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REASONS FOR AND RATIONAL METHODS OF MAGNETICALLY LEVITATED TRAINS DYNAMIC MODELLING

Background: the synthesis of high-quality dynamics and its analysis are the cardinal tasks within the issue relating to construction of a magnetically levitated train (MLT).

Aims of the work: to design the paradigm and method of rational solution to the specified tasks.

Methods: the synthesis of the supposed motion is suggested to be carried out by solving the inverse problem of the system's dynamics, and its preliminary analysis – by solving the same dynamic's direct problem.

Results: The reasons for MLT dynamics modelling are identified and substantiated. The paradigm and tensor method of its computer synthesis and analysis was developed.

Keywords: magnetically levitated train, dynamics, analysis and synthesis, computer modelling, tensor method.

Рубрика 2. НАУЧНЫЕ И ПРАКТИЧЕСКИЕ РАЗРАБОТКИ
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РЕЗОНЫ И РАЦИОНАЛЬНАЯ МЕТОДИКА МОДЕЛИРОВАНИЯ ДИНАМИКИ МАГНИТОЛЕВИТИРУЮЩЕГО ПОЕЗДА

Обоснование: Синтез качественной динамики и её анализ – кардинальные задачи проблемы создания магнитолевитирующего поезда (МЛП).

Цели работы: Разработать парадигму и методику рационального решения указанных задач.

Методы: Синтез желаемого движения предлагается осуществлять путем решения обратной задачи динамики системы, а его прелиминарный анализ – решением прямой задачи той же динамики.

Результаты: Выявлены и обоснованы резоны моделирования динамики МЛП. Разработана парадигма и тензорная методика её компьютерного синтеза и анализа.

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Ключевые слова: магнитолевитирующий поезд, динамика, анализ и синтез, компьютерное моделирование, тензорная методика.

INTRODUCTION

Magnetic levitation trains (MLT) travel at much higher speeds than other ground transportation means. The safety of such traffic should be guaranteed in all possible modes of operation and can be indicated by quality criteria. This makes it necessary to correctly and sufficiently complete modeling of dynamic processes taking place in MLTs. Therefore, among the tasks of constructing such trains the problem of synthesis of their qualitative dynamics and its analysis is cardinal.

A large-scale field experiment with MLT is expensive, time-consuming, dangerous and often impossible. Traditional analytical methods of studying their dynamics are usually ineffective as well. Therefore, probably the most part of such researches should be carried out by means of computer modelling [1], combining, as it is known, many advantages of analytical and experimental methods. At the same time, achieving the maximum possible efficiency of such modeling is ambivalent. It requires a significant improvement in the quality of the used mathematical models with the least resource intensity of their production and implementation. Traditional ways of solving the problems of MLT dynamics simultaneously do not allow satisfying the specified antagonistic requirements.

RESEARCH OBJECTIVES

The results of the analysis of the above facts reveal the need for the development of a paradigm and methodology for the rational solution to the problems of synthesis and analysis of the dynamics of MLTs.

RATIONALISATION OF MLT DYNAMICS RESEARCH ALGORITHM

The absolute majority of modern studies of MLT dynamics is based on the solution of its direct problem [2-4], i.e. - multiple integration of systems of nonlinear differential equations of the second order. Their number is usually large, and the expressions of elements are cumbersome and complex. The solution of the mentioned problem should be preceded by bringing the model equations to the first order, which is associated with the inversion of high dimensional matrices. If the parametric and/or structural non-stationarity of the system is taken into account, such a treatment should be performed at each step of integration. The operative synthesis of train motion requires an "onboard", predictive, multi-variant implementation of its model, the implementation of which is not realistic in the traditional way described above. But even if the model is supposed to be used for verification calculations in stationary - not "onboard" - conditions, and the computational scheme of the system is not too complex,

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stationary and holonomic, the integration of the equations of such a model is unreasonably resource-intensive.

The reason for this is an avalanche-like growth in volume and a catastrophically rapid drop in the accuracy of the required information transformations of modelling. The main irremovable drawback of a traditional way of the solution of problems of dynamics of MLT consists in the fact that the result of the solution of a direct problem of such dynamics has predictably-stating, cognitive, not creative character and essentially cannot guarantee achievement of demanded quality of investigated motion.

