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Калибровка магнитометра космического аппарата с учетом характера температурной зависимости матрицы чувствительности и вектора смещений нуля

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Аннотация. В настоящей работе предложен аналитический метод решения задачи калибровки магнитометра для модели, учитывающей вектор температурной зависимости смещений нуля для каждой из измерительных осей блока магнитометра и матрицу линейной температурной зависимости каждого из членов матрицы чувствительности, масштабирующей сигнал на основе реальной чувствительности каждой оси и включающей линейные внеосевые эффекты. При решении задачи определения калибровочных параметров блока магнитометра учитывается, что для измерений с любой пространственной ориентацией блока магнитометра величина измеряемого вектора магнитной индукции сохраняется и является известной модельной величиной. Вводится в рассмотрение штрафная функция 24 переменных, равная сумме квадратов невязок. Алгоритм решения задачи калибровки измерительных осей блока магнитометра сводится к поиску методом наименьших квадратов таких значений переменных этой функции, которые при заданном наборе векторов измерений магнитометра доставляют ей минимум. С этой целью указанная функция исследуется на экстремум. Исходя из необходимого условия экстремума штрафной функции, формируется система 24 уравнений относительно 24 неизвестных, которая для удобства разбивается на три системы (каждая из них есть система 8 линейных алгебраических уравнений относительно 8 неизвестных). Доказывается, что основная матрица каждой из этих трех систем не вырождена, откуда следует, что каждая из них имеет решение, и притом только одно. Компоненты решений этих систем (координаты стационарной точки штрафной функции) находятся по правилу Крамера. Доказывается, что второй дифференциал штрафной функции в найденной стационарной точке положителен, откуда следует, что эта точка действительно доставляет минимум указанной функции.

Ключевые слова: калибровка магнитометра, вектор магнитной индукции, метод наименьших квадратов, матрица Грама, правило Крамера.

Calibration of a spacecraft magnetometer taking into account the nature of the temperature dependence of the sensitivity matrix and the offset vector

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Abstract. The paper proposes an analytical method to solve the problem of magnetometer calibration for a model that considers the vector of temperature dependence of zero offsets for each of the measuring axes of the magnetometer unit and the matrix of linear temperature dependence of each of the members of the sensitivity matrix, scaling the signal based on the actual sensitivity of each axis and including linear off-axis effects. When solving the problem of determining the calibration parameters of the magnetometer unit, it is taken into account that for measurements with any spatial orientation of the magnetometer unit, the magnitude of the measured magnetic field strength vector is preserved and is a known model value. A penalty function of 24 variables equal to the sum of the squares of the residuals is introduced into consideration. The algorithm for solving the problem of calibrating the measuring axes of the magnetometer unit is reduced to searching by the method of least squares for such values of the variables of this function that, with a given set of vectors of magnetometer measurements, provide it with a minimum. For this purpose, the specified function is examined for an extremum. Based on the necessary condition for the extremum of the penalty function, a system of 24 equations in the 24 variables is formed, which, for convenience, is divided into three systems (each of them is a system of 8 linear algebraic equations in the 8 variables). It is proved that the main matrix of each of these three systems is an invertible, from which it follows that each of them has a solution, and only one. The components of the solutions of these systems (the coordinates of the stationary point of the penalty function) are found using Cramer's rule. It is proved that the second differential of the penalty function at the found stationary point is positive, from which it follows that this point really provides the minimum of the specified function.

Keywords: magnetometer calibration, the Earth's magnetic induction vector, the method of least squares, Gram matrix, Cramer's rule.

Introduction

Magnetometers are part of the orientation and stabilization system (OSS) of low-orbit small-sized spacecraft (LSS), where they are the main sources of information about the position of the LSS after separation from the booster block. Magnetometers measure the magnitude and direction of the magnetic induction vector of the Earth magnetic field. The obtained data are necessary to generate the control moments of LSS, while the duration of the calming mode largely depends on the accuracy of the instrument readings and the noise component.

Modern LSS OSS magnetometers are developed on the basis of the magnetoresistance effect and, due to the physical characteristics of the sensitive element, require mathematical calibration of the device. Currently, various methodologies for calibrating magnetometers have been proposed [1–16], in particular, article [11], which provides an overview of various methodologies to perform such operations.

Previously, the problem of calibrating a spacecraft magnetometer was solved using numerical methods. This paper proposes an analytical method to solve this problem for a model that takes into account the vector of temperature dependence of zero offsets for each of the measuring axes of the magnetometer block and the matrix of linear temperature dependence of each of the members of the sensitivity matrix, scaling the signal based on the actual sensitivity of each axis and including linear off-axis effects.

1. Error model in magnetic induction vector measurements

We could denote by $\boldsymbol{h} = (h_1, h_2, h_3)^T$ the value of the measured magnetic induction vector at a certain spatial position of the magnetometer unit (MU). We use the measurement model considered in [1]:

$$\boldsymbol{B} = (S + \tau K_S)\,\boldsymbol{h} + \boldsymbol{b} + \tau\,\boldsymbol{k_b}, \tag{1}$$

(1) uses the following notations:

$\boldsymbol{B} = (B_1, B_2, B_3)^T$ – true magnetic induction vector;

$\boldsymbol{b} = (b_1, b_2, b_3)^T$ – constant vector corresponding to the zero offsets for each of the MU measuring axes;

$\boldsymbol{k_b} = (\theta_1, \theta_2, \theta_3)^T$ – vector of temperature dependence of zero offsets for each of the MU measuring axes;

$S = \left(s_{ij}\right)_{i,j=1}^{3}$ – a sensitivity matrix that scales the signal based on the actual sensitivity of each axis and includes linear off-axis effects;

$K_S = \left(t_{ij}\right)_{i,j=1}^{3}$ – matrix of linear temperature dependence of each member of the sensitivity matrix;

τ – temperature transmitted by the sensor (scalar value).

