Rubric 2: SCIENTIFIC AND PRACTICAL DEVELOPMENT
UDC [УДК] 519.214+517.162
DOI 10.17816/transsyst20195142-53

© K. E. Voevodskii[1], V. M. Strepetov[2]
[1]St. Petersburg State University
[2]Emperor Alexander I St. Petersburg State Transport University
(St. Petersburg, Russia)

THE PROBABILITY OF THE CORRECT MAJORITY MADE DECISION

Aim: the probability of correctness of collective decision is studied in this paper, whereby the decision is made by majority vote of some team (group), consisting of an odd number of members, provided that the probability of correctness of individual decision of each member of the group is known.

Methods: the Bernoulli scheme, asymptotic representation, estimation by virtue of geometric progression, exponential series expansion, Wallis' product, a power scale of averages, Kolmogorov mean.

Result: it was found, that if for each member of the group the probability of the correct decision is more than ½, then with an unlimited increase of the number of members of the group, the probability of the right collective decision tends towards one. The asymptotic representation and a number of two-sided assessments which characterise the speed of this tending were obtained. For a non-homogeneous group (i.e. the group the members of which make the right individual decision with different probability) the notion of *a collective average* was introduced here as an averaged characteristic that can be used to replace individual probability of each group member saving the probability of the right collective decision. The existence and uniqueness of a collective average was proved.

A collective inequality was identified which shows that a collective average of some set of numbers is no less than the geometric mean of the same numbers, and the equality is present if and only if all members are equal at that. A collective inequality serves as analogue and addition to the known set of inequalities establishing connection between two different average values (for instance, the AM–GM inequality).

Conclusion: thus, the results of the study fully meet the aim of determining the probability of correct decision made by a majority of votes under the assumptions taken. As a result, asymptotic representation and bilateral estimates characterising the speed of tending to the correct decision was obtained. For a non-homogeneous group, the existence and uniqueness of the concept of collective average as an averaged characteristic were introduced and firmly proved, which can be used to replace an individual probability of each group member, whereby preserving the probability of correctness of the collective decision. It was found that the collective average is no less than the geometric mean. Potential applications of the results obtained can be the quantitative evaluation of election procedures and the solution of problems associated with improving the reliability of recognition of weak signals of control sensors in various transport systems, including high-speed transport systems on magnetic suspension.

Keywords: group of experts, the formula of full probability, geometric progression, recurrence relation, asymptotic representation, binomial series, collective average, geometric mean, exponential average, transport systems.

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

© К. Э. Воеводский[1], В. М. Стрепетов[2]
[1]Санкт-Петербургский государственный университет
[2]Петербургский государственный университет путей сообщения Императора Александра I
(Санкт-Петербург, Россия)

ВЕРОЯТНОСТЬ ПРАВИЛЬНОСТИ РЕШЕНИЯ, ПРИНИМАЕМОГО БОЛЬШИНСТВОМ ГОЛОСОВ

Цель. Исследуется вероятность правильности коллегиального решения, которое принимается большинством голосов некоторого коллектива (коллегии), состоящего из нечетного числа членов, если известна вероятность правильности индивидуального решения каждого члена коллегии.

Методы. Схема Бернулли, асимптотическое представление, оценка посредством геометрических прогрессий, разложение в степенной ряд, формула Валлиса, степенная шкала средних величин, среднее по Колмогорову.

Результаты. Установлено, что если для каждого члена коллегии вероятность правильности индивидуального решения больше ½, то при неограниченном росте числа членов коллегии вероятность правильности коллегиального решения стремится к 1. Получены асимптотическое представление и ряд двусторонних оценок, характеризующие скорость этого стремления. Для неоднородной коллегии (такой коллегии, члены которой принимают правильное индивидуальное решение с разной вероятностью) введено понятие *коллегиального среднего* как усредненной характеристики, которой можно заменить индивидуальную вероятность каждого члена коллегии с сохранением вероятности правильности коллегиального решения. Доказано существование и единственность коллегиального среднего.

Выведено *коллегиальное неравенство*, показывающее, что коллегиальное среднее некоторого набора чисел не меньше среднего геометрического тех же чисел, причем равенство имеет место в том и только том случае, когда все числа равны между собой. Коллегиальное неравенство служит аналогом и дополнением известному набору неравенств, устанавливающих связь между различными средними величинами (например, неравенство Коши для среднего арифметического и среднего геометрического).

