate of a low-flow centrifugal pump. Based on the results of theoretical research, an algorithm and a calculation program were developed that allows calculating local values. The considered approach is confirmed by verification of the results of mathematical modeling by graphical visualization of the flow and measurement of the power balance of a low-flow centrifugal pump. The obtained expressions for pressure gradient projections, determination of the derivative of the longitudinal velocity and experimental visualization play an important role in the calculation and analysis of the operation of centrifugal pumps. However, there is a need for further elaboration of the method to bring it to a form that allows calculating the three-dimensional flow of the working fluid in an arbitrary channel.*

*Keywords: centrifugal pump, impeller, head, optimization, speed gradient, pressure gradient, power balance.*

### Introduction

One of the most important points in the development and design of new models of rocket, space and aviation systems is the traditional provision of the highest possible parameters in terms of energy characteristics, service life and reliability of both individual units, structural elements and the apparatus as a whole. The criteria for maintaining the target performance indicators can be determined by means of sequential determination based on cause and effect relationships throughout the entire operational cycle of the technical system. This process includes substantiation of tactical and technical requirements, development of technical specifications for research and development work, preliminary design, development of design and technical documentation; final testing and production. Therefore, even a minor error at the stage of expected results of new developments can increase the cost of subsequent stages many times or terminate the project.

To achieve optimally high energy characteristics of a centrifugal pump, it is necessary to obtain and use the most complete energy conversion in the inter-blade channel of the impeller. In particular, it is the conversion of energy in the inter-blade channel of the impeller that plays a key role in ensuring the high efficiency of such installations. The research results and design recommendations presented in

various classical and modern works are of great importance for the development of efficient propulsion systems, including thermal power plants for aircraft. To ensure the supply of the required pressure of necessary flow parameters with a given increase in the pressure of the working fluid, centrifugal pumping units are mainly used, which have high energy characteristics with relatively small weight and dimensions. The propulsion system of the aircraft includes pumps that are responsible for feeding fuel into the combustion chamber with an electric drive and a turbo engine, which are used in liquid rocket stages of spacecraft. In hydraulic drive systems of actuators of thrust vector control systems and mechanization, centrifugal pumps are the source of power [1–5].

It should be emphasized that centrifugal pumps are common in many areas of industry (oil industry, energy, pharmaceuticals, transport, food, chemical industries, cryogenics, etc.), where the issue of reducing vibration, pressure pulsation, and noise is urgent. The use of a more advanced flow path in pumps helps solve these problems [6].

The performance of a centrifugal pump is determined by the high angular velocity of its impeller, for example, the Rocket Engine (RE)0146, which has a rotation speed of up to 123,000 rpm. When pumping highly viscous media, centrifugal pumps become less productive: high resistance and high pressure of the working medium reduce the ability to maintain a certain flow rate. The current task is the theoretical study of the flow movement in the channels of the flow part of centrifugal blade superchargers. It is this task that shows significant shortcomings, since a significant part of the channels have changing areas and complex spatial shapes with the presence of zones of low and high pressure gradient with the possible presence of a paired vortex, which leads to a bevel of the bottom current lines and curvature of the middle line. All these channels are in rotation, and the flow flowing through them directly interacts with the blades and thereby increases its specific energy [4]. The interblade channel in the working wheel is an important element. Uncooled blades are used in liquid rocket engine (LRE) turbines. In addition, different types of turbines are used in the direction of flow – radial, axial, diagonal – with different profiles of the interblade channels. Increasing the temperature of the working fluid also contributes to increasing the adiabatic work. Due to the design features and materials used, limitations are imposed on the temperature of the working fluid in an uncooled turbine, usually 1000–1200 °C for the reducing gas and 700–900 °C for the oxidizing gas [7].

The presence of a pressure gradient in the flow of the working fluid moving along the interblade channel, and consequently in the boundary layer, significantly complicates the computational problem of the latter. However, given the practical significance of this issue, it has attracted the attention of many researchers and various algorithmic approaches to the solution are currently being developed, based on approximate assumptions and empirical data [7–9].