The paradigm shift in the study of MLT dynamics towards solving its inverse task radically changes the situation. Realisation of model of motion becomes considerably less resource-consuming and quite possible directly in the course of building of movement – on onboard computers. Implementation of the results of such implementation allows guaranteeing the required quality of the designed motion and giving the system a very important property of rigidity [5]. If, moreover, the train is controlled by generalised accelerations, then there is an additional possibility of automatic provision of the traffic adaptability to the situation [6]. The research based on the proposed paradigm acquires a creative, heuristic character, which is not inherent in the traditional way of its implementation. This makes it possible to significantly facilitate the process and improve the result of the synthesis of the dynamics of the system under consideration.

A comprehensive analysis of the resulting dynamics of MTL requires field experiments. However, their volume should be reduced to an absolutely inevitable minimum. All preliminary experiments should be computer-based. They are quite feasible in stationary – not "onboard" – conditions, in an arbitrary, research-friendly time scale and can be rationally realised in a traditional way – by solving a direct problem of train dynamics.

MLT NATURAL DYNAMICS MODEL BUILDING

The basis for building the desired dynamics of the train is its natural – under the influence of objective disturbances – motion. They are subject to priority modeling. The main criteria for choosing its methodology should be: completeness, synthetics and convenience of organic recording of parameters, the structure of the system, as well as the processes taking place in it. The results of the analysis proceeding from these criteria, of such alternative methods testify to the advantages of tensor methods in comparison with classical ones. The main advantages are: invariance of the received equations concerning transformations of coordinates; geometrisation of processes of the analysis and synthesis of motion; radical increase of compactness and visibility of expressions; recurrence-block character of procedure of modelling with any required depth of an enclosure; possibility of high formalisation and automation of process of research on the basis of systematic use of computer equipment. The stated facts

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reveal the expediency of choosing the tensor type of method for the synthesis and analysis of MLT dynamics.

In solution of many tasks of MLT dynamics as the rated scheme of its mechanical subsystem (MS) it is expedient to accept the unit of absolutely firm – supporting –bodies connected by means of communications through pliable blocks. Therefore, as a basis for the method to be developed, we will accept the differential equations of spatial motion of such a body in the inertial coordinate system $OX_p \; \forall\, p \in [\overline{1,3}]$. The last equations are known [7], always constant and have the form

$$f_{ij\alpha\beta} \cdot \ddot{\varepsilon}^{\beta}_{ij} + \mathrm{E}_{ij\alpha,\beta\gamma} \cdot \dot{\varepsilon}^{\beta}_{ij} \cdot \dot{\varepsilon}^{\gamma}_{ij} = \mathrm{K}_{ij\alpha} \; \forall\, \alpha,\beta,\gamma \in [\overline{1,6}], \quad (1)$$

where $f_{ij\alpha\beta}, \mathrm{E}_{ij\alpha,\beta\gamma} \forall\, \alpha,\beta,\gamma \in [\overline{1,6}]$ – covariant metric tensor of j-th supporting body of calculation schematic of i-th MLT vehicle, and three index Christoffel symbol of the first kind of the same body in the coordinates $\varepsilon^{\beta}_{ij} \; \forall\, \beta \in [\overline{1,6}]$;

$\varepsilon^{\beta}_{ij}, \mathrm{K}_{ij\alpha} \; \forall\, \alpha,\beta \in [\overline{1,6}]$ – reference coordinates of the same body relatively to trihedral $OX_p \; \forall\, p \in [\overline{1,3}]$, as well as generalised forces corresponding to them.

Accepting, for instance,

$$\varepsilon^{1}_{ij} = x_{ijC1}; \quad \varepsilon^{2}_{ij} = x_{ijC2}; \quad \varepsilon^{3}_{ij} = x_{ijC3};$$

$$\varepsilon^{4}_{ij} = \lambda^{1}_{ij} = \tilde{\psi}_{ij}; \quad \varepsilon^{5}_{ij} = \lambda^{2}_{ij} = \tilde{\theta}_{ij}; \quad \varepsilon^{6}_{ij} = \lambda^{3}_{ij} = \tilde{\gamma}_{ij}, \quad (2)$$

where $x_{ijCp} \forall\, p \in [\overline{1,3}]$ – Cartesian coordinates of the point ijC – centre of the weights of the body in question – in trihedral $OX_p \; \forall\, p \in [\overline{1,3}]$;