In this case, the components of the vectors $\boldsymbol{B}$, $\boldsymbol{h}$, $\boldsymbol{b}$ and $\boldsymbol{k_b}$ are expressed in the same units of measurement.

The task of calibrating the measuring axes of the MU is reduced to finding the elements of the matrices S and K_S, as well as the components of the vectors $\boldsymbol{b}$ and $\boldsymbol{k_b}$.

2. Development of an algorithm to determine the calibration parameters of the MU

When solving the problem of determining the calibration parameters of the MU, we will use the fact that for measurements with any spatial orientation of the MU, the value of the measured magnetic induction vector $\boldsymbol{B}$ is preserved and is a known model value.

Let the results of magnetometer measurements at discrete moments in time be a set of vectors $\boldsymbol{h}^{(l)} = (h_1^{(l)}, h_2^{(l)}, h_3^{(l)})^T$, and as a result of measurements at the same discrete moments of time of the sensor transmitting temperature, a set of values $\tau^{(l)}$, $l = 1, 2, \ldots, N$ was obtained. Without loss of generality, we can assume that if $\left(\tau^{(l)}, h_1^{(l)}, h_2^{(l)}, h_3^{(l)}, \tau^{(l)}h_1^{(l)}, \tau^{(l)}h_2^{(l)}, \tau^{(l)}h_3^{(l)}\right)$ $(l = 1, 2, \ldots, N)$ is considered as point coordinates of 7-dimensional affine space, these points do not lie in the same hyperplane. We could designate these points as follows:

$$U_l\left(\tau^{(l)}, h_1^{(l)}, h_2^{(l)}, h_3^{(l)}, \tau^{(l)}h_1^{(l)}, \tau^{(l)}h_2^{(l)}, \tau^{(l)}h_3^{(l)}\right),\ l = 1, 2, \ldots, N. \tag{2}$$

We prove an auxiliary statement.

Lemma 1. If points

$$V_1\left(x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}\right),\ V_2\left(x_1^{(2)}, x_2^{(2)}, \ldots, x_n^{(2)}\right), \ldots,\ V_N\left(x_1^{(N)}, x_2^{(N)}, \ldots, x_n^{(N)}\right) \tag{3}$$

n-dimensional affine space do not lie in the same hyperplane ($N \geq n$), then among them there will be n affinely independent points.

Proof. We will prove the lemma by contradiction. We could assume that any n points from the set (3) are affinely dependent. Let m denote the maximum number of affinely independent points (3),

$1 \le m < n$. Let $V_{i_1}\left(x_1^{(i_1)}, x_2^{(i_1)}, \ldots, x_n^{(i_1)}\right)$, $V_{i_2}\left(x_1^{(i_2)}, x_2^{(i_2)}, \ldots, x_n^{(i_2)}\right)$, …, $V_{i_m}\left(x_1^{(i_m)}, x_2^{(i_m)}, \ldots, x_n^{(i_m)}\right)$ – affinely independent points, and Π_{n-1} – is any of the hyperplanes passing through these points. In hyperplane Π_{n-1} we could choose such points $W_1\left(y_1^{(1)}, y_2^{(1)}, \ldots, y_n^{(1)}\right)$, $W_2\left(y_1^{(2)}, y_2^{(2)}, \ldots, y_n^{(2)}\right)$, …, $W_{n-m}\left(y_1^{(n-m)}, y_2^{(n-m)}, \ldots, y_n^{(n-m)}\right)$, as points V_{i_1}, V_{i_2}, …, V_{i_m}, W_1, W_2, …, W_{n-m} are affinely independent. We will write the equation of the hyperplane Π_{n-1} [17]:

$$\begin{vmatrix} x_1 - x_1^{(i_1)} & x_2 - x_2^{(i_1)} & \ldots & x_n - x_n^{(i_1)} \\ x_1^{(i_2)} - x_1^{(i_1)} & x_2^{(i_2)} - x_2^{(i_1)} & \ldots & x_n^{(i_2)} - x_n^{(i_1)} \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots & \ldots\ldots\ldots \\ x_1^{(i_m)} - x_1^{(i_1)} & x_2^{(i_m)} - x_2^{(i_1)} & \ldots & x_n^{(i_m)} - x_n^{(i_1)} \\ y_1^{(1)} - x_1^{(i_1)} & y_2^{(1)} - x_2^{(i_1)} & \ldots & y_n^{(1)} - x_n^{(i_1)} \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots & \ldots\ldots\ldots \\ y_1^{(n-m)} - x_1^{(i_1)} & y_2^{(n-m)} - x_2^{(i_1)} & \ldots & y_n^{(n-m)} - x_n^{(i_1)} \end{vmatrix} = 0. \qquad (4)$$