Many scientific studies consider the properties of the turbulent boundary layer. Studies of the conservative properties of the boundary layer are of great importance. The research materials explain the properties of turbulent boundary layers, describe the problem with friction and heat exchange during the movement of the working fluid (liquid) through channels and flow parts (thermodynamically). The theoretical laws of limit friction and heat exchange of body surfaces are also considered. The main point is that the turbulence of the flow of the wall section has very little effect on external changes in several indicators of the average flow. Depending on the limiting relative laws of friction and heat exchange, mathematical calculation methods were proposed. In a turbulent flow with a longitudinal pressure gradient with heat exchange in the near-wall region, the velocity profile is represented by a logarithmic law and is practically independent of the pressure gradient, but the pressure gradient has an increased effect on the velocity distribution in the outer part of the turbulent boundary layer, which is more than 75% of its thickness. The influence of disturbance does not change the form of mathematical descriptions of the boundary layer, but is parametric. Understanding the physical features of flow motion in each element of the flow part of a centrifugal pump unit will allow developing calculation and design methods that take into account the flow features in complex spatial channels. The most difficult part of the study and mathematical description is the boundary layer on a curved (spherical) surface. On such a surface there is a separation point, a high pressure gradient, and the derivative of the flow velocity changes sign. The momentum balance method is used to determine the correction parameters in the flow [4; 6; 9–18].

When calculating the velocity field, it is necessary to conduct an additional analysis of the pressure field distribution. The pressure distribution in the interblade channel has a significant effect on the fluid flow dynamics and, consequently, on the energy characteristics of the centrifugal impeller. This analysis allows for a more complete and accurate determination of flow characteristics, which is important for developing effective optimization and design methods for impellers. There are no explicit equations for determining the pressure gradient, but the pressure parameter is included in the basic momentum equation. If the pressure field is defined, the equation can be solved without much difficulty, but there is no obvious way to determine the pressure field. It is possible to use the continuity equation to determine the pressure field. The pressure field must be defined in such a way that when it is used in the momentum equations, the resulting velocity field satisfies the continuity equation. The best way to determine the pressure field is to use discrete analogues of the momentum and continuity equations, since the others are not suitable for our solution. Because of the difficulties in finding pressures, methods have been developed that derive pressure from the system of constitutive equations [19–21].

### Objective

The objective is to develop a calculation method and analyze the influence of the pressure gradient on the velocity distribution in the near-wall part in the inter-blade channel of a centrifugal impeller with a finite number of blades, determine the energy characteristics of the impeller using the calculation method and perform optimization for a finite number of blades, as well as graphic visualization of the power balances of a low-flow centrifugal pump.

### Research methodology

The study of existing methods for calculating a turbulent boundary layer with a pressure gradient using the integral momentum equation allows us to obtain two approximate solutions of the boundary layer equation, including an equation that takes into account the features of a flow with a longitudinal pressure gradient. These solutions include an equation that takes into account the features of a flow with a longitudinal pressure gradient. Thus, it is possible to analyze the approximate equation of a turbulent boundary layer. In further calculations, methods for calculating a turbulent boundary layer with a pressure gradient will be used, taking into account that the obtained solutions of the criterial equations are valid only for a confusor flow process. These methods assume the use of the momentum loss thickness as a characteristic thickness of the boundary layer.

The introduction of additional parameters to describe the velocity profile is an important step in taking into account the strong dependence of the characteristics on the pressure gradient. These additional parameters can help to more fully and accurately describe the complex dependencies between pressure and velocity inside the interblade channel. This approach allows for a more realistic representation of the fluid flow dynamics and, accordingly, the energy characteristics of the centrifugal impeller. The momentum loss thickness for the calculation is determined by the momentum theorem, in which the shear stress on the wall is determined based on the law of resistance of a longitudinally flown plate. These methods are used to determine the friction resistance of bodies with different profiles and show positive results.

In this case, a special coordinate transformation is used to create a more uniform distribution of flow parameters throughout the space. When moving to the calculation of dynamic flow parameters in natural curvilinear coordinates for the streamline in the rotating interblade channel, an analysis of the steady-state flow of an ideal fluid is carried out taking into account friction. The graphical calculation is performed in a polar coordinate system, which is optimal in cases where the relationships between points are easier to depict as radii and angles.

### Projection of pressure gradient on the longitudinal axis $\varphi$

The longitudinal coordinate line  $\varphi$  is by definition the projection of the limiting line of the flow in the core of the flow onto the limiting surface. In our case, this is the inner surface of the cover disk at a

specific point of the curved line  $\varphi$ . The direction of the relative velocity  $\vec{W}$  is tangent to this line (Fig. 1).