$\tilde{\psi}_{ij}, \tilde{\theta}_{ij}, \tilde{\gamma}_{ij}$ – Euler angles determining mutual orientation of the body connected $ijCz_r \forall\, r \in [\overline{1,3}]$ and the same inertial $OX_p \; \forall\, p \in [\overline{1,3}]$ trihedral, we can show that

$$f_{ij\alpha\beta} = \begin{bmatrix} f_{ijt} & 0 \\ 0 & f_{ijr} \end{bmatrix} \forall\, \alpha,\beta \in [\overline{1,6}];$$

$$f_{ijt} = diag\{m_{ij}, m_{ij}, m_{ij}\};$$

$$f_{ijr} = \begin{bmatrix} f_{ij44} & f_{ij45} & f_{ij46} \\ f_{ij54} & f_{ij55} & 0 \\ f_{ij64} & 0 & f_{ij66} \end{bmatrix};$$

$$f_{ij44} = (I_{ij11} \cdot \cos^{(2)} \tilde{\gamma}_{ij} + I_{ij22} \cdot \sin^{(2)} \tilde{\gamma}_{ij}) \cdot \cos^{(2)} \tilde{\theta}_{ij} + I_{ij33} \cdot \sin^{(2)} \tilde{\theta}_{ij};$$

$$f_{ij45} = f_{ij54} = (I_{ij11} - I_{ij22}) \cdot \cos \tilde{\theta}_{ij} \cdot \sin \tilde{\gamma}_{ij} \cdot \cos \tilde{\gamma}_{ij};$$

$$f_{ij46} = f_{ij64} = I_{ij33} \cdot \sin \tilde{\theta}_{ij};$$

$$f_{ij55} = I_{ij11} \cdot \sin^{(2)} \tilde{\gamma}_{ij} + I_{ij22} \cdot \cos^{(2)} \tilde{\gamma}_{ij}; \quad f_{ij66} = I_{ij33}, \quad (3)$$

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where $m_{ij}, I_{ijpp}\ \forall\, p \in [\overline{1,\,3}]$ – the weight of the same reference body and its main central (relatively to the trihedral axes $ijCz_r\ \forall\, r \in [\overline{1,\,3}]$) moments of inertia.

MLT vehicle is a guided system. Therefore, as a rule, its dynamics is convenient to consider in relation to the guideway it travels on. This approach facilitates the interpretation of research results and increases their informativeness. At the same time, the coordinate trihedrals accompanying the vehicles in motion and tracking the surface of the guideway are non-inertial: their beginnings have non-zero absolute accelerations, and they themselves rotate. Thus, for MLTs moving along the spatial fracture of the track, the problem of relative motion dynamics arises.

The equations (1) – tensorial. Therefore, they are forminvariant relatively to transformations of coordinates in which they are put down. In the coordinates $\rho^{\kappa}_{ij}\ \forall\, \kappa \in [\overline{1,\,6}]$, which determine the positioning of the same reference body relatively to non-inertial trihedral $iQY_q\ \forall\, q \in [\overline{1,\,3}]$, accompanying i-th vehicle in its motion relatively to the guideway surface under it, the motion of the body can be described by the model, obtained from (1) through substitution

$$\varepsilon^{\beta}_{ij} = \frac{\partial \varepsilon^{\beta}_{ij}}{\partial \rho^{\kappa}_{ij}} \cdot \rho^{\kappa}_{ij}\ \forall\, \beta, \kappa \in [\overline{1,\,6}] \tag{4}$$

and multiplication of the resulting expressions by the transformation matrix

$$\tau^{\ \beta}_{ij\kappa} = \frac{\partial\, \varepsilon^{\beta}_{ij}}{\partial \rho^{\kappa}_{ij}}\ \forall\, \beta, \kappa \in [\overline{1,\,6}]. \tag{5}$$

Similarly (2), the position of the body in trihedral $iQY_q\ \forall\, q \in [\overline{1,\,3}]$ can be determined, for instance, by coordinates

$$\begin{gathered} \rho^{1}_{ij} = y_{ijC1};\quad \rho^{2}_{ij} = y_{ijC2};\quad \rho^{3}_{ij} = y_{ijC3}; \\ \rho^{4}_{ij} = \nu^{1}_{ij} = \psi_{ij};\quad \rho^{5}_{ij} = \nu^{2}_{ij} = \theta_{ij};\quad \rho^{6}_{ij} = \nu^{3}_{ij} = \gamma_{ij}, \end{gathered} \tag{6}$$

where $y_{ijC\xi}, \nu^{\xi}_{ij}\ \forall\, \xi \in [\overline{1,\,3}]$ – Cartesian coordinates of the point ijC in trihedral $iQY_q\ \forall\, q \in [\overline{1,\,3}]$, and the Euler angles determining orientation of trihedral relatively to it $ijCz_r\ \forall\, r \in [\overline{1,\,3}]$.

