ЕСТЕСТВЕННЫЕ И ТОЧНЫЕ НАУКИ

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THE DIOPHANTINE EQUATION FROM THE EYE OF PHYSICIST

Abstract. A Diophantine equation is an equation with integer coefficients, the solutions of which must be found among integers. The equation is named after the mathematician Diophantus of Alexandria (III century). Despite its simplicity, a Diophantine equation may have a nontrivial solution (several solutions) or has no solution at all. Fermat's Last Theorem and Pythagorean Theorem are the Diophantine equations too. In 1900 The German mathematician David Hilbert formulated the Tenth problem. After 70 years, the answer turned out to be negative: there is no general algorithm. Nevertheless, for some cases, schoolchildren can understand whether a Diophantine equation is solvable without resorting to calculations, relying on the methods of physics, symmetry and set theory.

Keywords: Diophantine equation, Fermat, physics, symmetry, hypercube, binary relations.

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ДИОФАНТОВО УРАВНЕНИЕ ГЛАЗАМИ ФИЗИКА

Аннотация. Диофантово уравнение – это уравнение с целыми коэффициентами, решения которого следует также искать в целых числах. Уравнение названо в честь математика Диофанта Александрийского (III век). Несмотря на свою простоту, такое уравнение может иметь нетривиальное решение (несколько решений) или вообще не иметь решения. Примеры таких уравнений: теорема Пифагора, Великая теорема Ферма. В 1900 году немецкий математик Давид Гильберт сформулировал «Десятую проблему», позволяющую сразу понять, разрешимо ли диофантово уравнение? Спустя 70 лет ответ оказался отрицательным: общего алгоритма не существует. Тем не менее, в ряде случаев школьники

могут понять, разрешимо ли диофантово уравнение, не прибегая к вычислениям, опираясь на методы физики, симметрии и теории множеств.

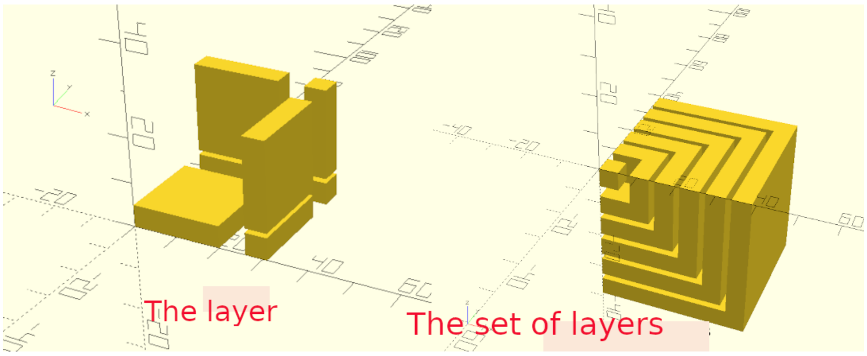
Ключевые слова: Диофантово уравнение; Ферма; физика; симметрия; гиперкуб; бинарные отношения.

Fermat's Last Theorem was formulated by Pierre de Fermat in 1672, it states that the Diophantine equation:

$$a^n + b^n = c^n \quad (1)$$

has no solutions in integers, except for zero values, for n > 2. The case degree of two is known in the school course under the name theorem Pythagoras. Euler in 1770 proved Theorem (1) for n=3, Dirichlet and Legendre in 1825 - for n = 5, Lame - for n = 7. In 1994 Prof. Princeton University Andrew Wiles proved (1), for all n, but this proof, contains over one hundred and forty pages, understandable only to high qualified specialists in the field of number theory [2]. But there is also a brief proof to the contrary:

If a triple of integers $a^n + b^n \equiv c^n$ exists, then it can map three nested integer edges hypercubes into each other (the centers of the nested hypercubes are aligned with the origin coordinates) while the volume of the small hypercube a^n is equal to the difference between the volumes $c^n - b^n$. Here the identity sign ‘≡’ means independence from the scale and set partition of our construction, i.e. a triple of integers in meters, decimeters, centimeters, millimeters. It is easy to prove that the condition for the equality of volumes and the properties of the central symmetry, continuity of the formed constuction mutually exclude each other. To understand this let’s mentally move the layer from set of points in space described by the formula $c^n - b^n$ into a small cube a^n and vice-versa.



