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MODEL OF A MAGNETICALLY LEVITATED TRAIN'S LEVITATION FORCE

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Annotation: The implementation of the magnetically levitated train's (MLT) levitation force (LF) occurs during the interaction between fields of superconductor train (STC) and short-circuited track contours (SCTC), which are the elements of levitation module (LM).

Purpose. Based on above, the purpose of this study is to obtain a correct description of such interaction. At the present stage, the main and the most universal tool for the analysis and synthesis of processes and systems is their mathematical and, in particular, computer modeling. At the same time, the radical advantages of this tool make even more important the precision of choosing a specific methodology for research conducting.

Methodology. This is particularly relevant in relation to such large and complex systems as MLT. For this reason, the work pays special attention to the reasoned choice of the selective features of the research paradigm. The analysis of existing versions of LF implementation's models show that each of them, along with the advantages, also has significant drawbacks.

Results. In this regard, one of the main result of the study should be the construction of this force implementation's mathematical model, which preserves the advantages of the mentioned versions, but would be free from their shortcomings. The rationality of application, for the train's LF researching, of an integrative holistic paradigm, which assimilates the advantages of the electric circuit and magnetic field theories, is reasonably justified in work.

The scientific novelty of the research. The priority of creation of such a paradigm and the corresponding version of the implementation of LF's model account for the novelty of the research.

Practical significance of the work. The practical significance consists in the possibility, in case of using its results, of significantly increasing the efficiency of dynamic MLT research while reducing their resource costs.

Keywords*:* magnetically levitated train (MLT), mathematical model of levitation, integrative paradigm of research.

Introduction

Currents and poles of contours of levitation junctions of MLT are the elements of one electromagnetic hyperprocess of electromechanical conversion of the energy. It is quite possible to simulate them [1] within the paradigms of electric circuits and magnetic field theories. Therefore, the existing versions of mathematical model of MLT's LF are built [1 − 3] basing upon these paradigms.

Analysis of the properties of the mentioned versions of models signifies that each of them possesses both advantages and disadvantages. Their common advantage is sufficient functionality. Yet the basic immanent drawback of these versions is non-stationary feature of differential equations, caused by cyclic variability of their coefficients, which correspond to self-inductance and mutual inductance of short-circuited track contours (SCTC) of LF both among each other and among superconductor train contours (STC), depending on the position of a train. This significantly complicates solution of tasks of the described dynamics [4], drastically decreasing practical value of model versions.

Purposes of the Studies

The introduction above reveals [5 − 7] relevance of creation of mathematical model of LF of MLT, which assimilates the advantages of both existing versions of the model, but free of their drawbacks. Creation of this model is the basic task of this work.

Methodology

Electromechanical conversion of LF of MLT is carried out in the process of interaction of poles and currents of SCTC and STC. Therefore, the pattern of LF of a train is an interaction of STC's element current with current pole of SCTC. This interaction may be described by Ampère's force law expression [8]:

$$f_{\beta\gamma} = l_{\beta\gamma} \cdot i^{\beta\gamma} \cdot B_{\beta\gamma} \cdot Sin\alpha_{\beta\lambda}, \qquad (1)$$

where $f_{\beta\gamma}$ – force, which exerts on γ^{th} element of β^{th} STC;

$l_{\beta\gamma}, i^{\beta\lambda}, B_{\beta\gamma}, \alpha_{\beta\gamma}$ – the length of the element, current in it, pole inductance, in which the element is located, and the angle between $\overline{i^{\beta\gamma}}$ and $\overline{B_{\beta\gamma}}$.

