Non-coplanar planetary-centric dynamics simulation of low thrust spacecraft

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Rationale. A solar sail is a device that uses the pressure of sunlight on a reflective surface to propel a spacecraft [1].

With the help of solar sails, the spacecraft can continue to accelerate as long as light pressure is applied to it. In a solar system, the pressure of light on the sail occurs throughout the flight. This means that spacecraft propelled by solar sails (SSS) can reach speeds that are almost impossible to achieve with chemical rockets.

Active control of the solar sail position is necessary to adjust the solar pressure force for trajectory changes and orbit control.

Objective. To develop a procedure for determining the optimum on/off control function of the SSS for the minimum flight time.

Methods. The starting orbit is a circular heliocentric trajectory, which coincides with the Earth's orbit.

A flat polar coordinate system is used to describe the heliocentric motion of the SSS.

Consider a solar sail with a perfectly reflecting surface with sides *a* and *b*, and with the presence of control surfaces of width *h* and length *a*. In this case there will be 2 variants of operation, control surfaces. At $\delta = -1$ the control surfaces completely absorb photons. At $\delta = +1$ the film is completely transparent and therefore a perfectly reflecting surface. At $\delta = 0$ both surfaces are in the same state and consequently no control momentum arises. In this way, by alternately switching on and off the relevant surfaces on the sail it is possible to perform the manoeuvres required for interplanetary flight.

The system of differential equations describing the controllable coplanar and non-coplanar motion of the SSS is as follows.

$$\begin{cases} \dfrac{dr}{dt} = V_r; \\ \dfrac{d\varphi}{dt} = \dfrac{V_\varphi}{r}; \\ \dfrac{dV_r}{dt} = \dfrac{V_\varphi^2}{r} - \dfrac{\mu}{r^2} + P_a \cdot \dfrac{ab}{M} \cdot \dfrac{1}{r^2} \cdot \cos^3 \lambda; \\ \dfrac{dV_\varphi}{dt} = -\dfrac{V_r V_\varphi}{r} + P_a \cdot \dfrac{ab}{M} \cdot \dfrac{1}{r^2} \cdot \cos^2 \lambda \cdot \sin\lambda; \\ \dfrac{d\lambda}{dt} = \omega_z - \dfrac{V_\varphi}{r}; \\ \dfrac{d\omega_z}{dt} = \dfrac{M_{\text{вн.сил}}\delta}{I_z} = \dfrac{3P_a \cdot h \cdot b}{M \cdot a^2 r^2} \cdot (a-h)\cos\lambda \cdot \delta. \end{cases}$$

$$\begin{cases} \dfrac{dr}{dt} = V_r; \\ \dfrac{d\varphi}{dt} = \dfrac{V_\varphi}{r} - \dfrac{\cos i \cdot \sin\varphi \cdot a_n}{\sin i \cdot V_\varphi}; \\ \dfrac{dV_r}{dt} = \dfrac{V_\varphi^2}{r} - \dfrac{\mu}{r^2} + a_r; \\ \dfrac{dV_\varphi}{dt} = -\dfrac{V_r V_\varphi}{r} + a_\varphi; \\ \dfrac{d\lambda_1}{dt} = -\dfrac{\omega_y}{\sin\lambda_1}; \\ \dfrac{d\omega_x}{dt} = \dfrac{M_x}{I_x}; \\ \dfrac{d\lambda_2}{dt} = \omega_x; \\ \dfrac{d\omega_y}{dt} = \dfrac{M_y}{I_y}; \\ \dfrac{d\Omega}{dt} = \dfrac{\sin\varphi \cdot a_n}{\sin i \cdot V_\varphi}; \\ \dfrac{di}{dt} = \dfrac{\cos\varphi \cdot a_r}{V_\varphi}. \end{cases}$$

r — the current distance from the spacecraft to the attraction centre; φ — current angular range of the spacecraft, V_r — spacecraft velocity projection onto the radius vector; V_φ — projection of the velocity of the spacecraft in the transversal direction; μ — the value of the Earth's gravitational parameter; a_r, a_φ — control acceleration components; λ — angle between the radius vector gravity centre — spacecraft and the normal to the sail plane; ω_z — current angular velocity of the spacecraft; ξ_z — angular acceleration.

Firstly, we consider a flat problem. The ballistic optimisation problem is formulated as follows [2].

Determine a vector of control function $\overline{u}(t) \in U$ and a vector of ballistic parameters of flight $\overline{b} \in B$, delivering for a given mass of spacecraft with a solar sail a minimum time of flight and ensuring the fulfillment of the target problem of the project, described by the set of allowed phase coordinates of the vehicle $\overline{x}(t) \in X$:

$$T_{opt} = \min_{\overline{u} \in U,\, \overline{b} \in B} T\left(M_0 = fixe, \overline{x}(t) \in X, \overline{u}(t), \overline{b}\right).$$

In order to determine the optimal law of change of the control angle of the acceleration vector $\overline{u}(t)$, and hence the on-off function of the control planes δ_{opt} it is necessary to proceed to the variational problem of speed-optimal flights between circular coplanar orbits.