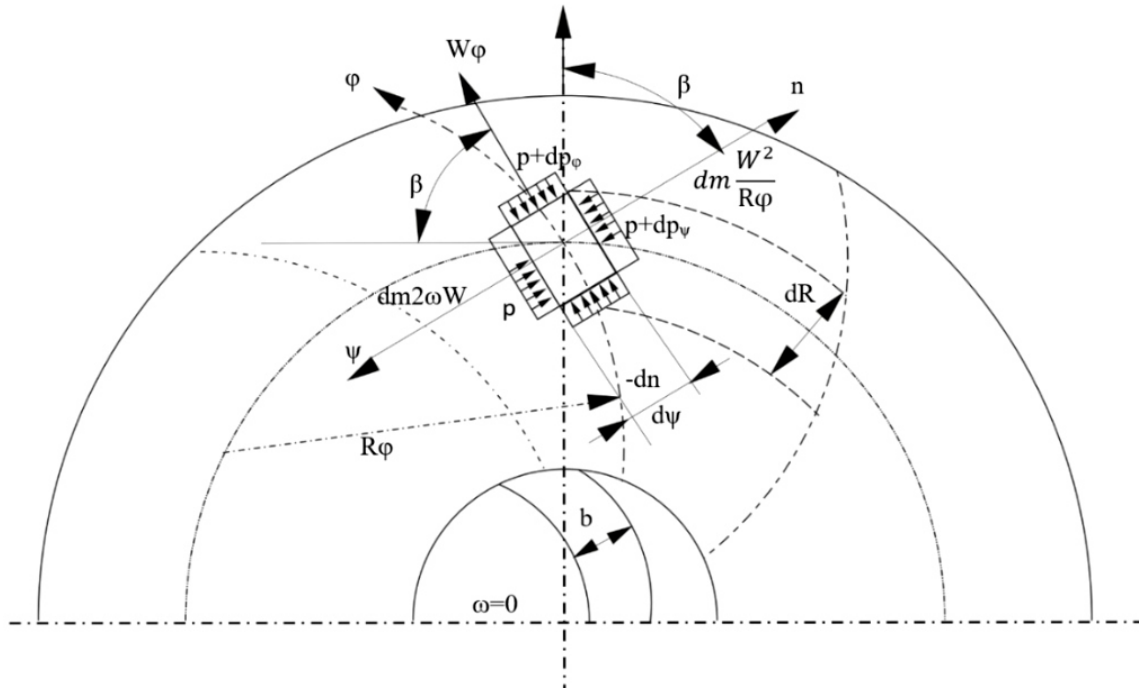


Рис. 1. Расчетная схема ядра потока

Fig. 1. Calculation scheme of the flow core

For steady relative motion without taking into account friction in the flow core, the equation of motion takes the form of the Euler equation, which, when projected onto the longitudinal coordinate line, is written as

$$\frac{\partial}{\partial \varphi} \left( \frac{W^2}{2} \right) = F_\varphi - \frac{1}{\rho} \frac{\partial p}{\partial \varphi}, \quad (1)$$

where  $\partial \varphi$  is the element of longitudinal coordinate line;  $F_\varphi$  is the projection of inertial forces sum on the coordinate  $\varphi$ , per unit mass, which corresponds to the inertial acceleration taken with the opposite sign.

In the case of relative motion in the rotating interblade channel of the impeller, inertial forces include:

- centrifugal forces of inertia from the rotation of the impeller  $\omega^2 R$ ;
- centrifugal force of inertia due to the curvature of the flow line (coordinate line  $\varphi$ )  $\frac{W^2}{R}$ ;
- Coriolis inertial force  $2\omega W$ .

We neglect the force of gravity and external inertial forces.

The component of the centrifugal force associated with the impeller from the rotation of the coordinates is equal to:

$$\omega^2 R \sin \beta = \omega^2 R \frac{dR}{d\varphi}, \quad (2)$$

where  $\sin \beta = \frac{dR}{d\varphi}$ , (see Fig.1).