The design schemes of SCTC and STC are taken as sets of galvanically isolated conducting rectangular frame and pairs of identical rectangular coils, connected in accordance with zero-flow scheme [1]. In that case, the levitation force (LF) of the train is determined as vector sum of quantities $\overline{f_{\lambda\chi}}\ \forall \lambda \in [\overline{1,N}],\ \chi \in [\overline{1,4}]$, each of which is a result of interaction of current of one element of STC with the pole of currents forming SCTC with it. In the last expression, N is the number of superconductor train contours (STC). The dynamics of electromagnetic component of such an interaction is determined by equations of Kirchhoff's second rule [8]. The subsystem of "STC− SCTC", as a rule, is degenerate [6], namely, their capacity indices are very low. Therefore, in the inertial system $Q\varepsilon^{\rho}\ \forall \rho \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}]$, the model of electromagnetic component of interaction of β^{th} STC with SCTC considered, has the form [8, 9]:

$$\sigma_{\rho\beta} = L_{\rho\rho} \cdot \frac{d}{dt} i^{\rho} + L_{\rho\mu} \cdot \frac{d}{dt} i^{\mu} + r_{\rho} \cdot i^{\rho} \ \forall \ \rho, \mu \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}]; \qquad (2)$$

$$\sigma_{\rho\beta} = \sigma^{u}_{\rho\beta} - \sigma^{l}_{\rho\beta}; \quad \sigma^{\kappa}_{\rho\beta} = -\frac{d}{dt}\left(M^{\kappa}_{\rho\beta} \cdot i^{\beta}_{s}\right)$$

$$\forall \rho \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}], \kappa = u \vee \kappa = l, \qquad (3)$$

where $\sigma^{\kappa}_{\rho\beta} \forall \rho \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}], \kappa = u \vee \kappa = l$ – electromotive force (EMF) in coils of ρ^{th} of short-circuited track contours with changes of coupling with subcontours of current i^{β}_{s} in circuit β^{th} superconductor train contours;

$L_{\rho\rho}, L_{\rho\mu}, r_{\rho} \ \forall \ \rho, \mu \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}]$ – own and mutual inductances and active resistances of short-circuited track contours;

χ_{β}– number (from the onset of a track section on which the magnetically levitated train runs) of the last short-circuited track contour the transverse section of which was passed by transverse section of β^{th} short-circuited track contour;

E – half of the figure of short-circuited track contours with which the electromagnetic interaction of each superconductor train contour is considered;

$i^{\rho}, i^{\mu} \ \forall \ \rho, \mu \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}]$ – short-circuited track contours currents;

$M^{\kappa}_{\rho\beta} \ \forall \rho \in [\overline{(\chi_{\beta} - \mathrm{E}), (\chi_{\beta} + \mathrm{E})}], \ \kappa = u \vee \kappa = l$ – mutual inductances between β^{th} superconductor train contour and coils of interaction with short-circuited track contours;

t – time.

Owing to the accepted design measures [1], values of currents i^{λ}_{s} $\forall \lambda \in [\overline{1, \mathrm{K}}]$, are changed rather slowly, and in intervals commensurate with the time of observing the train traffic, may be considered equal and constant

$$i^{\lambda}_{s} = i_{s} = const \ \ \forall \lambda \in [\overline{1, \mathrm{K}}], \qquad (4)$$

where K – a number of superconductor train contours installed in magnetically levitated train. The value of E should be chosen in a way that on both sides of each β^{th} superconductor train contour in short-circuited track contour, preceding and following the quantities considered, the values $\sigma^{\kappa}_{\rho\beta} \ \forall \rho < \chi_{\beta} - \mathrm{E} \vee \rho > \chi_{\beta} + \mathrm{E}, \ \kappa = u \vee, \kappa = l$ even in the non-equilibrium state of levitation module should be negligibly low.

Superconductor train contours and short-circuited track contour are mutually movable.