It is easily seen that any of the points (3) satisfies equation (4). Indeed, if we substitute the coordinates of any of the points U_l ($l = 1, 2, \ldots, N$) into the determinant appearing on the left side of this equation instead of x_1, x_2, …, x_n, we obtain the determinant

$$\begin{vmatrix} x_1^{(l)} - x_1^{(i_1)} & x_2^{(l)} - x_2^{(i_1)} & \ldots & x_n^{(l)} - x_n^{(i_1)} \\ x_1^{(i_2)} - x_1^{(i_1)} & x_2^{(i_2)} - x_2^{(i_1)} & \ldots & x_n^{(i_2)} - x_n^{(i_1)} \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots & \ldots\ldots\ldots \\ x_1^{(i_m)} - x_1^{(i_1)} & x_2^{(i_m)} - x_2^{(i_1)} & \ldots & x_n^{(i_m)} - x_n^{(i_1)} \\ y_1^{(1)} - x_1^{(i_1)} & y_2^{(1)} - x_2^{(i_1)} & \ldots & y_n^{(1)} - x_n^{(i_1)} \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots & \ldots\ldots\ldots \\ y_1^{(n-m)} - x_1^{(i_1)} & y_2^{(n-m)} - x_2^{(i_1)} & \ldots & y_n^{(n-m)} - x_n^{(i_1)} \end{vmatrix}. \qquad (5)$$

If $l = i_1$, then the first line of the determinant (5) is zero, and, therefore, it is equal to zero. If $l \in \{i_2, \ldots, i_m\}$, the determinant (5) is equal to zero due to the fact that its first row coincides with one of the rows with numbers 2, …, m. In case $l \notin \{i_1, i_2, \ldots, i_m\}$, due to the affine dependence of the points V_l, V_{i_1}, V_{i_2}, …, V_{i_m} the vectors $\overrightarrow{V_{i_1}V_l}$, $\overrightarrow{V_{i_1}V_{i_2}}$, $\overrightarrow{V_{i_1}V_{i_3}}$, …, $\overrightarrow{V_{i_1}V_{i_m}}$ are linearly dependent, therefore, the first m rows of the determinant (5) are linearly dependent, and, therefore, this determinant is equal to zero in this case too.

Thus, all N points (3) belong to the hyperplane Π_{n-1}, which contradicts the lemma condition. Therefore, our assumption that any n points from the set (3) are affinely dependent is not true, which means that among the points (3) there really are n affinely independent points. Lemma 1 is proven.

(1) results in:

$$\boldsymbol{B}^{(l)} = (S + \tau^{(l)} K_S)\, \boldsymbol{h}^{(l)} + \boldsymbol{b} + \tau^{(l)} \boldsymbol{k_b},\ l = 1, 2, \ldots, N, \qquad (6)$$

where $\boldsymbol{B}^{(l)} = (B_1^{(l)}, B_2^{(l)}, B_3^{(l)})^T$ – the true vector of magnetic induction at the same point in space as the measured vector $\boldsymbol{h}^{(l)}$, $l = 1, 2, \ldots, N$.

We rewrite the equality (6) in an expanded form:

$$\begin{pmatrix} s_{11}+\tau^{(l)}t_{11} & s_{12}+\tau^{(l)}t_{12} & s_{13}+\tau^{(l)}t_{13} \\ s_{21}+\tau^{(l)}t_{21} & s_{22}+\tau^{(l)}t_{22} & s_{23}+\tau^{(l)}t_{23} \\ s_{31}+\tau^{(l)}t_{31} & s_{32}+\tau^{(l)}t_{32} & s_{33}+\tau^{(l)}t_{33} \end{pmatrix}\begin{pmatrix} h_1^{(l)} \\ h_2^{(l)} \\ h_3^{(l)} \end{pmatrix}+\begin{pmatrix} b_1+\tau^{(l)}\theta_1 \\ b_2+\tau^{(l)}\theta_2 \\ b_3+\tau^{(l)}\theta_3 \end{pmatrix}=\begin{pmatrix} B_1^{(l)} \\ B_2^{(l)} \\ B_3^{(l)} \end{pmatrix}, \tag{7}$$

$l=1, 2, \ldots, N$. We write each of the N vector equalities (7) as a system of three scalar equalities:

$$\left(s_{i1}+\tau^{(l)}t_{i1}\right)h_1^{(l)}+\left(s_{i2}+\tau^{(l)}t_{i2}\right)h_2^{(l)}+\left(s_{i3}+\tau^{(l)}t_{i3}\right)h_3^{(l)}+b_i+\tau^{(l)}\theta_i=B_i^{(l)}, \quad i=1,2,3,$$

$l=1,2,\ldots,N$.

We could introduce into consideration a penalty function of 24 variables s_{ij}, t_{ij} (i, j = 1, 2, 3), b_i, θ_i (i = 1, 2, 3):

$$\Phi=\sum_{l=1}^{N}\sum_{i=1}^{3}\left[B_i^{(l)}-\tau^{(l)}\theta_i-b_i-\left(s_{i1}+\tau^{(l)}t_{i1}\right)h_1^{(l)}-\left(s_{i2}+\tau^{(l)}t_{i2}\right)h_2^{(l)}-\left(s_{i3}+\tau^{(l)}t_{i3}\right)h_3^{(l)}\right]^2. \tag{8}$$