Then from (2), (5) and (6) it follows that

$$\begin{gathered} \tau^{\ \beta}_{ij\kappa} = \begin{bmatrix} \varsigma_{ijpq} & 0 \\ 0 & \sigma^{\ u}_{ij\nu} \end{bmatrix};\quad \varsigma_{ijpq} = \frac{\partial\, x_{ijCp}}{\partial\, y_{ijCq}};\quad \sigma^{\ u}_{ij\nu} = \frac{\partial\, \lambda^{u}_{ij}}{\partial\, \nu^{\nu}_{ij}}; \\ \forall\, \beta, \kappa \in [\overline{1,\,6}];\ p, q, \nu \in [\overline{1,\,3}]. \end{gathered} \tag{7}$$

At the same time, for kinematic reasons, we conclude that

$$\lambda^{u}_{ij} = \lambda^{u}_{ij}\,(\nu^{\nu}_{ij}, \chi^{w}_{i})\ \forall\, u, \nu, w \in [\overline{1,\,3}], \tag{8}$$

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where $\chi_i^w \ \forall w \in [\overline{1,3}]$ – the Euler angles determining mutual orientation of trihedrals $iQY_q \ \forall q \in [\overline{1,3}]$ and $OX_p \ \forall p \in [\overline{1,3}]$.

It can be accepted that, for instance,

$$\chi_i^1 = \psi_i^*; \quad \chi_i^2 = \theta_i^*; \quad \chi_i^3 = \gamma_i^*, \tag{9}$$

where $\psi_i^*, \theta_i^*, \gamma_i^*$ – angles determining configuration of the guideway under i-th MLT vehicle.

Considering the guideway scleronomous, we have

$$\chi_i^\vartheta = \chi_i^\vartheta(w_{iQ}) \ \forall \vartheta \in [\overline{1,3}], \tag{10}$$

where w_{iQ} – distance covered by the point iQ along the guideway axis over the train tracking interval.

From (4) it follows that

$$\dot{\varepsilon}_{ij}^\beta = \frac{\partial \varepsilon_{ij}^\beta}{\partial \rho_{ij}^\kappa} \cdot \dot{\rho}_{ij}^\kappa = \tau_{ij\kappa}^\beta \cdot \dot{\rho}_{ij}^\kappa \ \beta, \kappa \in [\overline{1,6}]. \tag{11}$$

Then

$$\ddot{\varepsilon}_{ij}^\beta = \ddot{\rho}_{ij}^\kappa \cdot \tau_{ij\kappa}^\beta + \dot{\rho}_{ij}^\kappa \cdot \dot{\tau}_{ij\kappa}^\beta \ \forall \beta, \kappa \in [\overline{1,6}]. \tag{12}$$

In the last expressions, based on (5),

$$\dot{\tau}_{ij\kappa}^\beta = \frac{d}{dt}\frac{\partial \varepsilon_{ij}^\beta}{\partial \rho_{ij}^\kappa} = \dot{\rho}_{ij}^n \cdot \frac{\partial^{(2)} \varepsilon_{ij}^\beta}{\partial \rho_{ij}^\kappa \ \partial \rho_{ij}^n} = \dot{\rho}_{ij}^n \cdot \frac{\partial}{\partial \rho_{ij}^n} \tau_{ij\kappa}^\beta \ \forall \beta, \kappa, n \in [\overline{1,6}]. \tag{13}$$

After substitution of expressions (11)–(13) in the model (1), it acquires the form

$$f_{ij\alpha\beta} \cdot (\ddot{\rho}_{ij}^\kappa \cdot \tau_{ij\kappa}^\beta + \dot{\rho}_{ij}^\kappa \cdot \dot{\rho}_{ij}^n \cdot \frac{\partial}{\partial \rho_{ij}^n} \tau_{ij\kappa}^\beta) + \mathrm{E}_{ij\alpha,\beta\gamma} \cdot \tau_{ij\kappa}^\beta \cdot \dot{\rho}_{ij}^\kappa \cdot \tau_{ijn}^\gamma \cdot \dot{\rho}_{ij}^n = \mathrm{K}_{ij\alpha}$$
$$\forall \alpha, \beta, \gamma, \kappa, n \in [\overline{1,6}]. \tag{14}$$