Therefore, $L_{\rho\rho}, L_{\rho\mu}, M^{\kappa}_{\rho\lambda} \quad \forall \rho, \mu \in [\overline{(\chi_{\lambda} - \mathrm{E}), (\chi_{\lambda} + \mathrm{E})}], \quad \lambda \in [\overline{1, \mathrm{K}}]$, $\kappa = u \vee \kappa = l$ possesses cyclically varying values in time. This, in its turn, leads to non-stationary coefficients of equations (2), (3), and significantly decreases practical value of the version of the model. In order to eliminate this disadvantage, the realisation of elements of levitation force of MLT should be considered with respect to coordinate systems, in each of which the considered superconductor train contour and interaction with it short-circuited track contour are conditionally mutually immovable. In this capacity, it is most convenient to accept [5] counting systems $C_\lambda \eta^\mu \; \forall \lambda \in [\overline{1,\mathrm{K}}], \mu \in [\overline{1,3}]$, each of which is rigidly connected with λ^{th} superconductor train contour. Generally, $C_\lambda \eta^\mu \quad \forall \lambda \in [\overline{1,\mathrm{K}}], \mu \in [\overline{1,3}]$ are not inertial. At the same time, it is highly desirable [10] that equations describing dynamics of electromagnetic component of interaction between superconductor train contour and short-circuited track contour should have tensor property. These equations may be obtained [11] out of equations of type (2) by their derivatives $\frac{d}{dt}$ being replaced by absolute $\frac{D}{dt}$, as well as by means of transition in models (2), (3) to coordinates $\eta_\lambda^\mu \; \forall \lambda \in [\overline{1,\mathrm{K}}], \mu \in [\overline{1,3}]$. The relation between the above-mentioned derivatives, as it is known, is given as follows [11]:

$$\frac{D}{dt}\eta_\alpha^\mu = \frac{d}{dt}\eta_\alpha^\mu + e_{\mu\alpha\nu} \cdot \omega_\alpha \cdot \eta_\alpha^\nu \; \forall \mu, \nu \in [\overline{1,3}], \tag{5}$$

where $e_{\mu\alpha\nu} \forall \mu, \nu \in [\overline{1,3}]$, ω_α – Levi-Civita symbol and angular rotating velocity vector $C_\alpha \eta^\mu \; \forall \mu \in [\overline{1,3}]$.

After the mentioned replacement, relations obtained out of (2) acquire tensor character. Therefore, their form becomes invariant in relation to coordinates in which they are written, whereas transition to coordinates η_α^μ $\forall \mu \in [\overline{1,3}]$ is feasible according to the expressions:

$$\eta_\alpha^\mu = \vartheta_\rho^\mu \cdot \varepsilon^\rho \; \forall \; \rho \in [\overline{(\chi_\alpha - \mathrm{E}), (\chi_\alpha + \mathrm{E})}]; \mu \in [\overline{1,3}] \tag{6}$$

where ϑ_ρ^μ – coordinate transformation matrix:

$$\vartheta_\rho^\mu = \frac{\partial \eta_\alpha^\mu}{\partial \varepsilon^\rho} \; \forall \rho \in [\overline{(\chi_\alpha - \mathrm{E}), (\chi_\alpha + \mathrm{E})}]; \mu \in [\overline{1,3}]. \tag{7}$$

On the axis $\eta_\alpha^\mu \; \forall \mu \in [\overline{1,3}]$ and $\varepsilon^\rho \; \forall \rho \in [\overline{(\chi_\alpha - \mathrm{E}), (\chi_\alpha + \mathrm{E})}]$ any vector value may be projected, which characterise electrodynamics of interaction of superconductor train contour and short-circuited track contour in the counting systems respectively $C_\alpha \eta^\mu \; \forall \mu \in [\overline{1,3}]$ and $Q\varepsilon^\rho \; \forall \rho \in [\overline{(\chi_\alpha - \mathrm{E}), (\chi_\alpha + \mathrm{E})}]$. In particular, they can be vectors of currents, electromotive force and pole induction.

Expressions for connections

$$\eta_\alpha^\mu = \eta_\alpha^\mu(\varepsilon^\rho) \; \forall \; \rho \in [\overline{(\chi_\alpha - \mathrm{E}), (\chi_\alpha + \mathrm{E})}]; \mu \in [\overline{1,3}] \tag{8}$$

may be obtained assuming that [5] during the process of the described coordinate transformation, its invariants are amplitudes of currents in the analysed contours and their electromotive force.