The algorithm for solving the problem of calibrating the measuring axes of the MU is reduced to searching by the least squares method [18] for such values of variables s_{ij}, t_{ij} (i, j = 1, 2, 3), b_i, θ_i (i = 1, 2, 3) that deliver the minimum of function Φ at a given set of measurement vectors $\{\boldsymbol{h}^{(l)}\}$ ($l=1,2,\ldots,N$). For this purpose, it is necessary to study the function Φ for the extremum [19]. We could write down the necessary condition for the local extremum of this function:

$$\begin{cases} \dfrac{\partial\Phi}{\partial b_i}=-2\sum\limits_{l=1}^{N}\left[B_i^{(l)}-\tau^{(l)}\theta_i-b_i-\left(s_{i1}+\tau^{(l)}t_{i1}\right)h_1^{(l)}-\left(s_{i2}+\tau^{(l)}t_{i2}\right)h_2^{(l)}-\left(s_{i3}+\tau^{(l)}t_{i3}\right)h_3^{(l)}\right]=0, \ i=1,2,3, \\ \dfrac{\partial\Phi}{\partial \theta_i}=-2\sum\limits_{l=1}^{N}\tau^{(l)}\left[B_i^{(l)}-\tau^{(l)}\theta_i-b_i-\left(s_{i1}+\tau^{(l)}t_{i1}\right)h_1^{(l)}-\left(s_{i2}+\tau^{(l)}t_{i2}\right)h_2^{(l)}-\left(s_{i3}+\tau^{(l)}t_{i3}\right)h_3^{(l)}\right]=0, \ i=1,2,3, \\ \dfrac{\partial\Phi}{\partial s_{ij}}=-2\sum\limits_{l=1}^{N}h_j^{(l)}\left[B_i^{(l)}-\tau^{(l)}\theta_i-b_i-\left(s_{i1}+\tau^{(l)}t_{i1}\right)h_1^{(l)}-\left(s_{i2}+\tau^{(l)}t_{i2}\right)h_2^{(l)}-\left(s_{i3}+\tau^{(l)}t_{i3}\right)h_3^{(l)}\right]=0, \ i,j=1,2,3, \\ \dfrac{\partial\Phi}{\partial t_{ij}}=-2\sum\limits_{l=1}^{N}\tau^{(l)}h_j^{(l)}\left[B_i^{(l)}-\tau^{(l)}\theta_i-b_i-\left(s_{i1}+\tau^{(l)}t_{i1}\right)h_1^{(l)}-\left(s_{i2}+\tau^{(l)}t_{i2}\right)h_2^{(l)}-\left(s_{i3}+\tau^{(l)}t_{i3}\right)h_3^{(l)}\right]=0, \ i,j=1,2,3. \end{cases} \tag{9}$$

It is required to find the stationary points of the function Φ, that is the solution of system (9) – a system of linear algebraic equations concerning 24 unknowns s_{ij}, t_{ij} (i, j = 1, 2, 3), b_i, θ_i (i = 1, 2, 3).

For convenience, we will divide (9) into three systems

$$\begin{cases} Nb_i+\theta_i\sum\limits_{l=1}^{N}\tau^{(l)}+\sum\limits_{k=1}^{3}\left[s_{ik}\sum\limits_{l=1}^{N}h_k^{(l)}\right]+\sum\limits_{k=1}^{3}\left[t_{ik}\sum\limits_{l=1}^{N}\tau^{(l)}h_k^{(l)}\right]=\sum\limits_{l=1}^{N}B_i^{(l)}, \\ b_i\sum\limits_{l=1}^{N}\tau^{(l)}+\theta_i\sum\limits_{l=1}^{N}\left(\tau^{(l)}\right)^2+\sum\limits_{k=1}^{3}\left[s_{ik}\sum\limits_{l=1}^{N}\tau^{(l)}h_k^{(l)}\right]+\sum\limits_{k=1}^{3}\left[t_{ik}\sum\limits_{l=1}^{N}\left(\tau^{(l)}\right)^2h_k^{(l)}\right]=\sum\limits_{l=1}^{N}\tau^{(l)}B_i^{(l)}, \\ b_i\sum\limits_{l=1}^{N}h_j^{(l)}+\theta_i\sum\limits_{l=1}^{N}\tau^{(l)}h_j^{(l)}+\sum\limits_{k=1}^{3}\left[s_{ik}\sum\limits_{l=1}^{N}h_k^{(l)}h_j^{(l)}\right]+\sum\limits_{k=1}^{3}\left[t_{ik}\sum\limits_{l=1}^{N}\tau^{(l)}h_k^{(l)}h_j^{(l)}\right]=\sum\limits_{l=1}^{N}h_j^{(l)}B_i^{(l)}, \quad j=1,2,3, \\ b_i\sum\limits_{l=1}^{N}\tau^{(l)}h_j^{(l)}+\theta_i\sum\limits_{l=1}^{N}\left(\tau^{(l)}\right)^2h_j^{(l)}+\sum\limits_{k=1}^{3}\left[s_{ik}\sum\limits_{l=1}^{N}\tau^{(l)}h_k^{(l)}h_j^{(l)}\right]+\sum\limits_{k=1}^{3}\left[t_{ik}\sum\limits_{l=1}^{N}\left(\tau^{(l)}\right)^2h_k^{(l)}h_j^{(l)}\right]=\sum\limits_{l=1}^{N}\tau^{(l)}B_i^{(l)}h_j^{(l)}, \quad j=1,2,3, \end{cases} \tag{10}$$

i = 1, 2, 3.

We might note that the i-th system (10) is a system of linear algebraic equations relatively to eight unknowns b_i, θ_i, s_{i1}, s_{i2}, s_{i3}, t_{i1}, t_{i2}, t_{i3}, $i = 1, 2, 3$.