Заключение. Полученные результаты проведенного исследования полностью отвечают поставленной цели по определению вероятности правильности коллегиального решения, принятого большинством голосов при введенных допущениях. В результате получены асимптотическое представление и двусторонние оценки, характеризующие скорость стремления к правильному решению. Для неоднородной коллегии введены и строго доказаны существование и единственность понятия коллегиального среднего как усредненной характеристики, которой можно заменить индивидуальную вероятность каждого с сохранением вероятности правильности коллегиального решения. Установлено, что коллегиальное среднее не меньше среднего геометрического. Прикладными направлениями применения полученных результатов могут служить количественная оценка выборных процедур и решение проблем, связанных с повышением надежности распознавания слабых сигналов датчиков контроля различных транспортных систем, включая высокоскоростные транспортные системы на магнитном подвесе.

Ключевые слова: коллегия экспертов, формула полной вероятности, геометрическая прогрессия, рекуррентные соотношения, асимптотическое представление, биномиальный ряд, коллегиальное среднее, среднее геометрическое, среднее степенное, транспортные системы.

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

doi: 10.17816/transsyst20195142-53

INTRODUCTION

With the aim to increase the reliability of responsible decisions, the group decision making is used. For instance, significant calculations are tasked to several individuals, and the result is considered to be reliable if it is the same at all the individuals. Another example is obtaining experiment findings. In case of discrepancies in measuring sensors' weak signals values, the preference is given to the majority's data. This approach rests on the hypothesis that the decision that is made by the majority vote of some group (group of experts) is more correct than that of each expert's individually. The major aim of this work is to verify this hypothesis and assess quantitatively the collectiveness effect.

MAIN NOTIONS AND TASK SETTING

Let one assume that the group consists of (2*n*+1) experts (for short - (2*n*+1) -group) and is characterised by vector $X=(x_1,...,x_{2n+1})$, where $x_k \in [0;1]$ – probability that *k*-th expert takes the right decision. Then, by the law of total probability [1], the probability that the majority vote results in the right decision, is determined by the expression

$$\Pi(X)=\sum_{k=0}^{n}\sum_{\{i_1,..i_k\}\subset\{1,...,2n+1\}}(1-x_{i_1})...(1-x_{i_k})\cdot(x_1\cdot...\cdot x_{2n+1})/(x_{i_1}\cdot...\cdot x_{i_k}). \quad (1)$$

In the expression (1) the *k*-th number of the outer sum is the probability of the incorrect decision by *k* experts, and the sum is the probability that the incorrect decision has been made by less than half of the experts. If the fraction in (1) loses its literal sense as a result of some components equaling 0, it should be considered equal to multiplication of all components except for $x_{i_1}\cdot...\cdot x_{i_k}$.

If $X = (p,..., p)$, let the group be called homogeneous (a good one, if $p > 0,5$, and a bad one if $p < 0,5$). For this type of group, the expression (1) turns into the Bernoulli scheme [2]: $\Pi(X)=P_{2n+1}=\sum_{k=0}^{n}C_{2n+1}^{k}\,p^{2n+1-k}\,q^{k}$, where $q=1-p$. Further on, it is more convenient to deal with the probability of mistake $Q_{2n+1}=1-P_{2n+1}$, i.e. the probability that the group makes the incorrect decision. It is obvious,

$$Q_{2n+1}=\sum_{k=0}^{n}C_{2n+1}^{k}\,p^{k}\,q^{2n+1-k}. \quad (2)$$

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

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HOMOGENEOUS GROUP

Two-sided assessment of probability of mistake

Let the group be considered to be a good one. If the multiplication $q(pq)^n$ is put outside the brackets, from (2) $Q_{2n+1} = q(pq)^n \sum_{k=0}^{n} C_{2n+1}^{k} \cdot (q/p)^{n-k}$.is obtained. With $p >$ 0,5 it follows that $(q/p)^n \le (q/p)^{n-k} \le 1$. This implies that $q(pq)^n (q/p)^n \sum_{k=0}^{n} C_{2n+1}^{k} \le Q_{2n+1} \le q(pq)^n \sum_{k=0}^{n} C_{2n+1}^{k}$. Considering that $\sum_{k=0}^{n} C_{2n+1}^{k} = 0{,}5 \sum_{k=0}^{2n+1} C_{2n+1}^{k} = 2^{2n}$ [3], and making other obvious simplifications, we obtain

$$q\left(4q^2\right)^n \le Q_{2n+1} \le q(4pq)^n. \tag{3}$$

Let us introduce the parameter α = 2p –1. It is obvious that p =0,5(1+α), q = 0,5(1–α), whereby for a good group $0 < \alpha \le 1$, and for a bad one $-1 \le \alpha < 0$. Expressing p and q through α, the inequality (3) can be converted to

$$q\left((1-\alpha)^2\right)^n \le Q_{2n+1} \le q\left(1-\alpha^2\right)^n. \tag{4}$$