Multiplying these equations by τ_{ijp}^α, with convolution by α, the motion model of the reference body relatively to trihedral $iQY_q \ \forall q \in [\overline{1,3}]$ we obtain

$$g_{ijp\kappa} \cdot \ddot{\rho}_{ij}^\kappa + \Gamma_{ijp,\kappa n} \cdot \dot{\rho}_{ij}^\kappa \cdot \dot{\rho}_{ij}^n = \mathrm{T}_{ijp} \ \forall p, \kappa, n \in [\overline{1,6}]; \tag{15}$$

$$g_{ijp\kappa} = f_{ij\alpha\beta} \cdot \tau_{ijp}^\alpha \cdot \tau_{ij\kappa}^\beta;$$
$$\Gamma_{ijp,\kappa n} = f_{ij\alpha\beta} \cdot \tau_{ijp}^\alpha \cdot \frac{\partial}{\partial \rho_{ij}^n} \tau_{ij\kappa}^\beta + \mathrm{E}_{ij\alpha,\beta\gamma} \cdot \tau_{ijp}^\alpha \cdot \tau_{ij\kappa}^\beta \cdot \tau_{ijn}^\gamma;$$
$$\mathrm{T}_{ijp} = \mathrm{K}_{ij\alpha} \cdot \tau_{ijp}^\alpha \ \ \forall \alpha, \beta, \gamma, p, \kappa, n \in [\overline{1,6}], \tag{16}$$

where $g_{ijp\kappa}, \Gamma_{ijp,\kappa n}, \mathrm{T}_{ijp} \ \forall p, \kappa, n \in [\overline{1,6}]$ – covariant metric tensor of the body under

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consideration, its three index Christoffel symbol of the first kind in the coordinates $\rho_{ij}^{\kappa}\ \forall \kappa \in [\overline{1,\, 6}]$, as well as generalised forces respective to these coordinates.
In the model (15), (16), as it is known [7],

$$\Gamma_{ijp,\kappa n} = 0{,}5 \cdot \left(\frac{\partial g_{ijp\kappa}}{\partial \rho_{ij}^{n}} + \frac{\partial g_{ijpn}}{\partial \rho_{ij}^{\kappa}} - \frac{\partial g_{ij\kappa n}}{\partial \rho_{ij}^{p}} \right)\ \forall\, p, \kappa, n \in [\overline{1,\, 6}]. \tag{17}$$

So, the model (15)–(17) describes relative motion of a free reference body of the calculation schematic of MLT in the non-inertial trihedral $i\mathrm{Q}Y_q\ \forall q \in [\overline{1,\, 3}]$.

Before combining into the aggregate, which is the design scheme of the train, the bodies constituting it are not connected, their movement is not constrained, and the configuration of this combination in trihedral $i\mathrm{Q}Y_q\ \forall i \in [\overline{1,\, N}], q \in [\overline{1,\, 3}]$ can be determined by reference coordinates

$$\xi^{\beta}\ \forall \beta \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}], \tag{18}$$

где H, N – число опорных тел в расчётной схеме экипажа, а также таких экипажей в поезде.
После сопряжения в агрегат, на движения тел расчётной схемы МЛП накладываются ограничения, формализуемые уравнениями связей, которые будем считать склерономными голономными

$$\xi^{\beta} = \xi^{\beta}(\eta^{\lambda})\ \forall \beta \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}], \lambda \in [\overline{1,\, L}], \tag{19}$$

where $\eta^{\lambda}\ \forall \lambda \in [\overline{1,\, L}], L$ – generalised coordinates accepted for description of the system in question, and their number.
Thus, the method of combination of bodies into the aggregate, which is a calculation schematic of MLT, can be described by structural matrix [8] of this aggregate

$$s = \frac{\partial \xi^{\beta}}{\partial \eta^{\lambda}}\ \forall \beta \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}], \lambda \in [\overline{1,\, L}]. \tag{20}$$