Find the maximum speed control according to the Pontryagin maximum principle.

$$\delta_{opt} \to \max H \Rightarrow \delta_{opt} = \begin{cases} \psi_{\omega_z} > 0 \Rightarrow \delta = +1; \\ \psi_{\omega_z} < 0 \Rightarrow \delta = -1. \end{cases}$$

Consider the problem of orbiting Mars without velocity alignment.

$$\begin{cases} r = r_0 = 1 \ a.e.; \\ \varphi = \varphi_0; \\ V_r = V_{r_0} = 0; \\ V_\varphi = V_{\varphi_0} = 1; \\ \lambda = \lambda_0; \\ \omega_z = 0. \end{cases} \to \begin{cases} \psi_r = y_1; \\ \psi_\varphi = 0; \\ \psi_{V_r} = y_2; \\ \psi_{V_\varphi} = y_3; \\ \psi_\lambda = y_4; \\ \psi_{\omega_z} = y_5. \end{cases} \underset{t = T_{\text{к}} = y_6 \to \min}{} \overset{t \to \min}{\to} \begin{cases} r = r_n; \\ \varphi = unfix; \\ V_r = unfix; \\ V_\varphi = unfix; \\ \lambda = unfix; \\ \omega_z = unfix. \end{cases} \to \begin{cases} \psi_r = unfix; \\ \psi_\varphi = 0; \\ \psi_{V_r} = 0; \\ \psi_{V_\varphi} = 0; \\ \psi_\lambda = 0; \\ \psi_{\omega_z} = 0. \end{cases}$$

And a flight to Mars orbit with velocity compensation.

$$\begin{cases} r = r_0 = 1 \ a.e.; \\ \varphi = \varphi_0; \\ V_r = V_{r_0} = 0; \\ V_\varphi = V_{\varphi_0} = 1; \\ \lambda = \lambda_0; \\ \omega_z = 0. \end{cases} \to \begin{cases} \psi_r = y_1; \\ \psi_\varphi = 0; \\ \psi_{V_r} = y_2; \\ \psi_{V_\varphi} = y_3; \\ \psi_\lambda = y_4; \\ \psi_{\omega_z} = y_5. \end{cases} \underset{t = T_{\text{к}} = y_6 \to \min}{} \overset{t \to \min}{\to} \begin{cases} r = r_{\ddot{i}}; \\ \varphi = unfix; \\ V_r = V_n; \\ V_\varphi = V_n; \\ \lambda = unfix; \\ \omega_z = unfix. \end{cases} \to \begin{cases} \psi_r = unfix; \\ \psi_\varphi = 0; \\ \psi_{V_r} = unfix; \\ \psi_{V_\varphi} = unfix; \\ \psi_\lambda = 0; \\ \psi_{\omega_z} = 0. \end{cases}$$

Results.

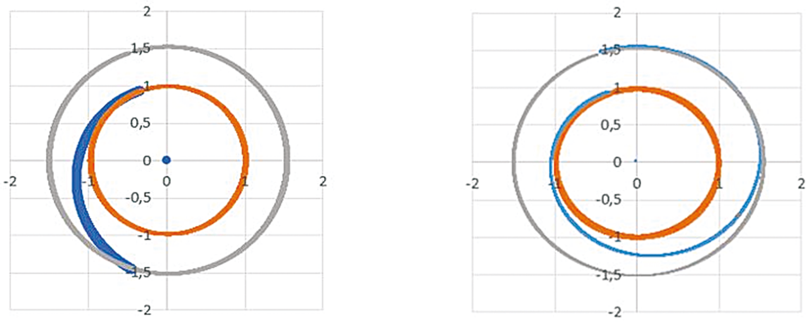

Fig. 1. SSS trajectory during Mars flyby and velocity compensation in Mars orbit

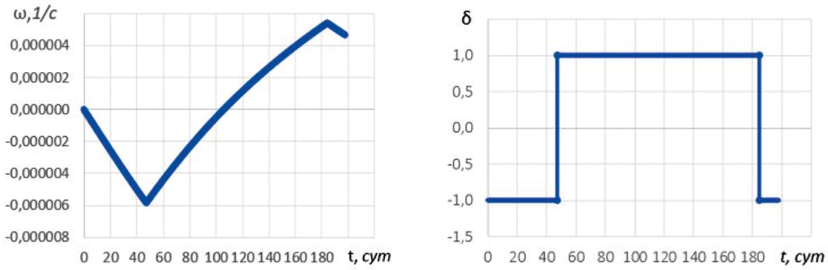

Fig. 2. Changing angular velocity ω and on-off function of the control planes δ during a Mars flyby

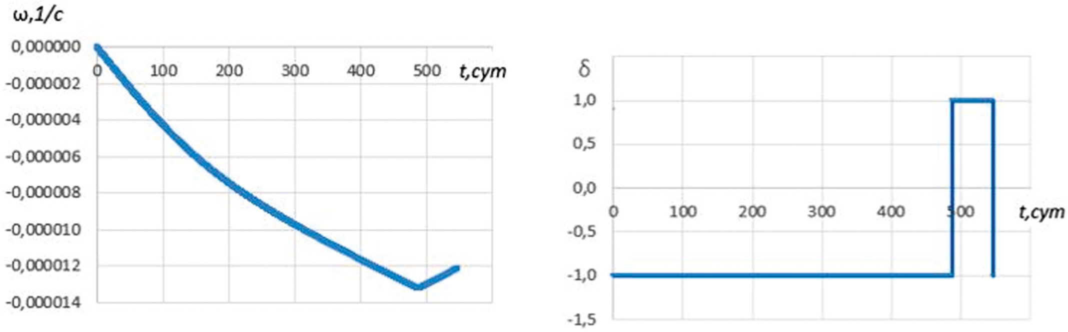

Fig. 3. Change of angular velocity ω and on-off function of the control planes δ during velocity compensation in Mars orbit

Conclusions. The paper describes a mathematical model of the guided motion of the SSS using control surfaces. The mathematical model of the angular motion of the SSS required for an optimal flight from the Earth's orbit to Mars orbit is described. The ballistic calculation of the SSS orbit of Mars, as well as the flight to Mars with velocity equalization is carried out.

Keywords: solar sail spacecraft; interplanetary flight; motion control; mathematical model.

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