For a good group $0 \le (1-\alpha)^2 < (1-\alpha^2) < 1$, means that the probability of mistake is assessed from both sides by infinitely decreasing geometric progressions. In particular, with unlimited growth in the number of experts, the probability of mistake tends towards 0.

Recurrence relations, decrease of probability of mistake

(2n+1)-group is a combination of (2n–1)-group and a pair of experts. Let $p_1(q_1)$ be the probability that (2n–1)-group have made the correct (incorrect) decision dominating by one vote. (2n+1)-group makes the incorrect decision in the following cases:

1) (2n–1)-group have voted incorrectly by a margin of more than one vote; the probability of this equals $Q_{2n-1} - q_1$;

2) (2n–1)-group have voted incorrectly by a margin of one vote, whereas a pair of experts has at least one that voted incorrectly; the probability of this event equals $q_1(1-p^2)$;

3) (2n–1)-group have voted correctly by a margin of one vote, and a pair of experts has two that have voted incorrectly; the probability of this event equals $p_1 q^2$.

Adding these probabilities together and considering that $p_1 = C_{2n-1}^{n} p^n q^{n-1}$, $q_1 = C_{2n-1}^{n-1} p^{n-1} q^n$, $p = 0{,}5(1+\alpha)$, $q = 0{,}5(1-\alpha)$ and $C_{2n-1}^{n} = C_{2n-1}^{n-1} = 0{,}5 C_{2n}^{n}$, we can obtain

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

Transportation Systems and Technology. 2019;5(1):42-53
doi: 10.17816/transsyst20195142-53

$$Q_{2n+1} - Q_{2n-1} = -(\alpha/2)C_{2n}^{n}\left(\left(1-\alpha^2\right)/4\right)^n . \tag{5}$$

For a good group (0< α ≤ 1) the right side (5) is negative, wherefrom it can be seen that the sequence Q_{2n+1} is a decreasing one, i.e. adding an additional pair of experts results in the decrease of probability of mistake.

Economical calculation formula

Having recorded Q_{2n+1} as Q_1+ (Q_3 – Q_1) +...+ (Q_{2n+1} – Q_{2n-1}), putting subtractions (Q_{2k+1}– Q_{2k-1}) from (5) and considering that Q_1= q = 0,5(1–α), we can obtain for Q_{2n+1} the following representation:

$$Q_{2n+1} = (1/2) - (\alpha/2)\sum_{k=0}^{n} C_{2k}^{k}\left(\left(1-\alpha^2\right)/4\right)^k = (1/2) - (\alpha/2)\sum_{k=0}^{n} A_k x^k , \tag{6}$$

where $x = 1-\alpha^2$, $A_k = C_{2k}^{k}/4^k$.

Let us also consider that, in accordance with [4] $A_k = C_{2k}^{k}/4^k = (2k-1)!!/(2k)!!$.

As n increases by 1, there is an additional of one term added to the addition in (6), and the already existing terms remain unchanged. In (2) as n increases, all the terms are changed. Therefore, the formula (6) *is more economical for numerical calculations.*

Asymptotic representation of the probability of mistake at $n\to\infty$

Considering the last representation for A_k, $\sum_{k=0}^{n} A_k x^k$ is a partial sum of binominal series with the facto r–1/2 [5]. Then $\sum_{k=0}^{n} A_k x^k = (1-x)^{-1/2} = 1/\alpha$, wherefrom $\sum_{k=0}^{n} A_k x^k = (1/\alpha) - \sum_{k=n+1}^{\infty} A_k x^k$. Putting this expression to (6) and putting the first member of the addition outside the brackets, we can obtain the following representation:

$$Q_{2n+1} = (\alpha/2)A_{n+1}x^{n+1}\sum_{k=0}^{\infty}\left(A_{k+n+1}/A_{n+1}\right)\cdot x^k . \tag{7}$$