With the help of the matrix, though

$$\vartheta_{\mu}^{\rho}=\frac{\partial\varepsilon^{\rho}}{\partial\eta_{\alpha}^{\mu}}=(\vartheta_{\rho}^{\mu})^{T}\ \forall\rho\in[\overline{(\chi_{\alpha}-\text{E}),(\chi_{\alpha}+\text{E})}];\mu\in[\overline{1,3}], \tag{9}$$

the irreversible transformation is feasible

$$\varepsilon^{\rho}=\vartheta_{\mu}^{\rho}\cdot\eta_{\alpha}^{\mu}\ \forall\ \rho\in[\overline{(\chi_{\alpha}-\text{E}),(\chi_{\alpha}+\text{E})}];\mu\in[\overline{1,3}]. \tag{10}$$

In the expressions (3) for $\sigma_{\rho\beta}^{\kappa}\ \forall\rho\in[\overline{(\chi_{\beta}-\text{E}),(\chi_{\beta}+\text{E})}],\ \kappa=u\vee\kappa=l$, $M_{\rho\beta}^{\kappa}$ $\forall\rho\in[\overline{(\chi_{\beta}-\text{E}),(\chi_{\beta}+\text{E})}],\ \kappa=u\vee\kappa=l$ they are rather dependent upon mutual location of the analysed β th superconductor train contour and short-circuited track contour, the interaction with which is considered for it. Therefore,

$$M_{\rho\beta}^{\kappa}=M_{\rho\beta}^{\kappa}(w_{\beta})\ \forall\ \rho\in[\overline{(\chi_{\beta}-\text{E}),(\chi_{\beta}+\text{E})}],\ \kappa=u\vee\kappa=l, \tag{11}$$

where w_{β} – coordinate determining the current location of the analysed β th superconductor contour in relation to the onset of magnetically levitated train movement along the axis of the track. In addition, since short-circuited track contour along the guidance is regularly located, last dependences have a galvanic character. At the same time, modern ways of measuring enable us, [12] by virtue of experimental and calculating methods, with quite acceptable accuracy to determine values of mutual induction of contours in magnetically connected electric circuits with their different location. This, in its turn, enables us, using the mentioned methods, to pointwise build the desired dependences (11) on the required net w_{β}. Furthermore, by means of polynomial regression [13], the realisation of which is affordable in a number of computer mathematics systems (for instance, Mathematica), the dependences of (11), with a preservation of quite a high accuracy of content, may be given a form of analytical expressions. Apart from that, considering equations (4), expressions (3) may be transformed into

$$\sigma_{\rho\beta}=\sigma_{\rho\beta}^{u}-\sigma_{\rho\beta}^{l};\quad \sigma_{\rho\beta}^{\kappa}=-i_{s}\cdot\dot{w}_{\beta}\cdot\frac{d}{dw_{\beta}}M_{\rho\beta}^{\kappa}$$

$$\forall\rho\in[\overline{(\chi_{\beta}-\text{E}),(\chi_{\beta}+\text{E})}],\kappa=u\vee\kappa=l, \tag{12}$$

where $\dot{w}_{\beta}$ – velocity of longitudinal (along the tangent to the axis) movement of the analysed β th superconductor contour with relation to the guidance.

Values $\frac{d}{dw_{\beta}}M_{\rho\beta}^{\kappa}\ \ \forall\rho\in[\overline{(\chi_{\beta}-\text{E}),(\chi_{\beta}+\text{E})}],\ \kappa=u\vee\kappa=l$ for substitution in the expressions (12) may be obtained using the described method in the form of analytical expressions, dependences of the (11). Thus, each of the vectors β $\overline{\sigma_{\rho\beta}}\ \forall\rho\in[\overline{(\chi_{\beta}-\text{E}),(\chi_{\beta}+\text{E})}]$ appears definite in the counting system $Q\varepsilon^{\rho}$

$\forall \rho \in [\overline{(\chi_\beta - \mathrm{E}),(\chi_\beta + \mathrm{E})}]$. Using the interactions of the (6) – (8), each of such vectors may be determined in the system $C_\beta \eta^\mu \ \forall \mu \in [\overline{1,3}]$ by projections $\sigma_{\mu\beta} \forall \mu \in [\overline{1,3}]$.

