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DETERMINATION OF INDUCTANCE OF VEHICLE ELECTROMAGNET IN MAGNETIC LEVITATION TRANSPRORT SYSTEM

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Objective. Of the work consists in receiving new analytic expressions for determination of inductance of vehicular electromagnets of transport levitation systems.

Methods. The key feature of calculation model of onboard coils in accordance with their inductance on the example of squared electromagnets lies an assumption about a smallness of the size «heights» in comparison with its other geometrical sizes. There is also an assumption that thickness of a skin layer of the conductor significantly surpasses cross sectional size of the wire making the coil. The single-turn coil is considered, inductance of an actual electromagnet is accepted proportional to a square of the number of turns. When calculating inductance the dimensionless quantities are used. As an absolute, a quarter of perimeter of the coil on the centerline is chosen. The condition of obtaining the approximate formula for calculation of inductance of the flat rectangular coil is the small thickness of a winding in comparison with the geometrical sizes of the coil.

Results. Accurate analytical expression for inductance of a «thin» source of square-shaped magnetic field as the algebraic sum of elementary functions has been received. Results of a numerical analysis show dependence of the relative inductance of the square-shaped flat coil on thickness relation to its minimum size. Influence of «elongation» of the coil (relation of length to width) is investigated. It is noted that inductance decreases with decrease of «elongation», and also with increase in the relation of thickness to the minimum size.

Practical importance. The approximate formula for calculation of value of inductance has been obtained. Its uncertainty does not exceed 14 % in the areas of changes of all geometrical parameters of an electromagnet is output. Borders of practical application of the received analytic expressions at the accepted assumptions have been set.

The received expressions are fairly simple structurally and are easily programmed

Inductance of onboard electromagnets, flat coil, skin layer, numerical analysis.

Introduction

Application of innovative technologies in transportation of both passengers and cargo on the basis of maglev transport systems facilitates increase of efficiency and sustainability of the above systems [1–3].

One of the options of magnetic suspension is a combined system of levitation and traction with a single-phase alternating current. In this case, electromagnets are powered by stationary frequency converters. Permissible values of voltage are limited by parameters of power semiconductor devices (PS) being used. The today's PSes have a permissible voltage of 5000 V [4–6].

The value of the required voltage in electromagnets is proportionate to their inductance. In this case, the accuracy of determination of inductance largely dictates the reliability of the results, obtained during researches of electromechanical systems.

Calculation of inductance of different conductor systems represents, as a rule, quite a sophisticated technical procedure, associated with awkward calculations even for simple forms of electromagnets.

A large number of referential books on calculation of the above values (see in, for example, Reference [7–9]) contain, mainly, sets of approximate formulae, whose accuracy and zones of application are far from being always specified.

The very article is devoted to determination of value of inductance L for coils of a specific configuration, in which value of "height" of electromagnet is negligibly low, compared to other geometrical sizes of the coil. Such electromagnets are hereafter referred to as infinitely thin (flat) sources of magnetic field.

It needs to be pointed out, that value of inductance is significantly dependent on frequency of the current running in a coil. Let us surmise, that current I, determining magnetic field of the system, changes slightly, namely, value of skin layer of a conductor significantly surpasses cross-sectional size of the wire, constituting the coil.

Methodology of Calculation

The common formula for determination of inductance of a flat source of magnetic field L_T is given by the following relation [7]:

$$L_T = \frac{\mu_0 W^2}{4\pi I^2} \int_S d\boldsymbol{\rho} \int_{S'} d\boldsymbol{\rho}' \left(\boldsymbol{i}, \boldsymbol{i}'\right) / \left|\boldsymbol{\rho} - \boldsymbol{\rho}'\right|, \tag{1}$$

where $d\boldsymbol{\rho} = dx\,dy, \quad d\boldsymbol{\rho}' = dx'\,dy'$;

$\left|\boldsymbol{\rho} - \boldsymbol{\rho}'\right|^2 = \left(x - x'\right)^2 + \left(y - y'\right)^2$;

(x, y) и (x', y') – suitable Cartesian system coordinates;

$\boldsymbol{i}$ и $\boldsymbol{i}'$ – linear current densities;

$S \equiv S'$ – current-carrying surface of a coil,

W – number of turns of a coil;

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m – vacuum permeability.

In this case, single-turn (W = 1) coil is considered, inductance of a real electromagnet is proportionate to the square of the number of turns. As a source of electromagnetic field, let us consider a square-shaped electromagnet. Its calculation scheme is seen from the fig. 1.