Picture 1. The figure of one layer (left) and set of layers in the octant (+, +, -)

Here below a layer is defined as a set of points of a multidimensional spaces of real numbers R^n between successively following hypercubes with integer edges. The layer, like the whole n-dimensional figure, consists of elementary hypercubes 1^n in whole number space denoted as Z^n.
The designed construction of three nested hypercubes can be filled of layers step-by-step from the periphery to the center and vise-versa like building a frame house. This is the method used Euclid’s

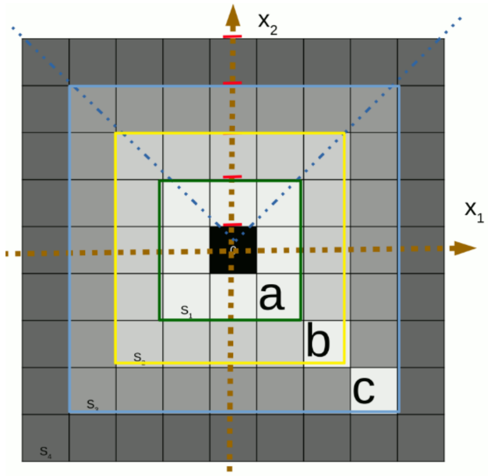
Elements [3]. A layer from the c-*Large* hypercube must fit an integer number of times in the a-*Small* hypercube (due to the excess of large over small - two or more times), otherwise the central symmetry of the construction or the continuity of the ordered layers will be lost.

Here understanding the structure of the layer gives the following formula

$$S_i = (i+1)^n - i^n = \sum_{k=0}^{k=n-1} \binom{n}{k} i^k 1^{n-k} \qquad (2)$$

The formula above is convenient to use for figure three inscribed in each other hypercubes, "origin of coordinate placed in vertices". Another view is "origin of coordinate placed in centers of the hypercubes". Both geometrical constructions are transformed into each other due to reflections from planes perpendicular to each of the n coordinate axes, or by cutting the figure and scaling.

Each layer of hypercube have elements of dimensions n-1, n-2, ... 1 (hyper)faces and edges such elements is described by formula $i^k 1^{n-k}$ - i.e. *cuboid*. "At the destination" volumes of elements of each dimension must be identically equal the volume of the corresponding moved element, by virtue of the principle incompressibility of the volume of a solid body and the equivalence of the quantity elementary hypercubes 1^n. These conditions lead to a system of n-1 equations that is not solvable for n > 2 not only in integers, but also in real numbers. To understand this we recall impossibility of constructing a right triangle, in which the hypotenuse is equal to the sum of the lengths of the legs. It is easy to verify that for these conditions, one of the legs will necessarily be equal to zero. Consequently, the construction of three nested hypercubes with integer edges is not exists in a space of whole numbers Z^n, n > 2 (*aporia* in terms of Ancient Greek philosophy), and there is no such triplet of numbers that would violate the Fermat's Last theorem.



Picture 2. Three nested hypercubes. *Piercing* by a two-dimensional plane. There is no parallax effect

The thesis about the piercing (or penetrating) rather than cutting plane of a two-dimensional hypercube is easy to understand the basics of linear algebra **AX = B** (matrix form). It follows from the Kronecker-Capelli theorem that the set of solutions X to a system of linear equations forms a hyperplane of dimension n - rank **A** in R^n. For example, for a three-dimensional space and a two-dimensional intersection plane: dim (**X**) = 3 - 2 = 1. For 4 dimensional space and more dim(X) = 4 -

2 = 2 and so on. Therefore, a two-dimensional probe can be covered by a closed loop in a plane orthogonal to the piercing one, and it is appropriate to speak of piercing rather than intersection.)
In XVII century described physical approach was enough for proof, but not in XXI. More formal approaches is required [1].