After transformation, the equations obtained form (2) and (3) by means of their transforming into trihedral $C_\beta \eta^\mu \ \forall \mu \in [\overline{1,3}]$, using the interactions (5) and (6) acquire the view of

$$\sigma_{\mu\beta} = L_{\mu\mu} \cdot \left(\frac{d}{dt} i^\mu + e_{\mu\beta\nu} \cdot \omega_\beta \cdot i^\nu \right) + L_{\mu\tau} \cdot \left(\frac{d}{dt} i^\tau + e_{\tau\beta\theta} \cdot \omega_\beta \cdot i^\theta \right) + r_\mu \cdot i^\mu$$

$$\forall \mu, \nu, \tau, \theta \in [\overline{1,3}]; \tag{13}$$

$$\sigma_{\mu\beta} = \vartheta_\rho^\mu \cdot \sigma_{\rho\beta} \ \forall \ \rho \in [\overline{(\chi_\beta - \mathrm{E}),(\chi_\beta + \mathrm{E})}]; \mu \in [\overline{1,3}]$$

$$\sigma_{\rho\beta} = \sigma_{\rho\beta}^u - \sigma_{\rho\beta}^l; \quad \sigma_{\rho\beta}^\kappa = -i_s \cdot \dot{w}_\beta \cdot \frac{d}{dw_\beta} M_{\rho\beta}^\kappa$$

$$\forall \rho \in [\overline{(\chi_\beta - \mathrm{E}),(\chi_\beta + \mathrm{E})}], \kappa = u \vee \kappa = l. \tag{14}$$

Equations (13) possess constant coefficients, are considered tensorial and describe the dynamics of currents of levitation modules in MLT, in coordinates $i^\mu \ \forall \mu \in [\overline{1,3}]$. After their (as a rule, numerical) resolution with respect to these derivatives, the latter, with the use of interactions (10), are transformed into coordinates $i^\rho \ \forall \rho \in [\overline{(\chi_\beta - \mathrm{E}),(\chi_\beta + \mathrm{E})}]$, the values of which are defined real currents in short-circuited contours circuits.

Magnetic circuit of levitation module is intended to be unsaturated [1]. Therefore it may be considered conditionally linear and, consequently, additivity may be applied to it. Based upon this, resulting field of currents in short-circuited track contour at any point of geometrical space $\mathrm{O}\Xi_\gamma \ \forall \gamma \in [\overline{1,3}]$, where superconductor contour moves relatively to short-circuited track contour, may be described as sum of fields created in this point by currents of separate modules of short-circuited track contour:

$$B_{\gamma\beta} = B_{\gamma\rho\beta} \cdot e^\rho; \ e^\rho = 1; \forall \rho \in [\overline{(\chi_\beta - E),(\chi_\beta + E)}], \ \gamma \in [\overline{1,3}], \tag{15}$$

where $B_{\gamma\beta}, B_{\gamma\rho\beta} \ \forall \rho \in [\overline{(\chi_\beta - E),(\chi_\beta + E)}], \gamma \in [\overline{1,3}]$ – space components of pole induction, created by all (those interacting with β^{th} superconductor track contour) modules of short-circuited track contour , and such separate modules in the analysed point of this space. In its turn, values of the components $B_{\gamma\alpha\beta} \ \forall \gamma \in [\overline{1,3}]$ for each α module of short-circuited track contour are determined by relations

$$B_{\gamma\alpha\beta}(i^\alpha) = B_{\gamma\alpha\beta}^u(i^\alpha) - B_{\gamma\alpha\beta}^l(i^\alpha) \ \forall \gamma \in [\overline{1,3}], \tag{16}$$

where $B^{\kappa}_{\gamma\alpha\beta}\ \forall\gamma\in[\overline{1,3}],\kappa=u\vee\kappa=l$ – space components of induction of pole of coil currents in α short-circuited track contour (interacting with β superconductor track contour).