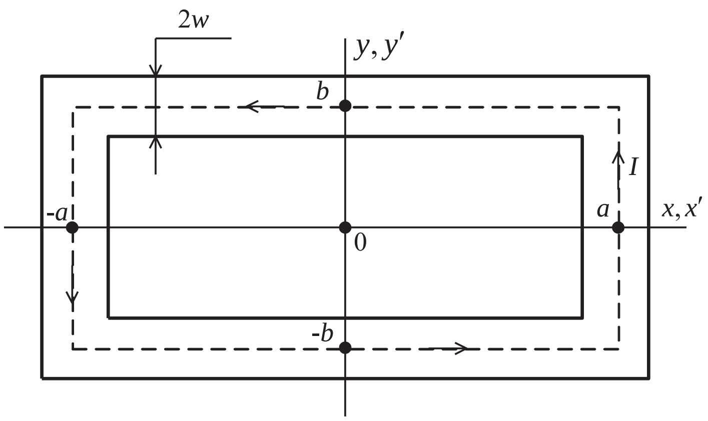

Fig. 1. Calculation scheme of vehicle electromagnet

It is to be pointed out, that natural limit are put on geometrical parameters of this flat electromagnet: $0 < w \le \min(a, b)$.

For the configuration of the source of magnetic field being considered, module of linear current density is defined by equation $i = i' = I/2w$. Proceeding to the point (1) of dimensionless coordinates and committing there a primary double integration, one may obtain the following expression for inductance of a flat square-shaped coil *L:*

$$L = \frac{2\mu_0 p}{\pi}\left(L()\alpha + L()\beta - M(\alpha,\beta) - M(0,0)\right), \tag{2}$$

where

$$L(\nu) = \frac{1}{2}\int_{-\delta}^{\delta} du \int_{-\delta+|u|+\nu}^{\delta-|u|+\nu} dv T(u,v), \nu = \alpha,\beta; \tag{3}$$

$$M(\alpha,\beta) = \int_{-\delta}^{\delta} du(\delta - |u|)\, T(u+\alpha,\, u+\beta); \tag{4}$$

$$2\delta^2 T(u,v) = u ln\frac{m+u}{m-u} + v ln\frac{m+v}{m-v} - 4m. \tag{5}$$

In equations (2)–(5) the following notation is accepted:

$$\alpha = a/p;\ \beta = b/p;\ \delta = w/p;\ p = a + b;\quad m^2 = u^2 + v^2. \tag{6}$$

As a normalising factor for transition to dimensionless coordinates in (2) the value 2*p* has been chosen – semiperimeter of a coil at the medium line.

Further integration of quadrature in (3) leads to the final formula for $L(v)$:

$$\frac{L(\nu)}{\nu} = ln\frac{1+r}{\varepsilon} - \frac{1}{\varepsilon}ln(\varepsilon + r) - \frac{(2+r)^2 - 2}{3(1+r)} - \\ -\frac{1}{3\sqrt{2}\,\varepsilon^2}\sum_{k=\pm 1}(1+k\varepsilon)^3 ln\frac{1+\sqrt{2}\,r - k\varepsilon}{(1+\sqrt{2})(1+k\varepsilon)}, \tag{7}$$

where $r^2 = 1+\varepsilon^2;\ \varepsilon = \delta/\nu;\ (0 < \varepsilon \le 1)$.

The summand in the sum of (7) with $k=-1$ and $\varepsilon = 1$ is further defined by zero value.

Expression for $M(\alpha,\beta)$ from (4) may also be obtained after corresponding integral transformations are conducted:

$$M(\alpha,\beta) = \sum_{k=\pm 1}\big(Q_k(\alpha,\beta) - Q_0(\alpha,\beta)\big), \tag{8}$$

where

$$Q_k(\alpha,\beta) \equiv Q(\alpha_k,\beta_k);\ \ \nu_k = \nu + k\delta;\ \ \nu = \alpha,\beta. \tag{9}$$

Function $Q(u,v)$ in (9) represents an addition of five summands:

$$2\delta^2 Q(u,v) = \frac{s^2 d}{8}ln\frac{u(m+u)}{v(m+v)} + \frac{sd^2}{\sqrt{2}}\times \\ \times ln\frac{\sqrt{2}m+s}{\sqrt{2}\,|d|} + \frac{5d^3}{12}ln\frac{m+d}{\sqrt{2uv}} + \frac{s}{6}(uv - 2d^2)\times \\ \times ln\frac{(m+u)(m+v)}{uv} - \frac{2m}{3}(d^2+uv), \tag{10}$$

where $s = u+v;\ d = u - v = \alpha - \beta$.

The case $u=0$ $(v=0)$ in (10) meets the situation when "the window" of a coil is an infinitively thin slot, that is $\delta = \min(\alpha,\beta)$, formula (10), in its turn, is further defined by continuity:

$$Q(u,0) = Q(0,u) = u^3\left(\frac{1}{\sqrt{2}}ln\left(1+\frac{1}{\sqrt{2}}\right) - \frac{2}{3}\right). \tag{11}$$

The second summand in (10) with $\alpha = \beta(d = 0)$ equals zero.