The set Theory and binary relations approaches

It should be noted without change generality that the natural numbers in formula (1) are related as $a < b < c$, and the situation of equality of edges $a = b$ is excluded due to the irrationality of $\sqrt[n]{2}$. The case of negative numbers can be considered by moving term into another part of the equation and substitution of variables - it is enough prove the theorem for the case of natural numbers a, b, c and generalize the result to whole numbers Z.

Let's consider inscribed hypercubes with edges, obtained from a series of consecutive natural numbers N_1, the centers which coincide with the origin of coordinates, and the faces are perpendicular to the axes coordinates Hypercubes e_i with edges i based on a series natural numbers inscribed in each other form an increasing chain sets and the inclusion relation in the set U which is understood as large hypercube with edge c:

$$e_0 \subseteq e_1 \dots \subseteq e_k \subseteq e_{k+1} \dots e_{k+l} \subseteq e_{k+l+1} \dots \subseteq e_{k+l+m} \subseteq U \qquad (3)$$

$$1^n \cup S_1 \cup S_2 \dots \cup S_k \cup S_{k+1} \dots \cup S_{k+l} \cup S_{k+l+1} \dots \cup S_{k+1+m} \subseteq U$$

A set partition one can see above. On the other hand, this formula describes a one-dimensional probe penetrating three nested hypercubes through a common center. The result of the *Cartesian Product* of two orthogonal probes can be seen in Picture 2 above, so the researcher can obtain a two-dimensional plane regardless of the space dimension. There is no parallax effect.

As mentioned in (3) the *a-Small* n-cube a^n is the set of layers from 1 to k, the *b-Medium* b^n is the set of layers from k+1 to $k + l$ and the *c-Large* c^n is the set of layers from $k+l+1$ to $k + l + m$. The layer is defined as the subset difference $S_i = e_i \setminus e_{i-1}$, $i > 1$. The first hypercube e_0 denote 1^n or 2^n, in parity, but given the enclosures below, this detail is not leads to qualitative differences. The mathematicians of ancient Greece introduced the concept of incommensurability of linear segments.

Table

The postulates of Euclid in the Digital epoch

Figure in Euclidean space (R^n) provided central symmetry	Analogue in Z^n set of hypercubes provided central symmetry	Dimension
Dot	1^n	**0** and at the same time **n** depending on the situation
Linear segment	$i^1 1^{n-1}$ set of hypercubes cardinality = i lined up in a row or column one	1
Plane	$i^2 1^{n-2}$ set of hypercubes cardinality = i^2 ordered in a square	2

The linear segments of length √2 and 1 are incommensurable. From these positions, each layer S_i is incommensurable with another S_j in the Z^n, n >2. It is easy to see that the analogous is true for sets of continuously following layers. The "uniqueness" of a layer can be formed by the condition: ∄ scale and set partition and natural i, j for which the *measure* $|S_i| = |S_j| \pm |S_{j-1}| + \ldots$ for n > 2, where |S| is *cardinality* i.e. quantity of 1^n in the investigated set. The *axiom of defining the measure* (volume in in terms of physics) over the set is violated. The measures of the set of layers S are not possess the *additivity* property in whole number multidimensional space Z^n for n > 2. The operations of addition, subtraction, reduction, other comparison of different layers are being prohibited. So formula (3) describing structure of hypercube and understanding *measure axiom* for S_i in Z^n are enough for proof.