The projection of this centrifugal force  $\frac{W^2}{R_\varphi}$ , which arises due to the presence of curvature along the flow line, and the Coriolis force  $2\omega W$  on the direction  $\varphi$  will be equal to zero due to the perpendicularity of the relative velocity. Then  $F_\varphi = \sum F_{\varphi in}$ , is written [22]:

$$F_\varphi = \omega^2 R \frac{\partial R}{\partial \varphi}. \quad (3)$$

Substituting (3) in (1), we will obtain

$$\frac{\partial \left( \frac{W^2}{2} \right)}{\partial \varphi} = \omega^2 R \frac{\partial R}{\partial \varphi} - \frac{1}{\rho} \frac{\partial p}{\partial \varphi}.$$

Transforming into an increment of kinetic energies, we obtain:

$$\frac{\partial \left( \frac{W_\varphi^2}{2} \right)}{\partial \varphi} - \frac{\partial \left( \frac{U^2}{2} \right)}{\partial \varphi} = -\frac{1}{\rho} \frac{\partial p}{\partial \varphi}.$$

We obtain an expression for the projection of the pressure gradient onto the longitudinal axis  $\varphi$

$$\frac{1}{\rho} \frac{\partial p}{\partial \varphi} = \frac{\partial}{\partial \varphi} \left( \frac{U^2}{2} - \frac{W_\varphi^2}{2} \right), \quad (4)$$

where  $W_\varphi$  is velocity in relative motion, tangent to the longitudinal coordinate  $\varphi$ ;  $U = \omega R$  is the transport velocity at a particular point on the streamline, or

$$-\frac{1}{\rho} \frac{\partial p}{\partial \varphi} = W \frac{\partial W}{\partial \varphi} - \frac{\omega^2 R dR}{d\varphi}. \quad (5)$$

### Projection of the pressure gradient onto the transverse axis $\psi$

We allocate in the interscapular channel an elementary volume of liquid with mass  $dm = \rho \cdot b \cdot d\psi \cdot d\varphi$ , where  $b$  is a channel width in the direction perpendicular to the plane (see Fig. 1).

We consider the equilibrium of an elementary volume of liquid in projection onto the transverse coordinate axis  $\psi$ , opposite to the normal  $n$  to the limiting line of flow in relative motion. The following components of forces act on the selected volume (see Fig. 1):

– surface pressure forces, the total component of which is equal to:

$$\partial p_\varphi \cdot b \cdot d\varphi;$$

– component of the centrifugal force of inertia arising from the curvature of the flow line  $\varphi$ :

$$-dm \cdot \omega^2 R \cdot \cos \beta = dm \cdot \omega^2 R \cdot \frac{\partial R}{\partial \varphi};$$

– component of the centrifugal force of inertia arising from the curvature of the flow line  $\psi$ :

$$-dm \cdot \frac{W^2}{R\varphi};$$

– component of the Coriolis force of inertia:

$$\partial m \cdot 2\omega W.$$

From the equilibrium condition, the sum of all components is equal to zero:

$$\partial p_{\varphi} \cdot b \cdot d\varphi - dm \cdot \omega^2 R \cdot \frac{\partial R}{\partial \varphi} - dm \frac{W^2}{R_{\varphi}} + dm \cdot 2\omega W = 0.$$

We divide and multiply the first term of equation (5) by  $\rho d\psi$ , we reduce all members to  $dm = \rho \cdot b \cdot d\psi \cdot d\varphi$  and we get the equation:

$$\frac{\partial p}{\rho \partial \psi} - \frac{\omega^2 R_{\varphi} dR}{\partial \psi} - \frac{W^2}{R_{\varphi}} + 2\omega W = 0 \quad (6)$$

or

$$-\frac{1}{\rho} \frac{\partial p}{\partial \varphi} = -\omega^2 R \cdot \cos \beta - \frac{W^2}{R_{\varphi}} + 2\omega W, \quad (7)$$

– derivative of static pressure with respect to coordinate  $\psi$ , where  $R_{\varphi}$  is a radius of curvature of the limiting streamline, in our calculation case  $R_{\varphi} = R_v = \text{const}$  scapular angle –  $\beta_v = f(\varphi)$ .

It is necessary to note the following: the direction of increase (growth) of static pressure coincides with the direction of the inertial force (and is opposite to the inertial acceleration of the elementary mass of the liquid).

Transverse waves arise from deformation of the shape, i.e., small rotational movements of the particles of the environment on a plane directed in the direction of propagation of the oscillations. The volume in the environment is unchanged, but a local deformation of a rectangular element of the environment occurs, and thus the S-wave is called a wave of motion. A transverse wave does not propagate in a liquid or gaseous environment, where the weak adhesion of the elements of the substance does not allow the transmission of shear deformations.