Expression for determination of values of $B^{\kappa}_{\gamma\rho\beta}(i^{\rho})$ $\forall\rho\in[\overline{(\chi_{\beta}-E),(\chi_{\beta}+E)}],\ \gamma\in[\overline{1,3}],\ \kappa=u\vee\kappa=l$ are of the form [14]:

$$B^{\kappa}_{1\rho\beta}=-\frac{\iota^{\rho}}{4\cdot\pi}\cdot\left\{\left[F_{12}(k_1',\varphi',\eta)+F_{12}(k_3',\varphi',\eta)\right]_{\varphi_1'=x_0-l}^{\varphi_2'=x_0-l-d}-\right.$$

$$\left.-\left[F_{12}(k_2',\psi',\eta)+F_{12}(k_4',\psi',\eta)\right]_{\psi_1'=x_0+l}^{\psi_2'=x_0+l+d}\right\}_{\eta_1=z_0+h}^{\eta_2=z_0-h};$$

$$B^{\kappa}_{2\rho\beta}=-\frac{\iota^{\rho}}{4\cdot\pi}\cdot\left\{\left[F_{12}(k_1,\varphi,\eta)+F_{12}(k_4,\varphi,\eta)\right]_{\psi_1'=x_0+l}^{\varphi_2'=x_0+l+d}-\right.$$

$$\left.-\left[F_{12}(k_2,\psi,\eta)+F_{12}(k_3,\psi,\eta)\right]_{\psi_1=y_0+a}^{\psi_2=y_0+a+d}\right\}_{\eta_1=z_0+h}^{\eta_2=z_0-h};$$

$$\left\{F_{12}(k,\varphi,\eta)\right\}_{\eta_1=z_0+h}^{\eta_2=z_0-h}=\left\{\eta\cdot arctg\frac{k\cdot\varphi-\eta^2}{\eta\cdot\sqrt{(k+\varphi)^2+\varphi^2+\eta^2}}-\right.$$

$$\left.-\varphi\cdot ar\,sh\frac{k+\varphi}{\sqrt{\varphi^2+\eta^2}}-\frac{k}{\sqrt{2}}\cdot ar\,sh\frac{k+2\cdot\varphi}{\sqrt{k^2+2\cdot\eta^2}}\right\}_{\eta_1=z_0+h}^{\eta_2=z_0-h};$$

$$B^{\kappa}_{3\rho\beta}=-\frac{\iota^{\rho}}{4\cdot\pi}\cdot\left\{\left[f_3^0(k_1,\varphi,\eta)+f_3^0(k_4,\varphi,\eta)\right]_{\varphi_1=y_0-a}^{\varphi_2=y_0-a-d}-\right.$$

$$-\left[f_3^0(k_2,\psi,\eta)+f_3^0(k_3,\psi,\eta)\right]_{\psi_1=y_0+a}^{\psi_2=y_0+a+d}+$$

$$+\left[f_{31}(k_1',\varphi',\eta)+f_{31}(k_3',\varphi',\eta)\right]_{\varphi_1'=x_0-l}^{\varphi_2'=x_0-l-d}-$$

$$\left.-\left[f_{31}(k_2',\psi',\eta)+f_{31}(k_4',\psi',\eta)\right]_{\psi_1'=x_0+l}^{\psi_2'=x_0+l+d}\right\}_{\eta_1=z_0+h}^{\eta_2=z_0-h}$$

$$\forall\rho\in[\overline{(\chi_{\beta}-E),(\chi_{\beta}+E)}],\ \kappa=u\vee\kappa=l;$$

$$f_{31}(k,\varphi,\eta)=-\eta\cdot ar\,sh\frac{k+\varphi}{\sqrt{\varphi^2+\eta^2}}+\varphi\cdot arctg\frac{(k+\varphi)\cdot\eta}{\varphi\cdot\sqrt{(k+\varphi)^2+\varphi^2+\eta^2}};$$