Let us prove that at $n \to\infty$ the sum in (7) tends towards $\sum_{k=0}^{\infty} x^k$, i.e. we will assess the subtraction of these sums. According to Wallis' product [6]

$$1/\sqrt{\pi(n+1/2)} < A_n < 1/\sqrt{\pi n} . \tag{8}$$

From this above, we can easily obtain: $A_{k+n}/A_n > \sqrt{n/(k+n+1/2)}$, wherefrom we have $1 - A_{k+n}/A_n < 1 - \sqrt{n/(k+n+1/2)}$. The right side is less than the value $(k+1/2)/2n$, consequently, 0 < 1– A_{k+n+1}/A_{n+1}< (k+1/2)/2(n+1). This means that

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

Transportation Systems and Technology. 2019;5(1):42-53
doi: 10.17816/transsyst20195142-53

$$0<\sum_{k=0}^{\infty}x^k-\sum_{k=0}^{\infty}\left(A_{k+n+1}/A_{n+1}\right)x^k=\sum_{k=0}^{\infty}\left(1-\left(A_{k+n+1}/A_{n+1}\right)\right)x^k<\sum_{k=0}^{\infty}(k+1/2)/2(n+1)\cdot x^k=$$

$$=1/2(n+1)\left(\sum_{k=0}^{\infty}k\,x^k+1/2\sum_{k=0}^{\infty}x^k\right), \qquad (9)$$

where $\sum_{k=0}^{\infty}x^k=1/(1-x)$ (the sum of geometric progression [7]); $\sum_{k=0}^{\infty}kx^k==x/(1-x)^2$ [8]; wherefrom, considering $x=1-\alpha^2$, we have: $\sum_{k=0}^{\infty}x^k=1/\alpha^2$, $\sum_{k=0}^{\infty}kx^k=\left(1-\alpha^2\right)/\alpha^4$. Adding this to (9) and expressing $\sum_{k=0}^{\infty}\left(A_{k+n+1}/A_{n+1}\right)x^k$ in the resultant inequality , we can obtain:

$$\left(1/\alpha^2\right)\left(1-1/\left(2(n+1)\right)\cdot\left(1/\alpha^2-1/2\right)\right)<\sum_{k=0}^{\infty}\left(A_{k+n+1}/A_{n+1}\right)x^k<1/\alpha^2\,. \qquad (10)$$

The sum in (10) differs from Q_{2n+1} by the multiplier $(\alpha/2)\ A_{n+1}\ x^{n+1}$ (see. (7)). Multiplying (10) by this multiplier and assessing A_{n+1} from both sides by virtue of (8), we can obtain the following two-sided assessment for Q_{2n+1}:

$$\left(\left(1-\alpha^2\right)^{n+1}/\left(2\alpha\cdot\sqrt{\pi(n+1/2)}\right)\right)\left(1-1/\left(2(n+1)\right)\left(1/\alpha^2-1/2\right)\right)<Q_{2n+1}<\left(1-\alpha^2\right)^{n+1}/\left(2\alpha\sqrt{\pi n}\right). \quad (11)$$

It is obvious that the second multiplier in the left part is $1+O(1/n)$. Therefore, the following asymptotic representation follows from (11):

$$Q_{2n+1}=\left(1-\alpha^2\right)^{n+1}/\left(2\alpha\sqrt{\pi n}\right)\cdot\left(1+O(1/n)\right). \qquad (12)$$

Let us compare the assessments (4) and (11). It is seen that the second one is asymptotically more precise, i.e. for each fixed $\alpha>0$ there exists n, starting from which the range of change Q_{2n+1}, in (11) is more narrow than the range in (4). However, the assessment (4) has the advantage that it is even along α, whereas in (11) the range expands without limits at $\alpha\to 0$. Therefore, the relevance of this or that assessment for certain calculations depends on numerical values of input data.