The limiting variant, defining a flat electromagnet as a “pierced” in the centre square $(\alpha = \beta = \delta)$ leads to relation $Q(0, 0) = 0$.

Since during deriving the expression $M(\alpha, \beta)$ form (8) the condition $\alpha + \beta = 1$ was used, the formula for $M(0,0)$ cannot be defined by means of equations (8)–(10), but should be calculated directly by integrating quadrature (4) taking into account equations $\alpha = \beta = 0$, thus

$$M(0, 0) = \frac{2\delta}{3}\left(ln(1+\sqrt{2}) - \sqrt{2}\right). \tag{12}$$

Equations (2)–(12) completely exhaust the problem of determination of an *accurate* formula for calculation of coefficient of self-inductance of a flat square-shaped coil.

Expression for L by completing the condition $\delta << \min(\alpha, \beta)$ is significantly simplified, and in the result of completion in (2)–(11) of a corresponding maximum transition, one can obtain

$$L\Big|_{\delta<<\min(\alpha,\beta)} = \frac{2\mu_0 p}{\pi}\left\{ln\frac{2\alpha\beta}{\delta} - \alpha ln(\alpha+\gamma) - \right.$$
$$\left. -\beta \ln(\beta+\gamma) - \frac{1}{2} + 2\gamma + \frac{2\delta}{3}\left(\sqrt{2} - ln(1+\sqrt{2})\right)\right\},$$

where $\gamma^2 = \alpha^2 + \beta^2$.

Despite some awkwardness of the relations (2)–(11), the formulae constituting them are elementary, are easy programmed and, which is very important, they represent themselves a procedure of calculation of an *accurate* value of the required self-inductance coefficient L.

It is advisable to conduct calculation of the value L in a dimensionless form. As a basic value, let us choose a value of inductance of a flat coil L_0, having a form of square, “pierced” in the centre. After calculations according to formulae (2)–(12) with $\alpha = \beta = \delta$ we can write down

$$L_D = \frac{\mu_0 P}{\pi} \cdot \frac{2}{3} \cdot \frac{1 + ln(1+\sqrt{2})}{1+\sqrt{2}} \approx 2{,}078 \cdot 10^{-7} P, \tag{13}$$

where $P = 4p$ – perimeter of a square coil on the medium line.

The results of a numerical analysis, showing the dependence of relative inductance L_T / L_D from the parameter e at different values of "elongation" of coil ς, are represented in the fig. 2 (figures on the curves correspond to the values of "elongation" of coil). Parameters e and ς, respectively, are equal: $e = w/\min(a, b)$, $\varsigma = \max(a, b)/\min(a, b)$. The dashed line corresponds to the approximate formula $L\big|_{\delta << \min(\alpha, \beta)} \big/ L_D$ with $\varsigma = 1$. In calculations, perimeter of a coil along the medium line $P = 4p$ is view as constant, L_D is defined in (13).

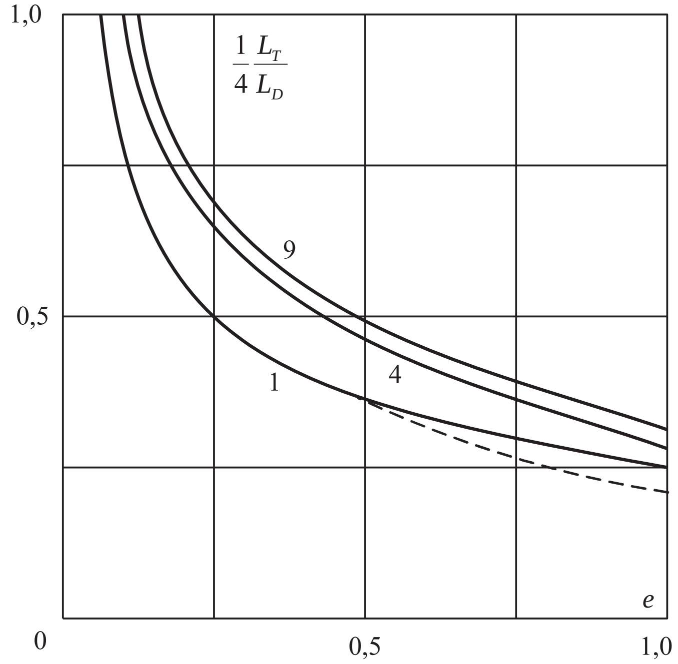

Fig. 2. Dependence of relative inductance of a flat square-shaped coil L_T/L_D from the parameter e

Conclusion

The accurate analytical expression for value of inductance of a "thin" source of a square-shaped magnetic field in the form of algebraic sum of elementary functions was obtained.

The approximate formula for calculation of value of inductance has been obtained. Its uncertainty does not exceed 14 % in the areas of changes of all geometrical parameters of an electromagnet is output.

The obtained expressions are fairly simple structurally and are easily programmed.

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