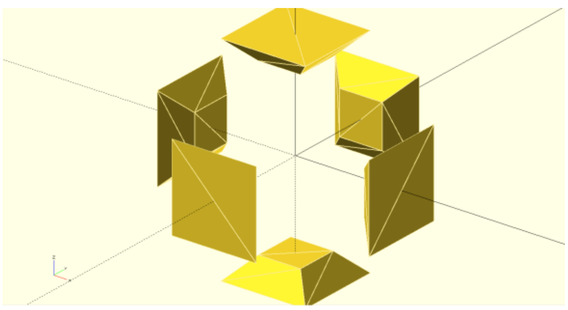
If ∃ the function F map the set of hypercube elements from the large hypercube c^n (the set U) into it F = { (y, z) | ∀ y ∈ U ∃! x ∈ U} then: ∀ F = G * H where: H = {(x, y) | x ∈ {Si} ∧ y ∈ {Si} } are equivalent relations within and G = {(x, y) | x ∈ {Si} ∧ y ∈ {Sj} : i ≠j } between different layers. Let us focus on the *restriction* of the relation G to one specific layer $G|_{Si}$ = {(x, y)| x ∈ {S_i} ∧ {S_i} ∈ $c^n \setminus b^n$, y ∈ *a-Small*}.

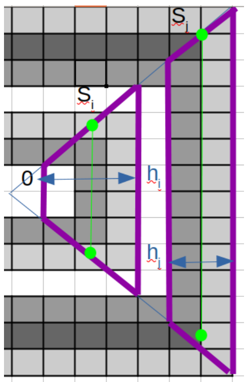
Rejecting options that violate the central symmetry of the constuction, one can get either a set of layers in *a-Small*, or the entire subset of it as a whole, depending on the ratio of powers $|S_i|$ and |*a-Small*| subsets. By scaling and decreasing the thickness of the layers, it is possible to achieve a situation where a single layer from $c^n \setminus b^n$ is mapped to a set of layers into a-*Small*. Obviously ∄ equivalence function F in Z^n, n > 2 maintaining the fundamental properties of the our construction: central symmetry and continuous succession of layers (because the function G should transfer *pairwise disjoint equivalence classes* of the elements $i^k 1^{n-k}$ – cuboid, but to ensure the simultaneous matching of the elements of the layer more than to one class is impossible due to the unsolvability for n > 2 of the stipulated below system of n-1 equations):

$$j^{n-1} = i^{n-1} + (i-1)^{n-1} + \ldots \text{ (two or more terms)} \quad (4)$$

$$j^{n-2} = i^{n-2} + (i-1)^{n-2} + \ldots \text{ (two or more terms)}$$

. . . *this series of equations continues from n-1 to 1 power. The observing construction has been filling of layers from the periphery to the center.*





Picture 3. ∄ equivalence function F in Z^n , n > 2 maintaining the fundamental properties of the Construction: central symmetry and continuous succession of layers except two-dimensional case (trapezoid)

For the special case Z^2 ∃ G, thanks to one equivalence class: comparison of trapezoid square is possible for ∀ i , j: ∃ h_i and h_j such as $S_i * h_i = S_j * h_j$ in Q numbers and by virtue of scaling for Z.

In the middle of the 20th century french mathematician Claude Chabauty in 1938 defended his doctoral dissertation on number theory and algebraic geometry, actively applied the methods of symmetry of subspaces in analysis of Diophantine equations. Minhyong Kim a mathematician from the University of Oxford, researching hidden arithmetic symmetry of the Diophantine equations, said: "It should be possible to use ideas from physicists to solve problems in number theory, but we haven't thought carefully enough about how to set up such a framework" (https://clck.ru/32nv3t). The algorithmic unsolvability of Hilbert's Tenth Problem was proved by Yuri Vladimirovich Matiyasevech in 1970 at the St. Petersburg branch of the Mathematical Institute. V. A. Steklov RAS [4]. From a philosophical standpoint, formula (1) has a contradiction between form (central symmetry) and content (volume) for n >2.

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