We can prove that each of the three systems of equations (10) has the only solution. To do this, it is sufficient to show that the fundamental matrix of each of these three systems is not degenerate. The main matrix of each of the indicated systems is the Gram matrix, composed of the scalar products of the following eight vectors:

$$\left(1,1,\ldots,1\right),\ \left(\tau^{(1)},\tau^{(2)},\ldots,\tau^{(N)}\right),\ \left(h_i^{(1)},h_i^{(2)},\ldots,h_i^{(N)}\right)\ (i=1,2,3),$$

$$\left(\tau^{(1)}h_i^{(1)},\tau^{(2)}h_i^{(2)},\ldots,\tau^{(N)}h_i^{(N)}\right)\ (i=1,2,3). \tag{11}$$

In this case, the scalar product of two vectors is defined as the sum of the products of their components with the same numbers.

We will prove that the system of vectors (11) is linearly independent, which will lead to the non-degeneracy of the Gram matrix of this system [20], that is the main matrix of each of these three systems of equations (10). There could be proof by contradiction. We assume that the system of vectors (11) is linearly dependent. Then the rank of the matrix is

$$\begin{pmatrix} 1 & \tau^{(1)} & h_1^{(1)} & h_2^{(1)} & h_3^{(1)} & \tau^{(1)}h_1^{(1)} & \tau^{(1)}h_2^{(1)} & \tau^{(1)}h_3^{(1)} \\ 1 & \tau^{(2)} & h_1^{(2)} & h_2^{(2)} & h_3^{(2)} & \tau^{(2)}h_1^{(2)} & \tau^{(2)}h_2^{(2)} & \tau^{(2)}h_3^{(2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots \\ 1 & \tau^{(N)} & h_1^{(N)} & h_2^{(N)} & h_3^{(N)} & \tau^{(N)}h_1^{(N)} & \tau^{(N)}h_2^{(N)} & \tau^{(N)}h_3^{(N)} \end{pmatrix},$$

its columns are composed of components of vectors (11), less than 8. Therefore, any minor of the 8th order of this matrix is equal to zero (we assume that $N \geq 8$), which implies the validity of the equality

$$\begin{vmatrix} 1 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ 1 & \tau^{(i_1)} & h_1^{(i_1)} & h_2^{(i_1)} & h_3^{(i_1)} & \tau^{(i_1)}h_1^{(i_1)} & \tau^{(i_1)}h_2^{(i_1)} & \tau^{(i_1)}h_3^{(i_1)} \\ 1 & \tau^{(i_2)} & h_1^{(i_2)} & h_2^{(i_2)} & h_3^{(i_2)} & \tau^{(i_2)}h_1^{(i_2)} & \tau^{(i_2)}h_2^{(i_2)} & \tau^{(i_2)}h_3^{(i_2)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots \\ 1 & \tau^{(i_7)} & h_1^{(i_7)} & h_2^{(i_7)} & h_3^{(i_7)} & \tau^{(i_7)}h_1^{(i_7)} & \tau^{(i_7)}h_2^{(i_7)} & \tau^{(i_7)}h_3^{(i_7)} \end{vmatrix} = 0, \tag{12}$$

$1 < i_1 < i_2 < \ldots < i_7 \leq N$, где $U_{i_l}\left(\tau^{(i_l)},h_1^{(i_l)},h_2^{(i_l)},h_3^{(i_l)},\tau^{(i_l)}h_1^{(i_l)},\tau^{(i_l)}h_2^{(i_l)},\tau^{(i_l)}h_3^{(i_l)}\right)$ – affinely independent points from the set of points (2), $l = 1, 2,\ldots, 7$ (seven such points exist due to Lemma 1), and $\left(x_1, x_2, \ldots, x_7\right)$ are coordinates of any of the remaining ($N - 7$) points of the set (2). Equality (12) is equivalent to the equality

$$\begin{vmatrix} x_1-\tau^{(i_1)} & x_2-h_1^{(i_1)} & x_3-h_2^{(i_1)} & x_4-h_3^{(i_1)} & x_5-\tau^{(i_1)}h_1^{(i_1)} & x_6-\tau^{(i_1)}h_2^{(i_1)} & x_7-\tau^{(i_1)}h_3^{(i_1)} \\ \tau^{(i_2)}-\tau^{(i_1)} & h_1^{(i_2)}-h_1^{(i_1)} & h_2^{(i_2)}-h_2^{(i_1)} & h_3^{(i_2)}-h_3^{(i_1)} & \tau^{(i_2)}h_1^{(i_2)}-\tau^{(i_1)}h_1^{(i_1)} & \tau^{(i_2)}h_2^{(i_2)}-\tau^{(i_1)}h_2^{(i_1)} & \tau^{(i_2)}h_3^{(i_2)}-\tau^{(i_1)}h_3^{(i_1)} \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots \\ \tau^{(i_7)}-\tau^{(i_1)} & h_1^{(i_7)}-h_1^{(i_1)} & h_2^{(i_7)}-h_2^{(i_1)} & h_3^{(i_7)}-h_3^{(i_1)} & \tau^{(i_7)}h_1^{(i_7)}-\tau^{(i_1)}h_1^{(i_1)} & \tau^{(i_7)}h_2^{(i_7)}-\tau^{(i_1)}h_2^{(i_1)} & \tau^{(i_7)}h_3^{(i_7)}-\tau^{(i_1)}h_3^{(i_1)} \end{vmatrix} = 0,$$

which is the equation of a hyperplane passing through affinely independent points U_{i_1}, U_{i_2}, …, U_{i_7}. From the above it follows that the coordinates of any of the N points of the set (2) satisfy this equation, and this contradicts the fact that the N points (2) do not lie in the same hyperplane.