COLLECTIVE AVERAGE

Let us return to $(2n+1)$-group of the general form, which is characterised by vector $X=(x_1,\ldots,x_{2n+1})$. Let us select such a homogeneous $(2n+1)$-group that has the same probability of the correct decision just the first one does. The value p of this homogeneous group (the probability of the right individual decision of its member) can be interpreted as an average value for a set of probabilities $x_1,\ldots,x_{2n+1}$. Let them be referred to as *a collective average* of the values $x_1,\ldots,x_{2n+1}$. In

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

doi: 10.17816/transsyst20195142-53

accordance with this definition, the collective average p of values $x_1,..., x_{2n+1}$ is a root of the expression

$$\Pi(X) = P_{2n+1}(p), \tag{13}$$

lying in the interval $0 \le p \le 1$, where $\Pi(X)$ is determined by formula (1), $P_{2n+1}(p) = \Pi(p, p...p) = \sum_{k=0}^{n} C_{2n+1}^{k} p^{2n+1-k}(1-p)^k$.

Existence and uniqueness of collective average

Let us study the monotony of $\Pi(X)$ by each variable, e.g. x_{2n+1}. Let us say that $2n$-group of experts with numbers 1, …, $2n$ have voted by a margin of k, if the right votes have been given by k more experts than incorrect ones. The margin acquires odd values from от $-2n$ to $2n$. With $k \neq 0$ the decision of $(2n+1)$-group does not depend on the decision of $(2n+1)$ expert, and with $k = 0$ it coincides with it. It means that $\Pi(X) = R_{2n}+...+ R_2+R_0\ x_{2n+1}$, where R_k – the probability of margin k. The probabilities $R_{2n},..., R_0$ do not depend on x_{2n+1}, and $R_0 \ge 0$, therefore $\Pi(X)$ is either strictly increasing function (at $R_0 \ge 0$), or a constant one (at $R_0 = 0$). The latter takes place when over half of the members of $2n$-group vote unanimously, and therefore their votes cannot be divided equally. In other words, among $x_1,..., x_{2n}$ there are more n components equalling 0, or more n, equalling 1. In the first case $\Pi(X) = 0$, in the second $\Pi(X) = 1$.

Thus, since $P_{2n+1}(p) = \Pi(p, p,\ldots p)$, then $P_{2n+1}(p)$ strictly increases at the interval (0; 1), where it is different from 0 and 1. But $P_{2n+1}(0) = 0$, $P_{2n+1}(1) = 1$. Thereby, $P_{2n+1}(x)$ strictly increases by [0; 1] from 0 to 1. It is uninterrupted, therefore the equation (13) has the sole solution [9].

Note: if in the set $(x_1,..., x_{2n+1})$ there are zeros or ones, for a collective average one of the properties of the Kolmogorov mean may be breached [10]: replacement of values of any subset in the set $(x_1,..., x_{2n+1})$ for the average value of this subset does not change the average value of the whole set. Therefore, for the collective average, generally speaking, the universal representation by Kolmogorov does not take place [10, 11] in the form $\varphi^{-1}(\varphi(x_1)+...+\varphi(x_n)/n)$, where φ is some strictly monotonic function.

COLLECTIVE INEQUALITY

Further on, it will be proved that the collective average is more than or equal to the geometric mean:

$$p \ge g = (x_1 \cdot ... \cdot x_m)^{1/m}, \text{ где } m = 2n+1. \tag{14}$$

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Let us start with special cases $g = 0$ и $g = 1$. In both cases, the inequality (14) is obvious, therefore it may be further assumed that $0 < g < 1$. This implies that all $x_1,..., x_{2n+1}$ are different from 0 and not all of them are equal to 1.

Due to strict increase of the function P_{2n+1} (14) $\Leftrightarrow$ $P_{2n+1}(p) \geq P_{2n+1}(g)$. But $P_{2n+1}(p) = \Pi(X)$ (13) and $P_{2n+1}(g) = \Pi(G)$, where $G=(g,...,g)$. Thus, (14) $\Leftrightarrow$ $\Pi(X) \geq \Pi(G)$. And this inequality, considering exclusion of special cases, is equipotent to the statement $(*)$, which is exactly what needs proving.

$(*)$ *Let* $g \in (0;1)$, *and* A *is a set of vectors* $X = (x_1,...,x_m)$, *the ones, that* $x_j \in (0;1]$, *and* $x_1 \cdot ... \cdot x_m = g^m$. *In this set, the function* $\Pi(X)$ *reaches the lowest value at* $X=G$, *where* $G=(g,..., g)$.

Note: due to equality of $x_1 \cdot ... \cdot x_m = g^m$ it is sufficient to prove that all components of the vector, delivering the minimum of $\Pi(X)$, are equal between each other.