Using the coordinates (18), the motion of a combination of bodies included in the calculation scheme of the MLT can be described by a model, which is a set of equations of the form (15)

$$b_{\alpha\beta} \cdot \ddot{\xi}^{\beta} + \mathrm{B}_{\alpha,\beta\gamma} \cdot \dot{\xi}^{\beta} \cdot \dot{\xi}^{\gamma} = U_{\alpha}\ \forall \alpha, \beta, \gamma \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}]; \tag{21}$$

$$b_{\alpha\beta} = diag\{\lfloor g_{ijp\kappa} \rfloor\};\quad \mathrm{B}_{\alpha,\beta\gamma} = diag\{\lfloor \Gamma_{ijp,\kappa n} \rfloor\};\quad U_{\alpha} = \left[\mathrm{T}_{ijp}\right]^{T}$$
$$\forall \alpha, \beta, \gamma \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}];\ i \in [\overline{1,\, N}];\ j \in [\overline{1,\, \mathrm{H}}];\ p, \kappa, n \in [\overline{1,\, 6}], \tag{22}$$

where $b_{\alpha\beta}, \mathrm{B}_{\alpha,\beta\gamma}\ \forall \alpha, \beta, \gamma \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}]$ – covariant metric tensor of the combination and its three-index Christoffel symbol of the first kind in the coordinates $\xi^{\beta}\ \forall \beta \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}]$; $U_{\alpha}\ \forall \alpha \in [\overline{1,\, 6 \cdot \mathrm{H} \cdot N}]$ – generalised forces corresponding to

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them.

Equations (21), as well as their components – equations (15), are tensorial. Therefore, they are form-invariant with respect to the transformations of the coordinates in which they are written. That is, they can be (without changing the form) converted to a record in the coordinates $\eta^{\lambda}\ \forall\lambda[\overline{1,\ L}]$. To do this (similarly to the transformation of equations (1) into the model (15), we use the structural matrix (20) of the calculation schematic of MLT, as well as expressions

$$\dot{\xi}^{\beta}=\frac{\partial\xi^{\beta}}{\partial\eta^{\lambda}}\cdot\dot{\eta}^{\lambda}\ \ \forall\beta\in[\overline{1,\ 6\cdot \mathrm{H}\cdot N}];\ \lambda\in[\overline{1,\ L}];\tag{23}$$

$$\ddot{\xi}^{\beta}=\frac{\partial\xi^{\beta}}{\partial\eta^{\lambda}}\cdot\ddot{\eta}^{\lambda}+\frac{\partial^{2}\xi^{\beta}}{\partial\eta^{\lambda}\,\partial\eta^{\mu}}\cdot\dot{\eta}^{\lambda}\cdot\dot{\eta}^{\mu}\ \ \forall\beta\in[\overline{1,\ 6\cdot \mathrm{H}\cdot N}];\lambda,\mu\in[\overline{1,\ L}],\tag{24}$$

directly following from the next equations (19) of connections, applied to the bodies of this schematic. After multiplying equations (21) by matrix (20) (with convolution by “mute” indices), and also substituting relations (23) and (24) into them, the MLT motion model in the coordinates $\eta^{\lambda}\ \forall\lambda[\overline{1,\ L}]$, that is, relatively non-inertial trihedral $iQY_q\ \ \forall i\in[\overline{1,\ N}],q\in[\overline{1,\ 3}]$, is obtained in the form

$$c_{\lambda\mu}\cdot\ddot{\eta}^{\mu}+C_{\lambda,\mu\nu}\cdot\dot{\eta}^{\mu}\cdot\dot{\eta}^{\nu}=Y_{\lambda}\ \ \forall\lambda,\mu,\nu\in[\overline{1,\ L}];\tag{25}$$

$$c_{\lambda\mu}=b_{\alpha\beta}\cdot\frac{\partial\xi^{\alpha}}{\partial\eta^{\lambda}}\cdot\frac{\partial\xi^{\beta}}{\partial\eta^{\mu}}\ \ \forall\alpha,\beta\in[\overline{1,\ 6\cdot \mathrm{H}\cdot N}];\lambda,\mu\in[\overline{1,L}];\tag{26}$$

$$C_{\lambda,\mu\nu}=b_{\alpha\beta}\cdot\frac{\partial\xi^{\alpha}}{\partial\eta^{\lambda}}\cdot\frac{\partial^{2}\xi^{\beta}}{\partial\eta^{\lambda}\,\partial\eta^{\nu}}+\frac{\partial\xi^{\alpha}}{\partial\eta^{\lambda}}\cdot\frac{\partial\xi^{\beta}}{\partial\eta^{\mu}}\cdot\frac{\partial\xi^{\gamma}}{\partial\eta^{\nu}}\cdot \mathrm{B}_{\alpha,\beta\gamma}$$
$$\forall\alpha,\beta,\gamma\in[\overline{1,\ 6\cdot \mathrm{H}\cdot N}];\ \lambda,\mu,\nu\in[\overline{1,L}];\tag{27}$$