Therefore, the system of vectors (11) is linearly independent, which means that the main matrix of each of the three systems of equations (10) is not degenerate. Consequently, each of these systems has a solution, and moreover, there is the only one, what was required to be proven.

We introduce the following notations:

$$A_i=\sum_{l=1}^{N}B_i^{(l)},\ C_i=\sum_{l=1}^{N}\tau^{(l)}B_i^{(l)},\ i=1,2,3;\ G_j=\sum_{l=1}^{N}\tau^{(l)}h_j^{(l)},\ H_j=\sum_{l=1}^{N}h_j^{(l)},\ L_j=\sum_{l=1}^{N}\left(\tau^{(l)}\right)^2h_j^{(l)},\ j=1,2,3;$$

$$M_{kj}=M_{jk}=\sum_{l=1}^{N}h_k^{(l)}h_j^{(l)},\ P_{kj}=P_{jk}=\sum_{l=1}^{N}\tau^{(l)}h_k^{(l)}h_j^{(l)},\ Q_{kj}=Q_{jk}=\sum_{l=1}^{N}\left(\tau^{(l)}\right)^2h_k^{(l)}h_j^{(l)},\ k,j=1,2,3;$$

$$D_{ij}=\sum_{l=1}^{N}B_i^{(l)}h_j^{(l)},\ F_{ij}=\sum_{l=1}^{N}\tau^{(l)}B_i^{(l)}h_j^{(l)},\ i,j=1,2,3;\ R=\sum_{l=1}^{N}\left(\tau^{(l)}\right)^2,\ T=\sum_{l=1}^{N}\tau^{(l)}.$$

Then each of the three systems of equations (10) can be presented in the form

$$\begin{cases}Nb_i+T\theta_i+\sum_{k=1}^{3}H_ks_{ik}+\sum_{k=1}^{3}G_kt_{ik}=A_i,\\ Tb_i+R\theta_i+\sum_{k=1}^{3}G_ks_{ik}+\sum_{k=1}^{3}L_kt_{ik}=C_i,\\ H_jb_i+G_j\theta_i+\sum_{k=1}^{3}M_{kj}s_{ik}+\sum_{k=1}^{3}P_{kj}t_{ik}=D_{ij},\quad j=1,2,3,\\ G_jb_i+L_j\theta_i+\sum_{k=1}^{3}P_{kj}s_{ik}+\sum_{k=1}^{3}Q_{kj}t_{ik}=F_{ij},\quad j=1,2,3,\end{cases}\tag{13}$$

i = 1, 2, 3. In each of the three systems (13) we express the first two equations b_i and θ_i via s_{i1}, s_{i2}, s_{i3}, t_{i1}, t_{i2}, t_{i3}, then we will exclude b_i and θ_i out of the remaining six equations of the system and multiply both parts of each of the last six equations by (T^2-NR):

$$\begin{cases}b_i=\left(T^2-NR\right)^{-1}\left[TC_i-RA_i+\sum_{k=1}^{3}\left(RH_k-TG_k\right)s_{ik}+\sum_{k=1}^{3}\left(RG_k-TL_k\right)t_{ik}\right],\\ \theta_i=\left(T^2-NR\right)^{-1}\left[TA_i-NC_i+\sum_{k=1}^{3}\left(NG_k-TH_k\right)s_{ik}+\sum_{k=1}^{3}\left(NL_k-TG_k\right)t_{ik}\right],\end{cases}\tag{14}$$

$$\sum_{k=1}^{3}\alpha_{j,k}s_{ik}+\sum_{k=1}^{3}\beta_{j,k}t_{ik}=\gamma_{j,i},\quad j=1,2,\ldots,6,\tag{15}$$

i = 1, 2, 3, where the coefficients of the unknowns and the free terms are determined by the equalities.

$$\alpha_{j,k}=\left(T^2-NR\right)M_{kj}+H_j\left(RH_k-TG_k\right)+G_j\left(NG_k-TH_k\right),$$

$$\alpha_{j+3,k}=\left(T^2-NR\right)P_{kj}+G_j\left(RH_k-TG_k\right)+L_j\left(NG_k-TH_k\right),$$

$$\beta_{j,k}=\left(T^2-NR\right)P_{kj}+H_j\left(RG_k-TL_k\right)+G_j\left(NL_k-TG_k\right),$$

$$\beta_{j+3,k}=\left(T^2-NR\right)Q_{kj}+G_j\left(RG_k-TL_k\right)+L_j\left(NL_k-TG_k\right),$$

$$\gamma_{j,i}=\left(T^2-NR\right)D_{ij}+H_j\left(RA_i-TC_i\right)+G_j\left(NC_i-TA_i\right),$$

$$\gamma_{j+3,i}=\left(T^2-NR\right)F_{ij}+G_j\left(RA_i-TC_i\right)+L_j\left(NC_i-TA_i\right),$$

i, j, k = 1, 2, 3.

We could note that at least one of the third-order minors located in the first three columns of the main matrix

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \beta_{11} & \beta_{12} & \beta_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \beta_{21} & \beta_{22} & \beta_{23} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha_{61} & \alpha_{62} & \alpha_{63} & \beta_{61} & \beta_{62} & \beta_{63} \end{pmatrix} \tag{16}$$

of each of the three systems of equations (15) is not equal to zero. Indeed, by virtue of Laplace's theorem, the determinant of matrix (16) is equal to the sum of the products of all the minors of the third order, located in the first three columns of this matrix, by their algebraic complements, and if all the indicated minors were equal to zero, then the determinant of matrix (16) would also be equal to zero, which would contradict the necessary and sufficient condition for the existence of a unique solution to each of the three systems of equations (15), and therefore to each of the three systems of equations (10).