$$f_{32}(k,\varphi,\eta)=\sqrt{2}\cdot\eta\cdot ar\,sh\frac{k+2\cdot\varphi}{\sqrt{k^2+2\cdot\eta^2}}-k\cdot arctg\frac{(k+2\cdot\varphi)\cdot\eta}{k\cdot\sqrt{(k+\varphi)^2+\varphi^2+\eta^2}};$$

$$f_3^0(k,\varphi,\eta)=f_{31}(k,\varphi,\eta)+f_{32}(k,\varphi,\eta);$$

$$k_1^{'}=-k_1=[(y_0-a)-(x_0-l)];$$

$$k_2^{'}=-k_2=[(y_0+a)-(x_0+l)];$$

$$k_3^{'}=k_3=-[(y_0+a)+(x_0-l)];$$

$$k_4^{'}=k_4=-[(y_0-a)+(x_0+l)], \qquad (17)$$

where $\iota^{\rho}\ \forall\rho\in[\overline{(\chi_{\beta}-E),(\chi_{\beta}+E)}]$ – current density in coils of short-circuited track contour;

$2\cdot h,\ d$ – height and thickness of each of this coil;

$2\cdot l,\ 2\cdot a$ – size of its internal space;

x_0, y_0, z_0 – coordinates of space point, in which the pole is described;

In expressions (17) besides there are:

$$\iota^{\rho}=0{,}5\cdot i^{\rho}\cdot q\cdot(h\cdot d)^{(-1)}\ \forall\rho\in[\overline{(\chi_{\beta}-\mathrm{E}),(\chi_{\beta}+\mathrm{E})}], \qquad (18)$$

where q – number of windings of short-circuited track contour coil.

Then, in (18) the values of currents $i^{\rho}\ \ \forall\rho\in[\overline{(\chi_{\beta}-\mathrm{E}),(\chi_{\beta}+\mathrm{E})}]$ are put and, according to (15) – (17), the components $B_{\gamma\beta}\ \forall\gamma\in[\overline{1,3}]$ of pole induction are located, which is created by currents of short-circuited track, considered in interaction with β superconductor track contour.

Space of the system $\mathrm{O}\Xi_{\gamma}\ \ \forall\gamma\in[\overline{1,3}]$ – Euclidean. Therefore, instant value of vector of module of full induction of pole, whose components have been determined, may be expressed as follows:

$$B_{\beta}=\sqrt{B_{\gamma\beta}^{(2)}\cdot e^{\gamma}};\ e^{\gamma}=1\ \forall\gamma\in[\overline{1,3}]. \qquad (19)$$

Relation of the mentioned values of components $B_{\gamma\beta}\ \forall\gamma\in[\overline{1,3}]$ determines the direction of vector $\overline{B_{\beta}}$.

Since geometrical size of the elements of windings of superconductor track contour are determined by their construction, and the direction of vector of currents of such elements may be considered congruous with their longitudinal axis, all values become known, which are included in expression for determination of levitation force of magnetically-levitated train. Thus it results in design of this sought model of force.

Result

The integrative paradigm of simulation of levitation force of magnetically-levitated train has been created. It assimilates advantages of circuit and field theories, yet is free of their drawbacks. The mathematical model of such levitation force has been designed, which does not possess any drawbacks of previous version of the model. This is what has solved the present part of the study.

Scientific Novelty and Practical Significance of the Studies

The scientific novelty of the studies is represented by priority of creating an integrative holistic paradigm of simulation of levitation force of magnetically-levitated train as well as the corresponding version of the model of realisation of levitation force. The core significance of the studies is, in case it is implemented, significant increase of efficiency of dynamics research of MLT with a simultaneous decrease of their resource-intensity.

Conclusion

The holistic and integrative model that has been created enabled us to significantly enhance the quality of the model of MLT's LF, created by virtue of such a paradigm. Its application during researches of MLT will enable us to increase their efficiency and decrease their resource-intensity.

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