### Derivative of longitudinal velocity in transverse direction

Using expression (5) for the longitudinal static pressure gradient, multiplying the parts by  $d\varphi$ , we obtain

$$-\frac{1}{\rho} dp = W dW - \omega^2 R dR. \quad (8)$$

By integrating the equations along the current stream from section 1-1 to section 2-2, we obtain an expression for the static pressure of an incompressible fluid:

$$\frac{p_2 - p_1}{\rho} = \frac{U_2^2 - W_2^2}{2} - \frac{U_1^2 - W_1^2}{2}. \quad (9)$$

Assuming that the energy of the jets per step is constant [3], from equation (8) we obtain the following relationship:

$$-\frac{1}{\rho} \frac{\partial p}{\partial \varphi} = W \frac{dW}{d\psi} - \omega^2 R \frac{dR}{d\psi}. \quad (10)$$

Using expression (8) for the first number (1), we obtain:

$$-\frac{\omega^2 R dR}{d\psi} - \frac{W^2}{R_{\varphi}} + 2\omega W = W \frac{dW}{d\psi} - \omega^2 R \frac{dR}{d\psi}.$$

After canceling the terms and dividing by  $W$ , we obtain an expression for the derivative of the longitudinal velocity  $W_u$  in the transverse direction  $\Psi$ :

$$\frac{\partial W_u}{\partial \Psi} = 2\omega - \frac{W}{R_\varphi}. \quad (11)$$

Longitudinal waves always propagate faster than transverse waves in the same space, caused by a change in volume during the translational motion of particles in the direction of propagation of elastic oscillations. It is also known that longitudinal waves propagate at a speed  $V_p$  that is determined by the elasticity and density of the media. Thus, the medium (solid, homogeneous, isotropic) will spread regardless of time and space.

### Power balance

According to the experiment conducted with the distribution of pump power, we obtain numerical integral hydraulic losses. Part of the expended power is converted into useful work, the rest goes into losses [3; 9; 10].

Thus, the balance of effective power of the pump will be

$$N_{useful} = N_{cons} - (N_{fr} + N_h + N_l + N_{mech}^{fix} + N_{mech}^{ip}).$$

Power balance components in a supercharger (pump):  $N_{useful}$  is useful pump power;  $N_{cons}$  is power expended;  $N_{fr}$  is power expended on friction (hydraulic);  $N_h$  is losses (hydraulic) in the flow part of the pump;  $N_l$  is power losses due to leaks of working fluid;  $N_{mech}^{fix}$  is losses due to contact with fixed parts – supports, seals;  $N_{mech}^{ip}$  is mechanical power losses in the impeller.

Fig. 2 and 3 show the graphical dependences of the loss values. The obtained energy characteristics coincide with the results of the study with an error not exceeding 3–5%. This makes it possible to assert that the developed methodology is correct and verified.

Fig. 2 shows the results of numerical and experimental visualization: power balance, dependence of pressure and flow rate  $K_z$  of a low-flow centrifugal pump for cylindrical blades,  $\beta_{1v} = \beta_{2v} = 60^\circ$ . Fig. 3 shows the power balance  $K_z$ , the dependence of the pressure and the flow rate of a low-flow centrifugal pump for tangential blades  $\beta_{2v} = 77^\circ$ .