Let j_1, j_2, j_3 $(1 \le j_1 < j_2 < j_3 \le 6)$ be the numbers of the rows of the matrix (16) at the intersection of which with the first three columns of this matrix a non-zero minor of the third order is located. We could swap the equations of each of the three systems (15) so that the equations with numbers j_1, j_2, j_3 are the first, second and third, respectively. The numbers of the equations that ended up in fourth, fifth and sixth places will be designated as j_4, j_5, j_6, respectively. We could also introduce the following notations:

$$\Omega_{11} = \left(\alpha_{j_n,k}\right)_{n,k=1}^{3}, \quad \Omega_{12} = \left(\beta_{j_n,k}\right)_{n,k=1}^{3}, \quad \Omega_{21} = \left(\alpha_{j_{n+3},k}\right)_{n,k=1}^{3}, \quad \Omega_{22} = \left(\beta_{j_{n+3},k}\right)_{n,k=1}^{3},$$

$$\mathbf{\Gamma}_{1i} = \left(\gamma_{j_1,i}, \gamma_{j_2,i}, \gamma_{j_3,i}\right)^T, \quad \mathbf{\Gamma}_{2i} = \left(\gamma_{j_4,i}, \gamma_{j_5,i}, \gamma_{j_6,i}\right)^T,$$

$i = 1, 2, 3$. Then the extended matrix of the system obtained from the i-th system (15) as a result of the above rearrangement of equations will take the form

$$\left(\begin{matrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{matrix}\middle| \begin{matrix} \mathbf{\Gamma}_{1i} \\ \mathbf{\Gamma}_{2i} \end{matrix}\right), \tag{17}$$

$i = 1, 2, 3$. In this case, Ω11 is a non-singular matrix due to its definition and the conditions for choosing the numbers j_1, j_2, j_3 of the rows of matrix (16). From the second row of the i-th block matrix (17) we subtract its first row, multiplied from the left by $\Omega_{21}\Omega_{11}^{-1}$:

$$\left(\begin{matrix} \Omega_{11} & \Omega_{12} \\ O & \Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12} \end{matrix}\middle| \begin{matrix} \mathbf{\Gamma}_{1i} \\ \mathbf{\Gamma}_{2i} - \Omega_{21}\Omega_{11}^{-1}\mathbf{\Gamma}_{1i} \end{matrix}\right), \tag{18}$$

$i = 1, 2, 3$, where O where O is a zero square matrix of the third order. The resulting matrix (18) is equivalent to matrix (17) [21]. For each $i = 1, 2, 3$, the last three equations of the system corresponding to the augmented matrix (18) form a system whose matrix notation looks as follows:

$$\left(\Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12}\right)\mathbf{\Lambda}_i = \mathbf{\Gamma}_{2i} - \Omega_{21}\Omega_{11}^{-1}\mathbf{\Gamma}_{1i}, \tag{19}$$

где $\mathbf{\Lambda}_i = (t_{i1}, t_{i2}, t_{i3})^T$, $i = 1, 2, 3$.

We have to underline that the main matrix $\left(\Omega_{22}-\Omega_{21}\Omega_{11}^{-1}\Omega_{12}\right)$ each of the three systems of equations (19) is not degenerate, since otherwise the necessary and sufficient condition for the existence of a unique solution to each of the three systems (19), and therefore to each of the three systems (15), and, consequently, to each of the three systems (10), would be violated.

The values of the unknowns t_{i1}, t_{i2}, t_{i3} for each i = 1, 2, 3 are found from the system of equations (19) using Cramer's formulae

$$t_{ik}=\frac{\Delta_{i,k}}{\Delta},\quad i,k=1,2,3, \tag{20}$$

where

$$\Delta=\det\left(\Omega_{22}-\Omega_{21}\Omega_{11}^{-1}\Omega_{12}\right),$$

and each of the determinants $\Delta_{i,k}$ is obtained from the determinant Δ by replacing its k-th column with the column of free terms

$$\mathbf{\Gamma}_{2i}-\Omega_{21}\Omega_{11}^{-1}\mathbf{\Gamma}_{1i},$$

i, k = 1, 2, 3.

Substituting for each i = 1, 2, 3 into the first three equations of the system corresponding to the extended matrix (18), instead of t_{i1}, t_{i2}, t_{i3} the values of these unknowns calculated by formulae (20), we obtain a system of three equations for three unknowns s_{i1}, s_{i2}, s_{i3}, the matrix notation of which has the form

$$\Omega_{11}\mathbf{\Sigma}_i=\mathbf{\Gamma}_{1i}-\Omega_{12}\tilde{\mathbf{\Lambda}}_i, \tag{21}$$

where $\tilde{\mathbf{\Lambda}}_i$ is column-vector of solutions of the system of equations (19), $\mathbf{\Sigma}_i=(s_{i1}, s_{i2}, s_{i3})^T$, i = 1, 2, 3. The values of the unknowns s_{i1}, s_{i2}, s_{i3} for each i = 1, 2, 3 are found from the system of equations (21) using Cramer's formulae.

$$s_{ik}=\frac{\tilde{\Delta}_{i,k}}{\tilde{\Delta}},\quad i,k=1,2,3,$$

where

$$\tilde{\Delta}=\det\left(\Omega_{11}\right),$$

а каждый из определителей $\tilde{\Delta}_{i,k}$ получаем из определителя $\tilde{\Delta}$ заменой его k-го столбца на столбец свободных членов

$$\mathbf{\Gamma}_{1i}-\Omega_{12}\tilde{\mathbf{\Lambda}}_i,$$

i, k = 1, 2, 3.