$$Y_{\lambda}=\frac{\partial\xi^{\alpha}}{\partial\eta^{\lambda}}\cdot U_{\alpha}\ \ \forall\alpha,\beta,\gamma\in[\overline{1,\ 6\cdot \mathrm{H}\cdot N}];\ \lambda\in[\overline{1,L}],\tag{28}$$

where $c_{\lambda\mu},C_{\lambda,\mu\nu},Y_{\lambda}\ \ \forall\lambda,\mu,\nu\in[\overline{1,\ L}]$ – covariant metric tensor of the calculation schematic of the train, its three index Christoffel symbol of the first kind in the coordinates $\eta^{\lambda}\ \forall\lambda\in[\overline{1,\ L}]$, as well as respective generalised forces.

Similarly (17),

$$C_{\lambda,\mu\nu}=0{,}5\cdot\left(\frac{\partial c_{\lambda\mu}}{\partial\eta^{\nu}}+\frac{\partial c_{\lambda\nu}}{\partial\eta^{\mu}}-\frac{\partial c_{\mu\nu}}{\partial\eta^{\lambda}}\right)\forall\lambda,\mu,\nu\in[\overline{1,\ L}].\tag{29}$$

Building of model (25), (26), (28), (29) requires only matrix algebra operations. They have been implemented programme-wise using the symbolic component of Mathematica computer mathematics system. Therefore, such modelling takes place in a fully automatic mode, having as its final result explicit

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expressions of all the elements of the specified model (which are not given because of the cumbersomeness). At the same time, due to the synthetics of the functional-structural organisation of MLT, the obtained equations of the model of its relative motion automatically take into account all the members reflecting the true dynamic force equilibrium in the system. Consideration of pseudoforces of inertia, as it is assumed, for instance, when using the Lagrange equations [9-11], is not required. The model equations obtained using the proposed technique are again tensorial. Therefore, if such a need arises, they are transformable (according to the described algorithm) not only to any other system of generalized coordinates convenient for any research, but also to any other structure of the calculation scheme of the object under study.

MLT CONTROLLED DYNAMICS BUILDING METHOD

The dynamics of the train, determined by its natural disturbances and described by the model (25), in the vast majority of cases does not have the desired properties. In turn, these properties determine the quality of motion and can be represented by its objectives, and then - in the form of programmes - by objectives, so that they become available for use. In order for MLT to make the desired motion, the describing model should not contradict the objective laws of mechanics and, at the same time, should be compatible with the programmes of controlled dynamics [5]. Simultaneous observance of both these conditions is achievable by any change of the natural dynamics model, which makes it joint with such programmes. The way of transformation of the model is not limited in principle and is determined mainly by the convenience of natural realisation of the changes of the system under study and/or its effects. In practice, the most common is the introduction of additive control actions on the system into the model of motion [12].

In order to specify the consideration, we will take one of the most flexible and acceptable for MLT method of natural realisation of its desired movement, i.e. its terminal management [13, 14]. Then the task of train movement is to bring it, upon reaching the accepted independent variable of the set values, into an a priori set sequence of states. And the spaces of phase coordinates, natural disturbances and controls are subject to real restrictions $\Omega_x(t)$, $\Omega_w(t)$ and $\Omega_u(t)$, respectively. Depending on the goals and objectives of the study, these sets can physically be interpreted in a different way.

Proceeding from the model (25) of natural dynamics of mechanical subsystem of MLT, in case of its terminal control, the model of controlled motion of this subsystem has form of

$$\dot{x}(t) = f\,[x(t), u(t), w(t), t] \;\; \forall t \in [t_s, \theta]; \;\; x(t_s) = x_s, \qquad (30)$$

where $x(t), u(t), w(t) \; \forall t \in [t_s, \theta]$ – subsystem's state vectors, as well as contolling and disturbing influences;

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t – convenient for research independent variable, e.g. time;
$[t_s, \theta]$ – motion building integral;
x_s – its initial conditions.

The aim of the train movement is formalised by the programme

$$x(\tau) = x_f, \tag{31}$$

where τ, x_f – values of independent variable and vector of state of MLT magnetic field in the finite moment at the terminal management interval under consideration.