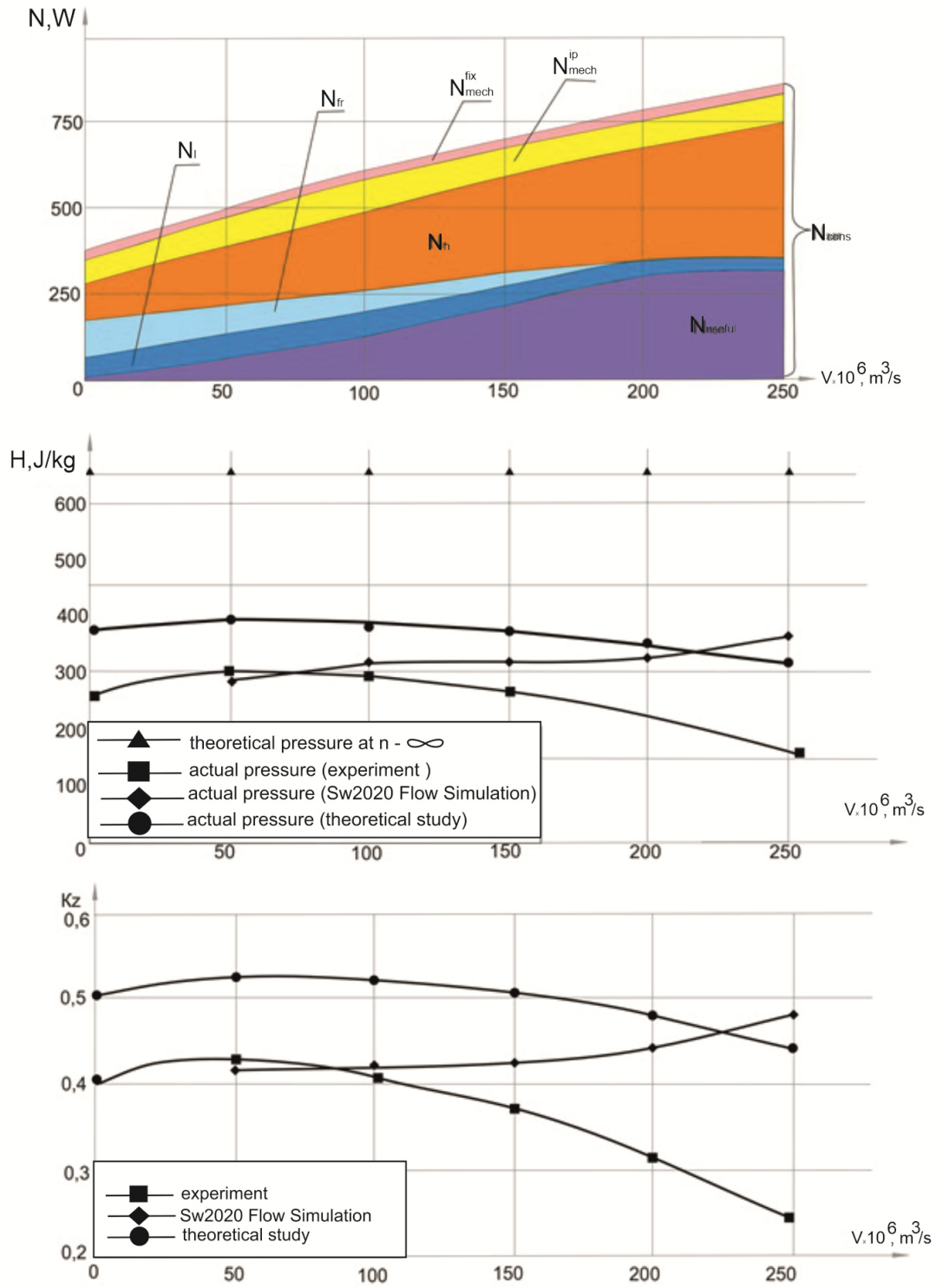


Рис. 2. Баланс мощностей, зависимость напора и  $K_z$  от расхода малорасходного центробежного насоса (цилиндрические лопасти,  $\beta_{1v} = \beta_{2v} = 60^\circ$ )

Fig. 2. Power balance, dependence of pressure and  $K_z$  on the flow rate of a low-flow centrifugal pump (cylindrical blades,  $\beta_{1v} = \beta_{2v} = 60^\circ$ )