Let us proceed to the proof. Expressing x_m from the condition $x_1 \cdot ... \cdot x_m = g^m$ and putting it to $\Pi(X)$, we acquire rational and fractional function of the variables $x_1,..., x_{m-1}$, an uninterrupted one since its denominator $x_1 \cdot ... \cdot x_{m-1}$ differs from 0 due to $0 < g < 1$. And $x_k \in [g^m;1]$ for all $k = 1,...,m-1$. Thus, the function is determined at the compactum [12], meaning it reaches the minimal value [13]. Consequently, the vector constituting the minimum $\Pi(X)$, exists. Now, let us prove that it cannot have irregular components.

Let us consider the vector $X \in A$, that has more than n components equalling 1. For this $\Pi(X)=1$, whereas $\Pi(G) < 1$ (because $g < 1$). It means that the vector, which constitutes the minimum of the function $\Pi(X)$, has no more than n components equalling 1. Let X be such a vector, and let not all of its components be equal to each other. Without limiting the generality, it can be assumed that $x_1 \neq x_2$. Let us prove that then the value of $\Pi(X)$ is not the lowest at A.

Since all the components are different from 0, the fraction in (1) can be perceived literally and the multiplication of all components $x_1 \cdot ... \cdot x_{2n+1} = g^m$. can be put outside the brackets. Now, $\Pi(X) = g^m \sum_{k=0}^{n} \sum_{\{i_1,...i_k\} \subset \{1,...,m\}} (t_{i_1} - 1)...(t_{i_k} - 1)$, where $t_j = 1/x_j$, and the term of the outer sum, corresponding to $k = 0$, equals 1, and the vector $T = = (t_1,..., t_m)$ possesses the following properties:

1) $t_1 \neq t_2$;

2) among $t_1,..., t_m$ no more than n units equal 1;

3) the vector T delivers the function $\Theta(T) = \sum_{k=0}^{n} \sum_{\{i_1,...i_k\} \subset \{1,...,m\}} (t_{i_1} - 1)...(t_{i_k} - 1)$ the minimum at the set of vectors, which have $t_j \geq 1$, and $t_1 \cdot ... \cdot t_m = r^m > 1$, where $r = 1/g$.

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019
Transportation Systems and Technology. 2019;5(1):42-53
doi: 10.17816/transsyst20195142-53

Let us be view Θ (T) as the function of t_1, t_2 at constancy of the other variables. Separating the terms and co-multipliers, containing variables t_1, t_2, expressed by Θ (T) one can convert them to

$$\Theta(T) = C_2 t_1 t_2 + (C_1 - C_2)(t_1 + t_2) + C_0, \tag{15}$$

where coefficients C_1, C_2, C_0 are not dependent on t_1, t_2, and

$$C_1 = \sum_{k=0}^{n=1} \sum_{\{i_1,\ldots i_k\}\subset\{3,\ldots,m\}} (t_{i_1} - 1)\ldots(t_{i_k} - 1),\; C_2 = \sum_{k=0}^{n-2} \sum_{\{i_1,\ldots i_k\}\subset\{3,\ldots,m\}} (t_{i_1} - 1)\ldots(t_{i_k} - 1). \tag{16}$$

For further work, the signs of coefficients in (16) are significant at $t_1 \cdot t_2$ and $t_1 + t_2$. In the sum for C_2 all the terms are non-negative, with one of them being equal to 1. It means, $C_2 > 0$. Let us prove that $C_1 - C_2 > 0$. In accordance with (16) the outer sum for C_1 differs from the outer sum for C_2 on additional term with the number $k = n - 1$. Therefore, $C_1 - C_2 = \sum_{\{i_1,\ldots i_{n-1}\}\subset\{3,\ldots,m\}} (t_{i_1} - 1)\ldots(t_{i_{n-1}} - 1) \geq 0$ as the sum of non-negative terms. Let us assume that this sum equals 0. Then all the variables are equal to 0, i.e. any multiplication of $(t_{i_1} - 1)\cdot\ldots\cdot(t_{i_{n-1}} - 1)$, where $\{i_1,\ldots i_{k_{n-1}}\}\subset\{3,\ldots m\}$, is equal to 0. Thus among the subtractions and $(t_3 - 1),\ldots,(t_m - 1)$ there are no more than n–2 that are different from 0, and it means there are at least $2n - 1 - (n-2) = n+1$ equalling 0. Or, which is the same, among the components $t_3,\ldots,t_m$ there are n+1, equalling 1. It is especially true for a full set of components t_1, ..., t_m. However, this contradicts the above-mentioned property 2) of the vector T. So, $C_1 - C_2 > 0$.