Having found the values of the unknowns t_{i1}, t_{i2}, t_{i3}, s_{i1}, s_{i2}, s_{i3}, we find the values of the unknowns b_i and θ_i using formulas (14), i = 1, 2, 3.

We prove that the found stationary point of the function Φ, defined by equality (8), that is the solution of the system of equations (9), obtained in the manner described above, provides the minimum of the function Φ. We derive the expression for the second differential $d^2\Phi$ of the function Φ:

$$d^2\Phi=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{m=1}^{3}\left[\frac{\partial^2\Phi}{\partial s_{ij}\partial s_{km}}ds_{ij}ds_{km}+2\frac{\partial^2\Phi}{\partial s_{ij}\partial t_{km}}ds_{ij}dt_{km}+\frac{\partial^2\Phi}{\partial t_{ij}\partial t_{km}}dt_{ij}dt_{km}\right]+$$

$$+\sum_{i=1}^{3}\sum_{m=1}^{3}\left[2\frac{\partial^2\Phi}{\partial b_i\partial\theta_m}db_id\theta_m+\frac{\partial^2\Phi}{\partial b_i\partial b_m}db_idb_m+\frac{\partial^2\Phi}{\partial\theta_i\partial\theta_m}d\theta_id\theta_m\right]+$$

$$+2\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\left[\frac{\partial^2\Phi}{\partial s_{ij}\partial b_k}ds_{ij}db_k+\frac{\partial^2\Phi}{\partial s_{ij}\partial\theta_k}ds_{ij}d\theta_k+\frac{\partial^2\Phi}{\partial t_{ij}\partial b_k}dt_{ij}db_k+\frac{\partial^2\Phi}{\partial t_{ij}\partial\theta_k}dt_{ij}d\theta_k\right]=$$

$$=2\left\{\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{m=1}^{3}\sum_{l=1}^{N}\left[h_j^{(l)}h_m^{(l)}\Delta s_{ij}\Delta s_{im}+2\tau^{(l)}h_j^{(l)}h_m^{(l)}\Delta s_{ij}\Delta t_{im}+\left(\tau^{(l)}\right)^2 h_j^{(l)}h_m^{(l)}\Delta t_{ij}\Delta t_{im}\right]+\right.$$

$$\sum_{i=1}^{3}\sum_{l=1}^{N}\left[2\tau^{(l)}\Delta b_i\Delta\theta_i+\left(\Delta b_i\right)^2+\left(\tau^{(l)}\right)^2\left(\Delta\theta_i\right)^2\right]+$$

$$\left.+2\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{l=1}^{N}\left[h_j^{(l)}\Delta s_{ij}\Delta b_i+\tau^{(l)}h_j^{(l)}\Delta s_{ij}\Delta\theta_i+\tau^{(l)}h_j^{(l)}\Delta t_{ij}\Delta b_i+\left(\tau^{(l)}\right)^2 h_j^{(l)}\Delta t_{ij}\Delta\theta_i\right]\right\}=$$

$$=2\sum_{i=1}^{3}\sum_{l=1}^{N}\left\{\Delta b_i+\tau^{(l)}\Delta\theta_i+\sum_{j=1}^{3}h_j^{(l)}\Delta s_{ij}+\sum_{j=1}^{3}\tau^{(l)}h_j^{(l)}\Delta t_{ij}\right\}^2,$$

where $\Delta s_{ij} = ds_{ij}$, $\Delta t_{ij} = dt_{ij}$ $(i, j = 1, 2, 3)$, $\Delta b_i = db_i$, $\Delta\theta_i = d\theta_i$ $(i = 1, 2, 3)$.
где $\Delta s_{ij} = ds_{ij}$, $\Delta t_{ij} = dt_{ij}$ $(i, j = 1, 2, 3)$, $\Delta b_i = db_i$, $\Delta\theta_i = d\theta_i$ $(i = 1, 2, 3)$. Provided that at least one of the terms is not equal to zero, the obtained expression for $d^2\Phi$ can result in

$$\left\{\Delta b_i+\tau^{(l)}\Delta\theta_i+\sum_{j=1}^{3}h_j^{(l)}\Delta s_{ij}+\sum_{j=1}^{3}\tau^{(l)}h_j^{(l)}\Delta t_{ij}\right\}^2\quad\left(i=1,2,3,\ l=1,\ldots,N\right)$$

In the last double sum (from a physical point of view this is quite natural) the inequality $d^2\Phi > 0$ takes place at any point, and therefore, at the found stationary point of the function Φ. Consequently, the obtained solution of the system of equations (9) actually provides a minimum of the function Φ [19].

Conclusion

Therefore, we have obtained an analytical solution to the problem of calibrating a spacecraft magnetometer for a model that considers the vector of temperature dependence of zero offsets for each of the measuring axes of the magnetometer unit and the matrix of linear temperature dependence of each of the members of the sensitivity matrix, scaling the signal based on the real sensitivity of each axis and including linear off-axis effects.

The procedure for calculating the calibration parameters of the MU using the derived formulae has the following obvious advantages compared to numerical methods to solve this problem:

– the number of arithmetic operations is significantly reduced;
– the problem of possible instability of the method disappears.

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