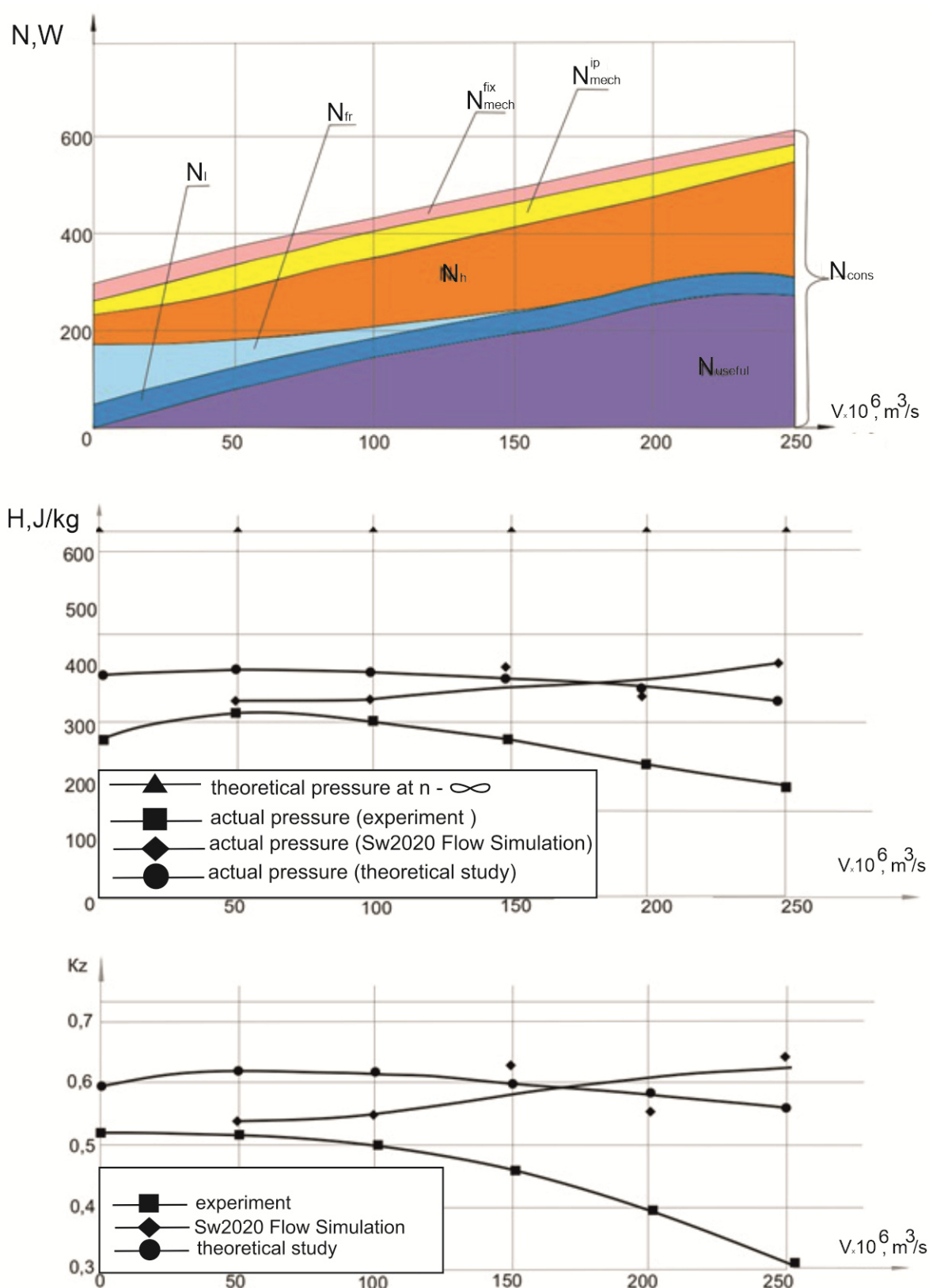


Рис. 3. Баланс мощностей, зависимость напора и  $K_z$  от расхода малорасходного центробежного насоса (тангенциальные лопатки,  $\beta_{2v} = 77^\circ$ )

Fig. 3. Power balance, dependence of pressure and  $K_z$  on the flow rate of a low-flow centrifugal pump (tangential blades,  $\beta_{2v} = 77^\circ$ )

## Conclusion

Based on the above, the following conclusions can be drawn:

- a new form of the equation of momentum of the spatial boundary layer for a laminar relative flow with a longitudinal-transverse pressure gradient was obtained, combined with the friction law determined from the classical parabolic profile;
- based on the research results, expressions were obtained for the projection of the pressure gradient onto the longitudinal axis  $\varphi$ ;
- the derivative of the static pressure with respect to the coordinate  $\psi$  is obtained;
- expressions for the derivative of the longitudinal velocity  $W_u$  in the transverse direction  $\psi$  in the natural coordinate system for flow in a circular sector are obtained ;
- graphs of power balances, the dependence of pressure and  $K_z$  on the flow rate of a low-flow centrifugal pump (cylindrical blades,  $\beta_{1v} = \beta_{2v} = 60^\circ$ ) and (tangential blades,  $\beta_{2v} = 77^\circ$ ) were constructed.

However, the method requires further development in order to bring it to a form that allows calculating the three-dimensional flow of the impeller in a channel of arbitrary shape. The results of all parts of the study will be used to calculate the optimization of the final number of blades in the impeller of the pump.

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