Now, let us change t_1 and t_2 leaving the rest of the variables constant. Then the multiplication $t_1 t_2 = r^m/(t_3 \cdot\ldots\cdot t_m)$ will also be constant, meaning in the expression (16) only the second term will be changed, which equals (with the precision of up to the positive multiplier) $t_1 + c/t_1$, where $c = r^m/(t_3 \cdot\ldots\cdot t_m)$. The sum $t_1 + c/t_1$ acquires the minimal value at $t_1 = \sqrt{c}$ [14]. But then $t_2 = c/\sqrt{c} = \sqrt{c}$ as well, i.e. $t_1 = t_2$. Meanwhile, as to the assumption, $t_1 \neq t_2$, it means we have found the vector different from the initial one, at which the function Θ acquires the lesser value. This contradicts the assumption that the initial vector delivers the function Θ the minimum.

Returning to the initial variables $x_1,\ldots,t_m$, we obtain the statement $(*)$. Indeed, the vector that has not all components equal to each other, is not a vector which delivers the function $\Pi(X)$ minimum. But this vector does exist, and it means that this is a vector having different components. And this is nothing more like the vector G. Thus *the collective inequality has been proved*.

Let us add that due to the well-known properties of the power scale of averages [15], it additionally follows from this inequality that the collective average is no less than any power mean with non-positive value.

Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

doi: 10.17816/transsyst20195142-53

The notions of the collective average and collective inequality introduced here provide simplified lower bound assessment of the quality of decisions by the non-homogeneous group. The probability of its correct decision is no less than that of the homogeneous group comprising the same number of experts, where the probability of the correct individual decision by one expert equals the geometric mean of similar probabilities in the initial non-homogeneous group.

CONCLUSION

The expressions (6, 11, 12 и 14–16) fully resolve the issue stated to determine the probability of the right majority made decision, with the assumptions accepted. The asymptotic representation and two-sided assessment, which, characterise the speed of tending towards the right decision.

For a non-homogeneous group, the existence and uniqueness of the concept of collective average as an averaged characteristic were introduced and firmly proved, which can be used to replace an individual probability of each group member, whereby preserving the probability of correctness of the collective decision

Potential applications of the results obtained can be the quantitative evaluation of election procedures and the solution of problems associated with improving the reliability of recognition of weak signals of control sensors in various transport systems, including high-speed transport systems on magnetic suspension [16-17].

The Author (-s) hereby state that:

1. They have no conflict of interests;
2. The present article does not contain any researches involving humans as objects of research.

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Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

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Received: 02.12.2018. Revised: 14.02.2019. Accepted: 01.04.2019

doi: 10.17816/transsyst20195142-53

Information about authors:
Konstantin E. Voevodskii, Ph.D., Assistant Professor;
eLibrary SPIN: 2579-75410000-0000; ORCID: 0000-0002-0519-5527;
E-mail: kv5832@mail.ru
Vladimir M. Strepetov, Ph.D., Assistant Professor;
eLibrary SPIN: 4649-2141; ORCID: 0000-0002-4072-4519;
E-mail: strepetov.vm@mail.ru

Сведения об авторах:
Воеводский Константин Эммануилович, к.т.н., доцент;
eLibrary SPIN: 2579-7541; ORCID: 0000-0002-0519-5527;
E-mail: kv5832@mail.ru
Стрепетов Владимир Михайлович, к.т.н., доцент;
eLibrary SPIN: 4649-2141; ORCID: 0000-0002-4072-4519;
E-mail: strepetov.vm@mail.ru

To cite this article:
Voevodskii KE, Strepetov VM. The Probability of the Correct Majority Made Decision. *Transportation Systems and Technology*. 2019;5(1):42-53. doi: 10.17816/transsyst20195142-53

Цитировать:
Воеводский К.Э., Стрепетов В.М. Вероятность правильности решения, принимаемого большинством голосов // Транспортные системы и технологии. – 2019. – Т. 5. – № 1. – С. 42–53. doi:10.17816/transsyst20195142-53