At the same time, proceeding from the physical sense of the motion process, the ratios should be observed

$$x(t) \in \Omega_x(t),\ u(t) \in \Omega_u(t)\ \ \forall t \in [t_s, \tau] \tag{32}$$

and, besides, some a priori information should be known

$$w(t) \in \Omega_w(t)\ \ \forall t \in [t_s\ \tau]. \tag{33}$$

The programme (31) puts restrictions only on finite (at the interval $[t_s, \tau]$) state of mechanical subsystem of MLT. Therefore, the model (30), together with the conditions (31)–(33), determines the ensemble of phase trajectories of the subsystem showing the points in the space of its states

$$\begin{gathered} X[u(\bullet), w(\bullet), x_s, x_f] = \{x[\bullet, u(\bullet), w(\bullet), x_s, x_f] \in \Omega_x(t): \\ u(\bullet) \in \Omega_u(t),\ w(\bullet) \in \Omega_w(t),\ t \in [t_s, \tau]\}; \\ u(\bullet) = \{u(t)\ \ \forall t \in [t_s, \tau]\};\ w(\bullet) = \{w(t)\ \ \forall t \in [t_s, \tau]\}; \\ \{x[\bullet, u(\bullet), w(\bullet), x_s, x_f]\} = \{x[t, u(\bullet), w(\bullet), x_s, x_f]\ \forall t \in [t_s, \tau]\}, \end{gathered} \tag{34}$$

each of which meets the edge conditions

$$x(t_s) = x_s, \quad x(\tau) = x_f. \tag{35}$$

If, in addition to meeting these conditions, there are no other requirements for the motion of mechanical subsystem of MLT, it is synthesised in a purely terminal statement, implying, depending on the specific values of disturbances (33), the possibility of the realisation of any trajectory of the ensemble (34). Thus, in this case, the motion is determined only with the accuracy up to this possible ensemble.

From the above-stated, it follows that, at purely terminal statement of a problem of construction of movement of a train MF of a train, concerning its concrete realisation there is an essential design unreasonability. It can be used to give this motion additional useful properties. For instance, if we demand that, in addition to meeting the conditions (32) and (35), the implemented phase trajectory of the subsystem has the following ratio

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$$I = \inf_{u(\bullet)} \sup_{w(\bullet)} \{ \int_{t_s}^{\tau} \lambda[u(\bullet), w(\bullet)] \cdot dt:$$
$$u(\bullet) \in \Omega_u(t),\ w(\bullet) \in \Omega_w(t),\ t \in [t_s, \tau]\}, \quad (36)$$

where I – integral values of the motion quality;
λ – set function of own arguments,
then the task of the synthesis of the motion has a game minimalistic character, based on the concept of guaranteed result [15]. At the same time, the mentioned motion acquires the properties optimal by the criterion I, and the only trajectory extreme by this criterion is realised from the ensemble (34). The motion of mechanical subsystems of MLT is defined unambiguously and guaranteed to be optimal in the desired sense under any possible perturbations. Such approach allows, depending on necessity, in various operational modes to optimise required characteristics of the specified movement.

Thus, by classifying and parameterising the medium in which the motion of mechanical subsystems of MLT takes place [15], as well as by constructing for each typical situation an optimal, in the desired sense, control of this motion, the problem of its terminal synthesis [13] can be solved, i.e. - the inverse problem of train dynamics in the terminal formulation. In practice, the development of constructive systems of such synthesis inevitably raises the problem of the need to simultaneously meet a set of heterogeneous, often - antagonistic - requirements for the quality of the synthesised motion. In this case, the rational way of its optimisation is to introduce a vector criterion I, the elements of which are the specific criteria indicating the optimised partial properties of such a motion [16, 17].

CONCLUSION

Research into MLT dynamics, conducted within the framework of the developed paradigm and the methodology based on it, are comprehensively computerised on the basis of modern systems of computer mathematics [18-20]. These studies are much more heuristic and resource-economical than conventional methods. The results obtained by the proposed method are creative. Their use allows us to guarantee the required quality of train movement, as well as to give it a very important property of rigidity. In this case, if the control of the synthesised motion is closed by the generalised acceleration of the system, this movement acquires the property of adaptability to the situation. The above facts reveal the expediency of using the proposed paradigms and methods in the study of MLT dynamics. It will allow us to facilitate considerably the process and to improve the result of such researches.

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